braneworld gravity

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papers: hep-ph/0506305 + ...

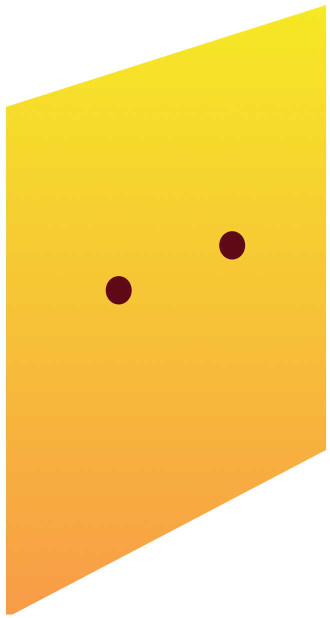
outline

- gravity in various 5d braneworld orbifolds
- problems with DGP
- can we make gravity decouple?
- the interval approach to braneworld gravity
- straight gauges
- some general results
- ghosts, strong coupling, and vDVZ

what is missing from this talk

- codimension>1 braneworlds
- anything beyond pure gravity
- dynamical branes, strings
- SUSY
- proper attribution to all the brilliant+charming people who have contributed to this subject

a brane in 5d



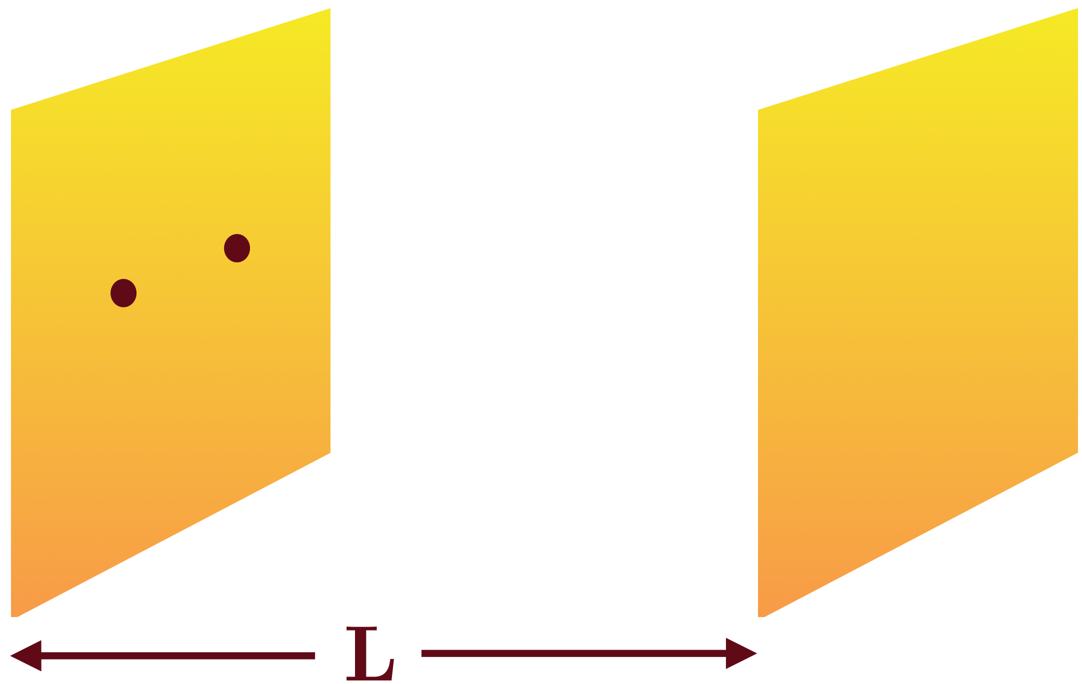
$$2M^3 \int d^5x \sqrt{-G}\, R + \int d^4x^0 \sqrt{-g^0} (\mathcal{L}_m - 2M^3 U_0)$$

brane tension $U_0 = 0 \Rightarrow$ flat 5d background

gravitational potential between two brane sources: $\propto \frac{1}{M^3 r^2}$

gravity on an orbifold

same as previous case, but compactify the 5th dimension on a $\mathbf{Z_2}$ orbifold of a circle of circumference $\mathbf{2L}$



