Recent Developments in Non-Perturbative Gauge Theory

Matthew J. Strassler
U. of Washington

Late 1960s: String Theory born from hadronic data.

Regge spectrum $M^2 \propto J$

Regge behavior in small-angle scattering

etc.

But failed in large-angle scattering, Bjorken scaling

1973: Asymptotic Freedom (re)discovered.

QCD hailed as theory of strong interactions.

String Theory abandoned…

1974: String Theory reborn as theory of gravity.

etc., etc.

Question: Why did string theory work at all in context of hadronic physics?

Long history ('t Hooft, Polyakov, many others)

1997 — Maldacena: Gauge Theory = Superstring Theory

$$3+1 \text{ dim. gauge theory} = 4+1\ (+5) \text{ dim. string theory}$$

we'll call extra "fifth" coordinate "r", $\propto$ energy scale μ.

Precise conjecture for certain supersymmetric theories.

A "duality": quantum equivalence of classically different theories.

Multiple descriptions of one physical theory

The catch: In any regime, at most one description is simple.

How does this work?

Gauge theory has number of colors N, gauge coupling g.

- If $g^2 N \ll 1$, Feynman diagrams ok.

("ok" means "can be used to calculate!")

- If $1 \ll g^2 N \ll N$, Feynman graphs fail, String diagrams ok.

In QCD,

- N not large
- $g^2 N \sim 1$ only for $\mu \sim \Lambda$
- no evidence $g^2 N \gg 1$ for $\mu \ll \Lambda$.

Thus string theory for QCD is not simple...

So why care?

TWO REASONS:

1) These are the best toy models known for QCD.

- Exist in 3+1 *Minkowski* dimensions
- Can exhibit IR confinement and UV scaling
- Allow study of interesting non-pert dynamics not accessible to Feynman diagrams or lattice gauge theory
- Useful for identifying universal/nonuniversal aspects of confining gauge theories
- Useful for supporting/disproving folk theorems, speculations
- May be useful for developing new methodologies

So why care?

TWO REASONS:

2) These are interesting and natural gauge theories in their own right.

- Might be responsible for electroweak symmetry breaking
- Dual to Randall-Sundrum 5d-gravity models
- May be observed at LHC – would we know?
- Might be responsible for other physics (flavor, supersymmetry breaking, inflation, etc.)
- Provide examples of novel phenomena previously believed impossible

Today: Two Case Studies

I) The properties of rho mesons:

The relation between the universality of the ρ's couplings to other hadrons to "ρ-dominance" in form factors. (Hong, Yoon, MJS 05)

II) The properties of fast hadrons:

BFKL and other approaches to the Regge regime in QCD and elsewhere. (Brower, Polchinski, MJS, Tan 05; see also Andreev; Andreev and Seigel; Janik and Peschanski)

Other examples not covered today:

Structure and rigidity of hadronic spectrum, couplings in QCD. (... Erlich, Katz, Son, Stephanov 05; Sakai, Sugimoto 05)

Black hole dynamics and RHIC physics: low viscosity fluids and horizons. (Herzog, Son 04; Kovtun, Son, Starinets 04; Nastase 04)

Example: Properties of the Rho

ρ Universality

An apparent but unverifiable general principle of QCD

Data indicate that

$$g_{\rho\pi\pi} \approx g_{\rho NN} \approx m_\rho^2/f_\rho$$

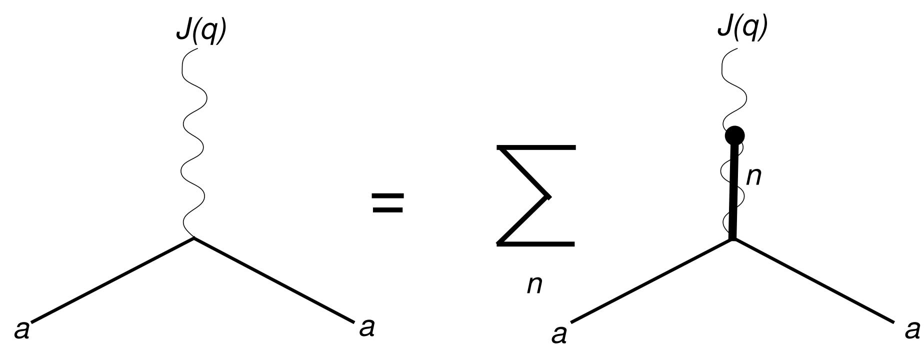
Are the equality, and the value, indicating something important?