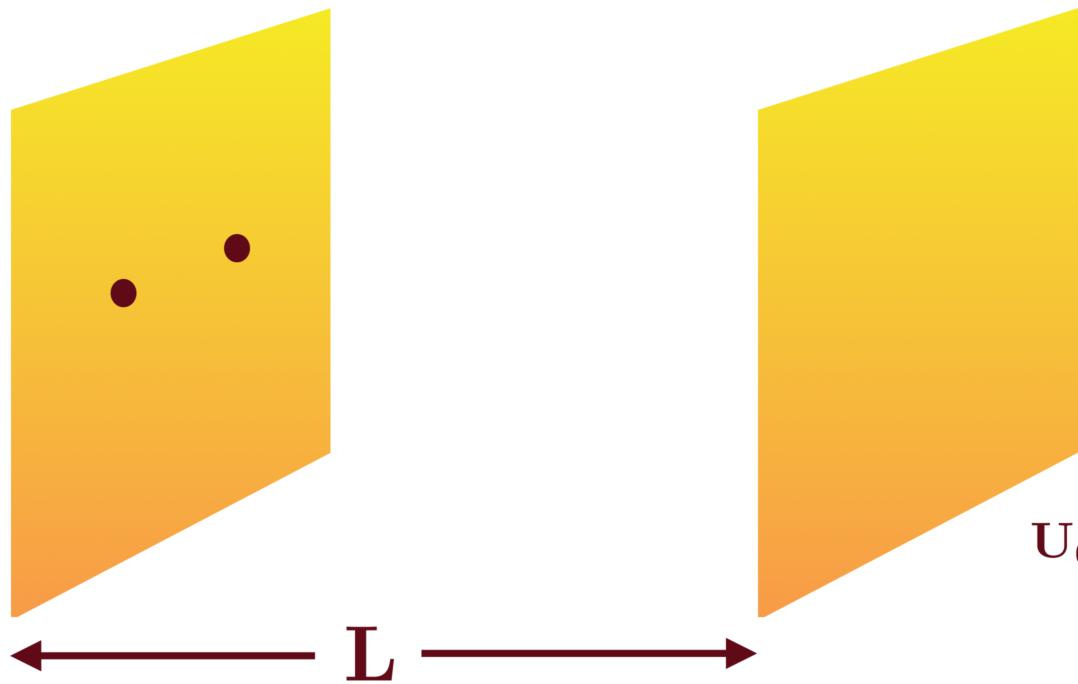
orbifolding means that graviton KK modes obey Neumann b.c. at both branes

gravitational potential between two brane sources: $\propto \frac{1}{M_4^2 r} + \sum_n \frac{1}{M_4^2 r} e^{-\frac{nr}{L}}$ where $M_4^2 \equiv M^3 L$

RSI model

same as previous case, but add a bulk AdS cosm. constant $\Lambda = -24\mathbf{M}^3\mathbf{k}$
where $\mathbf{k} \simeq \mathbf{M}$ and tune the brane tensions to make 4d slices flat

Planck brane



TeV brane

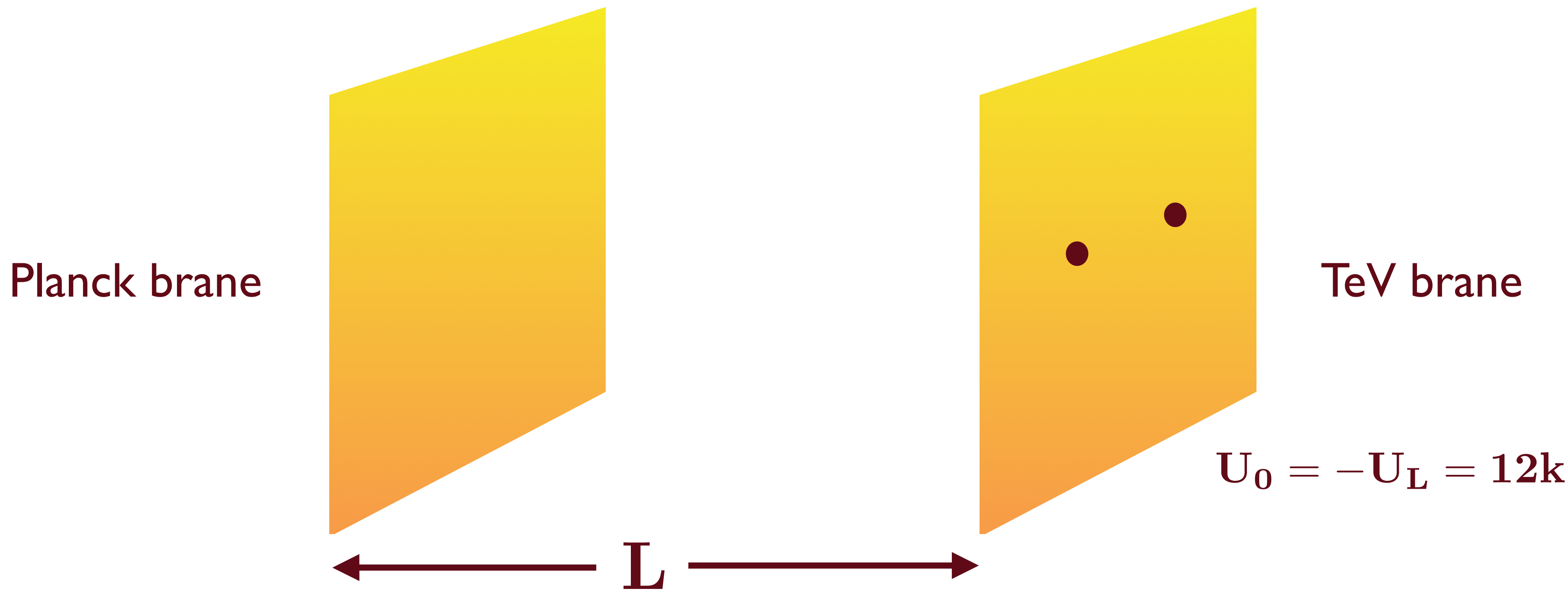
$\mathbf{U_0 = -U_L = 12k}$

there is a normalizable graviton zero mode + a tower of massive KK modes

gravitational potential between two Planck brane sources: $\propto \dfrac{1}{\mathbf{M_4^2 r}} + \dfrac{1}{\mathbf{M_4^2 k^2 r^3}}$ where $\mathbf{M_4^2 \simeq M^3/k}$

RSI model

due to warping, at the TeV brane $\mathbf{M},\ \mathbf{k} \rightarrow \mathbf{T},\ \ \mathbf{T} = \text{TeV}$



but the graviton zero mode exchange also has a wave function suppression $\sim \mathbf{T^2/k^2}$

gravitational potential between two TeV brane sources:

$$\propto \frac{\mathbf{1}}{\mathbf{M_4^2 r}} + \frac{\mathbf{1}}{\mathbf{T^4 r^3}}$$

LHC!

RSII and LR models

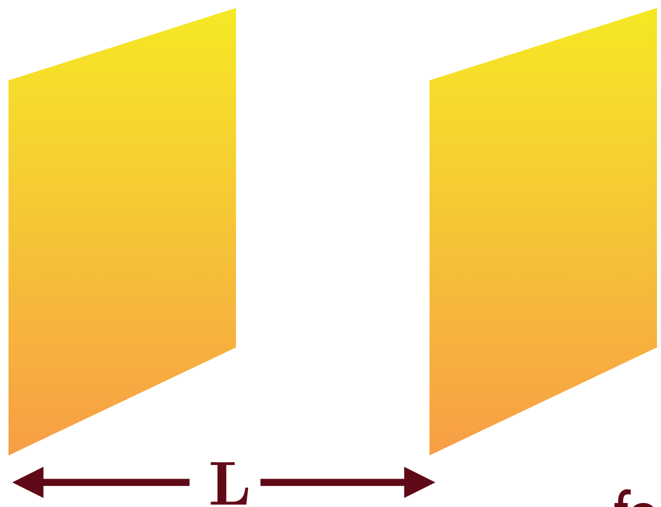
take $L \to \infty$ limit, look at gravity on the Planck brane (RSII) or on a probe brane (LR)

due to warping, graviton zero mode is still normalizable.
there is a continuum of massive KK graviton modes
gravitational potential on the Planck brane is ~ same as in RSI

gravitational potential between two probe brane sources: $\propto \dfrac{1}{M_4^2 r} + \dfrac{1}{T^8 r^7}$ LHC!

detuned warped models

Planck brane



TeV brane

gives $\mathbf{AdS_5}\ \mathbf{AdS_4}$

for any $-12 \leq U_i \leq 12$

$$g_{\mu\nu} = \frac{a(y)^2}{(1 - \frac{H^2 x^2}{4})^2}\eta_{\mu\nu}$$

warp factor: $a(y) = \dfrac{\cosh k(y - y_0)}{\cosh k y_0}$ where:

$$\tanh k y_0 = \frac{U_0}{12k}$$

$$\tanh k(L - y_0) = \frac{U_L}{12k}$$

$$H = \frac{k}{\cosh k y_0}$$

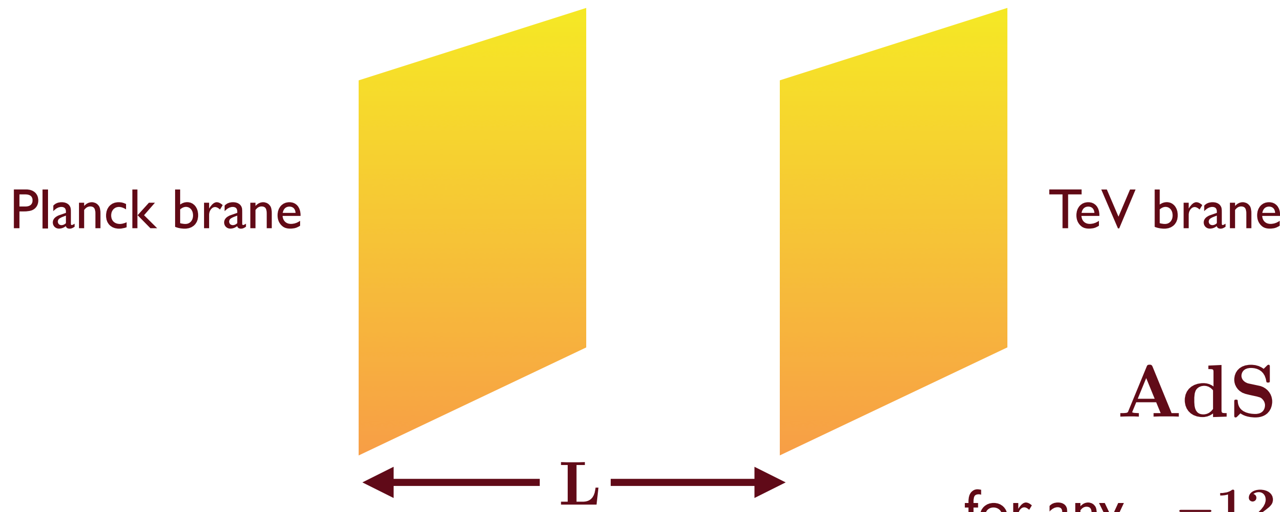
* L is now fixed

* warp factor blows up for large y, instead of damping out as in RS11 or LR

* in the "bigravity" region, $U_0 = U_L <\simeq 12k$, there is ***both*** a zero mode graviton and an ultralight KK graviton, with mass $\sim H^2/k$

Kogan, Mouslopoulos, Papazoglou,Ross, Santiago, Kaloper, etc

detuned warped models



$$\mathbf{AdS_5\,AdS_4}$$

for any $-12 \leq U_i \leq 12$

warp factor: $a(y) = \dfrac{\cosh k(y - y_0)}{\cosh ky_0}$ where:

$$\tanh ky_0 = \frac{U_0}{12k}$$

$$\tanh k(L - y_0) = \frac{U_L}{12k}$$

$$H = \frac{k}{\cosh ky_0}$$

* $L \rightarrow \infty$ removes the TeV brane, but also removes the graviton zero mode, which is no longer normalizable.

* this leaves a model with only the ultralight KK mode to mock up 4d gravity on the Planck brane (Karch-Randall)

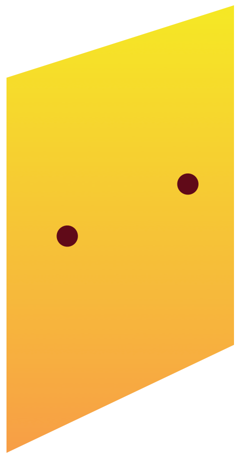
DGP model

$$\mathbf{2M^3 \int d^5x \sqrt{-G}\, R + 2M_4^2 \int d^4x^0 \sqrt{-g^0}\, \tilde{R}}$$

- a single tensionless brane (U=0)
- flat 5d Minkowski background
- add brane kinetic term for gravity
- the two Planck constants compete against each other

Dvali, Gabadadze, Porrati

DGP model



$$\mathbf{2M^3 \int d^5x \sqrt{-G}\, R + 2M_4^2 \int d^4x^0 \sqrt{-g^0}\, \tilde{R}}$$

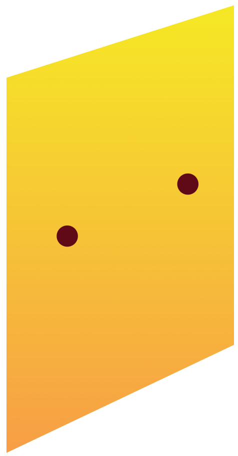
$$\lambda \equiv \mathbf{M_4^2/M^3}$$

graviton propagator: $$\frac{\mathbf{P}_{\mu\nu\rho\lambda}}{\mathbf{M^3 p + M_4^2 p^2}}$$

- the two Planck constants compete against each other
- for brane observer, gravity looks 5d on distance scales $> \lambda$; could self-accelerate the expansion of our (brane) universe!
- at shorter scales, it looks like 4d gravity + an extra scalar interaction

$$\mathbf{P}_{\mu\nu\rho\lambda} = \frac{\mathbf{1}}{\mathbf{2}}(\eta_{\mu\rho}\eta_{\nu\lambda} + \eta_{\mu\lambda}\eta_{\nu\rho}) - \frac{\mathbf{1}}{\mathbf{3}}\eta_{\mu\nu}\eta_{\rho\lambda}$$

problems with the DGP model



graviton propagator: $\frac{\mathbf{P}_{\mu\nu\rho\lambda}}{\mathbf{M^3 p + M_4^2 p^2}}$

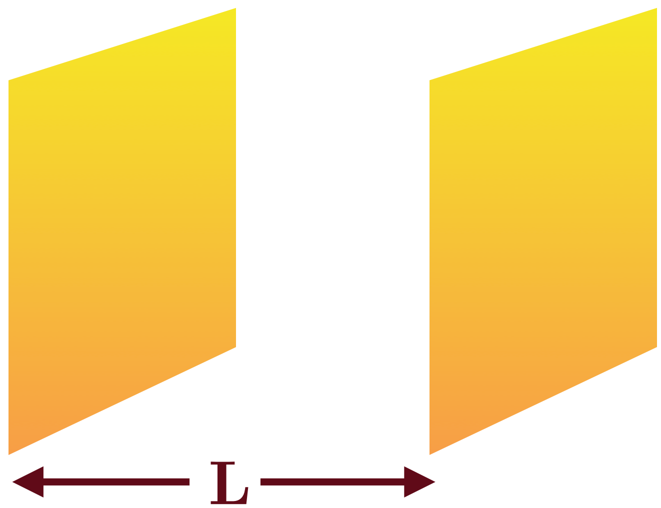
$$\mathbf{P}^{\mathbf{DGP}}_{\mu\nu\rho\lambda} = \frac{\mathbf{1}}{\mathbf{2}}(\eta_{\mu\rho}\eta_{\nu\lambda} + \eta_{\mu\lambda}\eta_{\nu\rho}) - \frac{\mathbf{1}}{\mathbf{3}}\eta_{\mu\nu}\eta_{\rho\lambda}$$

$$\mathbf{P}^{\mathbf{4d}}_{\mu\nu\rho\lambda} = \frac{\mathbf{1}}{\mathbf{2}}(\eta_{\mu\rho}\eta_{\nu\lambda} + \eta_{\mu\lambda}\eta_{\nu\rho}) - \frac{\mathbf{1}}{\mathbf{2}}\eta_{\mu\nu}\eta_{\rho\lambda}$$

- the "4d" graviton has the tensor structure of a 5d graviton; related to the vDVZ discontinuity: in flat space, the massless limit of a massive graviton is *not* a massless graviton
- there is an extra scalar mode whose kinetic term in the "brane effective action" vanishes as $\lambda = \mathrm{M}_4^2/\mathrm{M}^3 \to \infty$; this implies that strong coupling sets in at a (low) energy scale $\mathbf{M^2/M_4}$, linearized gravity breaks down, and the low energy effective field theory is inconsistent

Luty, Porrati, Rattazzi; Rubakov, etc

can we make gravity decouple?

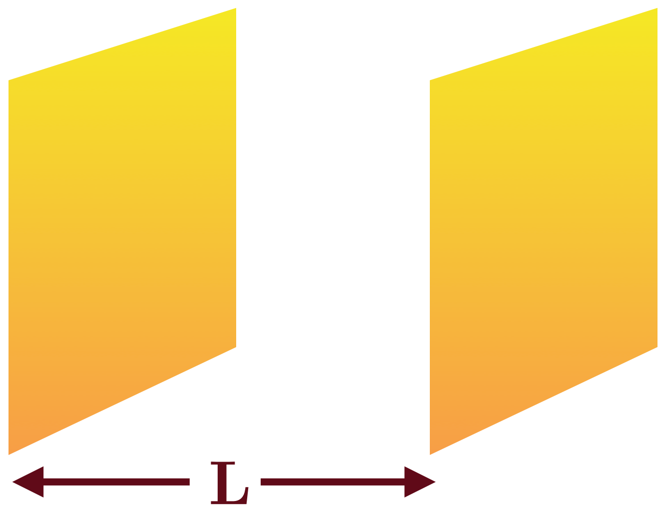


- first do it for gauge theory:
- go back to the Z_2 orbifold, warped or flat
- put an abelian gauge theory in the bulk
- add brane kinetic terms on both branes

$$\frac{1}{4g_5^2}\int d^5x\, F_{MN}F^{MN} + \sum_i \frac{1}{4g_4^{(i)\,2}}\int d^4x\, F^{(i)}_{\mu\nu}F^{\mu\nu\,(i)}$$

Carena, Ponton, Tait, Wagner
Davoudiasl, Hewett, Rizzo

decoupling gauge theories



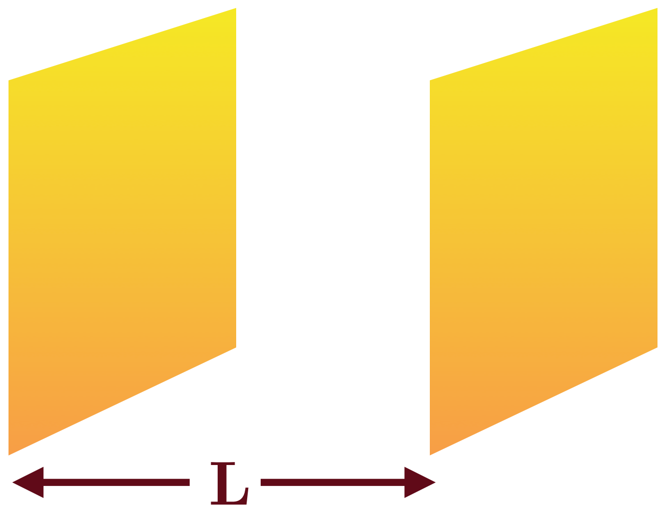
- in addition to the zero mode photon, there is an ultralight massive KK photon

- charged sources on the Planck brane couple to only one linear combination of these two modes

- charged sources on the TeV brane couple only to the orthogonal linear combination

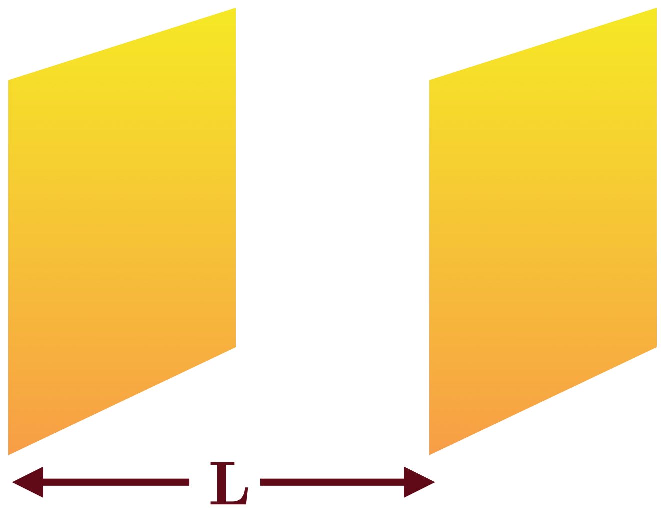
- in a certain energy range there are two decoupled 4d gauge theories

decoupling gauge theories



- this is the beginning of deconstructing or latticizing 5d gauge theory
- there is no problem with having two photons in the effective 4d theory, since we can have two different gauge symmetries
- and there is no vDVZ discontinuity for photons

decoupling gravity?



- however we *do* expect problems in deconstructing or latticizing 5d gravity

  Arkani-Hamed, Georgi, Schwartz, Randall, etc

- there is a big problem with having two *gravitons* in the effective 4d theory, since there is only one 4d general covariance
- and there is a vDVZ discontinuity for gravitons (in flat space)
- but if you *could* decouple a 4d graviton from (ordinary!) matter sources on the "wrong" brane, it would be a big deal

how to make progress on these issues?

- many worthy researchers have proposed many brilliant and novel approaches to this complex of problems/opportunities
- let’s ignore all of them in favor of the most simple-minded approach imaginable:

a straight ahead approach

- write the action for the most general class of 5d warped brane gravity models that you have the patience to analyze
- gauge-fix them
- integrate out the extra dimension to get the gauge-fixed 4d effective gravity theory
- in the big parameter space, try to generalize potentially interesting "limits"
- in the big parameter space, explore issues of strong coupling, vDVZ, decoupling, and their relation e.g. to kinetic ghosts (an older but better-understood problem)

why is it hard?

- write the action for the most general class of 5d warped brane gravity models: need the "interval picture" for braneworld orbifold gravity
- gauge-fix them: need to invent "straight gauges"
- integrate out the extra dimension to get the gauge-fixed 4d effective gravity theory: nasty calculations, especially if you want to go beyond quadratic order to examine strong coupling issues

gravity on orbifolds?

- general relativity is defined on manifolds, not orbifolds; orbifolds have singular fixed points
- this is not a problem in string theory, because the strings don't care about the singularities, and anyway they are just limits of smooth manifolds in the bigger moduli space
- does this mean we have to "blow up" the fixed points of our orbifold braneworlds, in order to have a well-defined gravity action?

the interval approach to braneworld gravity

- replace the 5d orbifold by two 5d manifolds with boundaries; the branes are wedged "in between" adjacent boundaries
- a general 5d warped gravity model is then described by an action like

$$\mathbf{S} = \left(\int_{\mathcal{M}_1} \mathbf{d^5x} + \int_{\mathcal{M}_2} \mathbf{d^5x}\right) \sqrt{-\mathbf{G}}\left(\mathbf{2M^3R} - \mathbf{\Lambda}\right)$$

$$+\mathbf{2M^3} \sum_{\mathbf{i}} \int_{\mathbf{\Phi_i = 0}} \mathbf{d^4x^{(i)}} \sqrt{-\mathbf{g^{(i)}}} (\lambda_{\mathbf{i}} \tilde{\mathcal{R}}^{(\mathbf{i})} - \mathbf{U_i}) + \mathbf{4M^3} \oint_{\partial\mathcal{M}_1 + \partial\mathcal{M}_2} \mathbf{K}$$

brane-boundary equations

- this representation of the gravity orbifold has a well-defined action principle
- but the bulk Einstein equations have supplemented by additional "brane-boundary" equations:

bulk: $$\mathbf{R_{MN} - \frac{G_{MN}}{2}\left(R - \frac{\Lambda}{2M^3}\right) = 0}$$

brane-boundary:

$$\left[\left(\lambda_{\mathbf{i}}\tilde{\mathcal{R}}^{(\mathbf{i})}_{\alpha\beta} - \frac{\mathbf{g}^{(\mathbf{i})}_{\alpha\beta}}{\mathbf{2}}(\lambda_{\mathbf{i}}\tilde{\mathcal{R}}^{(\mathbf{i})} - \mathbf{U_i})\right)\mathbf{e}^{(\mathbf{i})}{}^{\bar{\alpha}}_{\mathbf{M}}\,\mathbf{e}^{(\mathbf{i})}{}^{\bar{\beta}}_{\mathbf{N}} + \mathbf{2P}^{(\mathbf{i})}{}^{\mathbf{P}}_{\mathbf{M}}\nabla_{\mathbf{P}}\mathbf{N}^{(\mathbf{i})}_{\mathbf{N}} - \mathbf{2P}^{(\mathbf{i})}_{\mathbf{MN}}\nabla_{\mathbf{P}}\mathbf{N}^{(\mathbf{i})\mathbf{P}}\right]_{\Phi_{\mathbf{i}}=\mathbf{0}} = \mathbf{0}$$

where $\mathrm{N}^{(\mathrm{i})}_{\mathrm{M}}$ is the outward normal to the *i*th brane/boundary and $\mathrm{P}^{(\mathrm{i})}_{\mathrm{MN}}$ is the projection operator onto the *i*th brane/boundary

a problem with gauge-fixing

- to get the effective 4d action for the physical modes, we need to do a complete gauge fixing of the 5d general coordinate invariance
- the usual gauge choices (harmonic, de Donder, axial) that simplify the equations of motion, have a big problem:
- in these gauges at least one of the branes looks “bent”

straight gauges

- possible solutions are: (1) don't try to do a complete gauge-fixing, (2) analyze the 4d theory with bent branes, and (3) analyze the 4d theory in two different coordinate patches
- instead we have defined a family of "straight gauges"
- straight gauge = a choice of 5d bulk coordinate system such that:

  - both of the branes are described by straight slices $\mathbf{y} = \mathbf{y_i}$

  - $\left[\mathbf{G}^{(\mathbf{i})}_{\mu \mathbf{4}}\right]_{\mathbf{y}=\mathbf{y_i}} = \mathbf{0}$

straight gauges

- good thing about straight gauges: the branes are straight, and the brane-boundary equations simplify a lot:

$$\left[\lambda_{\mathbf{i}}\tilde{\mathcal{R}}_{\mu\nu} - \frac{\mathbf{1}}{\mathbf{2}}\mathbf{g}_{\mu\nu}(\lambda_{\mathbf{i}}\tilde{\mathcal{R}} - \mathbf{U}_{\mathbf{i}}) + \theta_{\mathbf{i}}\sqrt{\mathbf{G}^{\mathbf{44}}}(\mathbf{g}'_{\mu\nu} - \mathbf{g}_{\mu\nu}\mathbf{g}'_{\rho\sigma}\mathbf{g}^{\rho\sigma})\right]_{\mathbf{y}=\mathbf{y}_{\mathbf{i}}} = \mathbf{0}$$

- bad thing about straight gauges: the full analysis of the equations of motion is tedious

some general results

- looked at most general interval picture action for 5d braneworld gravity, with two branes, up to second order in derivatives

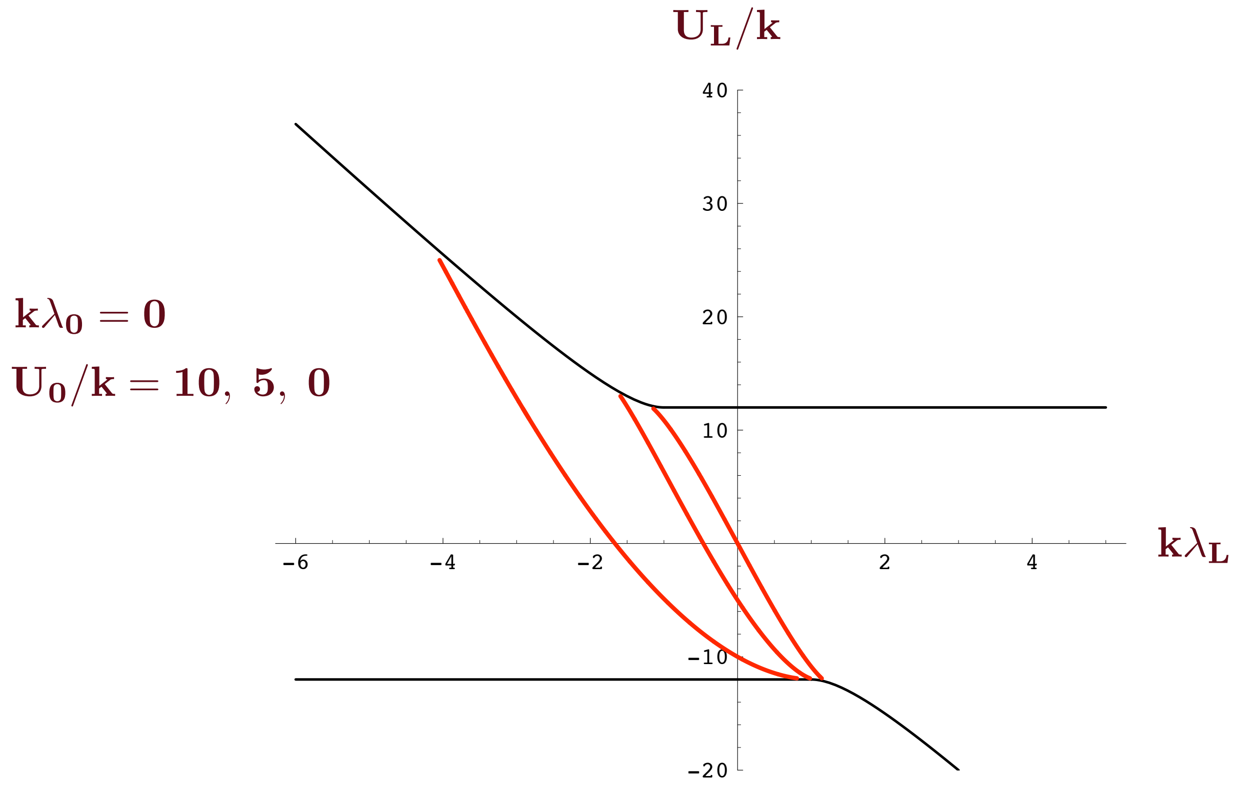
- expanded around a general $\mathrm{AdS}_{5\,\mathrm{AdS}_4}$ background geometry

- gauge-fixed to a straight gauge, and computed the quadratic 4d effective action for the physical modes

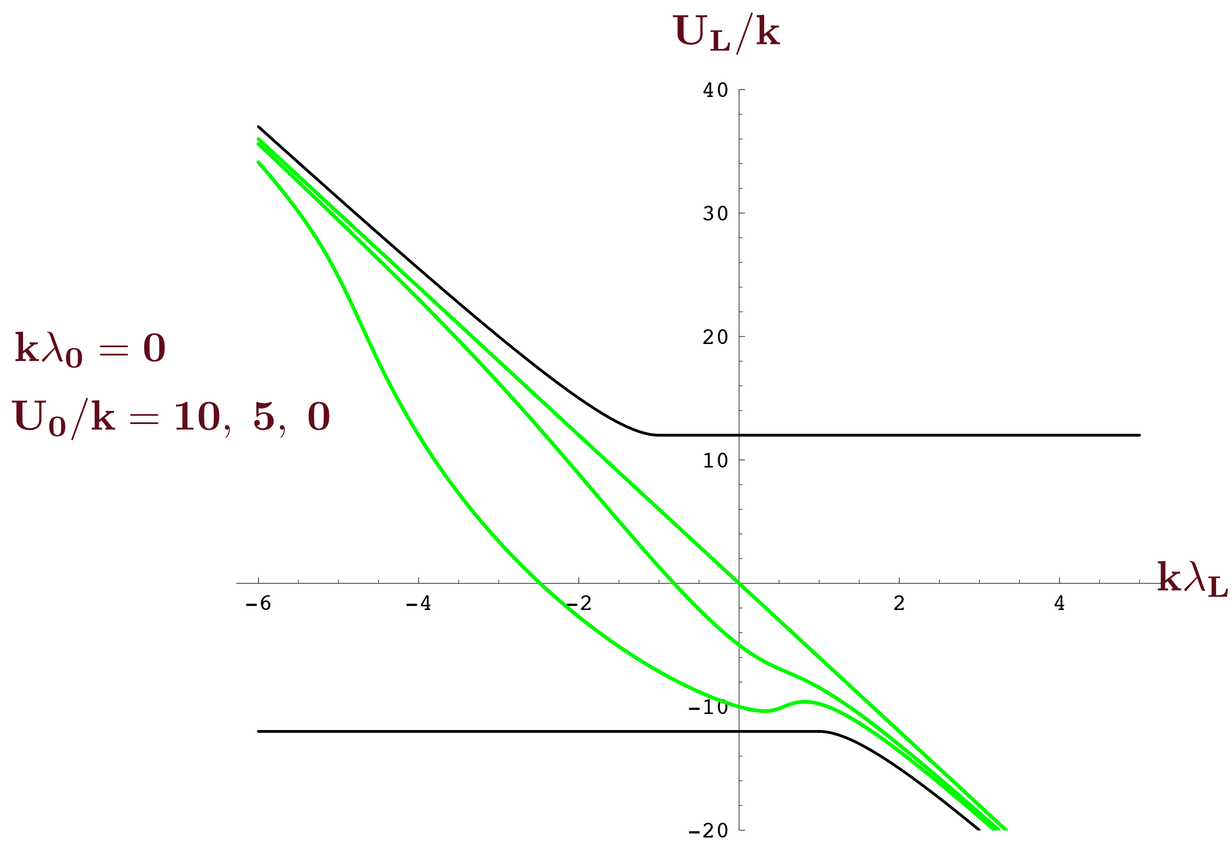
- the only physical modes are the graviton and a radion

ghosts, strong coupling, vDVZ

- there are 4 input parameters which define the $\mathbf{AdS_{5}}_{\mathbf{AdS_4}}$ background geometry: two brane tensions $\mathbf{U_0},\ \mathbf{U_L}$, and the coefficients of the brane kinetic terms $\lambda_\mathbf{0},\ \lambda_\mathbf{L}$
- we compute the coefficients in the 4d effective action of the kinetic terms for the radion and for the zero mode of the graviton
- in each case, these coefficients can be positive (that's good), negative (that's a kinetic ghost), or zero (that's strong coupling).
- we can also check for vDVZ problems in various limits



- inside the black lines, there is an $AdS_{5_{AdS_4}}$ background solution
- to the left of the red lines, the zero mode graviton is a kinetic ghost
- at the red lines, there is strong coupling



- inside the black lines, there is an $AdS_{5_{AdS_4}}$ background solution
- to the left of the green lines, the radion is a kinetic ghost
- at the green lines, there is strong coupling

is this general enough to include DGP-like limits?

- yes, but this is work in progress
- note that the DGP-like limits are living in the same parameter space as lots of other models, including RS, KR, and bigravity.

outlook

- we still have a lot to learn about braneworld models that mock up 4d gravity over some range of length/energy scales

- this is an important topic for both LHC and cosmology