Explain by ρ Dominance?

Form factors: $F_a(q^2)$ for hadron $|a\rangle$

Decomposition: Approximate in QCD, exact in large N limit

$$F_a(q^2) = \sum_n \frac{f_n g_{naa}}{q^2 + m_n^2}$$



ρ created by conserved current $\Rightarrow F_a(0) = 1$ exactly for all $|a\rangle$.

If ρ *dominates* sum, then, for all $|a\rangle$,

$$F_a(0) = 1 \approx \frac{f_\rho g_{\rho aa}}{m_\rho^2} \Rightarrow g_{\rho aa} \approx m_\rho^2 / f_\rho$$

True or False?

So ρ dominance $\Rightarrow$ ρ universality with $g_{\rho aa} = m_\rho^2/f_\rho$.

But are these principles and arguments correct? are they generic?
is universality and/or dominance exact in some theories to which QCD
is an approximation?

Perfect question to explore in our new set of calculable, non-QCD
$3+1$–dim. gauge theories.

We find

- approximate (never exact) ρ universality is generic
- *but ρ dominance is not necessary*
- A new form of reasoning replaces the dominance argument

Sketch of Argument

Hadron $|a\rangle \Rightarrow$ cavity mode in 5 dim. $\psi_a(r)e^{ikx}$
satisfying a simple 2nd-order diff. eq.

$\rho \Rightarrow$ lowest mode of its diff. eq. $\Rightarrow$
$\psi_\rho(r)$ positive, real, monotonic, no nodes; $\psi(r_{min}) \neq 0$

Three-hadron coupling

$$g_{\rho aa} = \int_{r_{min}}^{\infty} dr\ \psi_a^*(r)\psi_\rho(r)\psi_a(r) \approx \psi_\rho(r_{min}) \times \langle a|a\rangle$$

Integrand positive definite, which allows estimate.

Now, orthogonality of vector meson modes implies

$$1 = \int dr\ S(r)\psi_\rho^2(r) \approx S(r_{min})\psi_\rho^2(r_{min})$$

$S(r)$ a measure factor

$$1 = \int dr\ S(r)\psi_\rho^2(r) \approx S(r_{min})\psi_\rho^2(r_{min})$$

Current J^μ creates vector mesons $|n\rangle$ including ρ.

$J(q) \Rightarrow \Psi(q^2, r)$, satisfies same diff. eq. as $\psi_n(r)$ but with different boundary conditions $\Rightarrow$

$$\Psi(q^2, r) = \sum \frac{f_n \psi_n(r)}{q^2 + m_n^2} \qquad \text{Son 04}$$

J^μ conserved $\Rightarrow \Psi(0, r) = 1$

Take $q^2 = 0$, integrate both sides against $\int\ dr\ S(r)\psi_\rho(r) \Rightarrow$,

$$S(r_{min})\psi_\rho(r_{min}) \sim \frac{f_\rho}{m_\rho^2}\ ,$$

using orthogonality. Therefore

$$g_{\rho aa} \sim \psi_\rho(r_{min}) \sim m_\rho^2/f_\rho$$

as an approximate, generic statement (could fail in isolated cases.)

I have dropped certain measure factors; they all work out of course.

Comments

Thus ρ universality is true without the assumption of ρ dominance.

Not only is ρ dominance unnecessary, it is violated in some examples that still have universal couplings.

Argument fails for the excited vector mesons.

Also fails for any hadron created by an operator which is not conserved.

Thus: applies only to the ρ (and its $SU(3)$ partners) and to lowest spin–2 glueball.

Future

Now that the argument is known to apply at large g^2N, can it be formulated in QCD itself?

Use of light-cone wave functions justified by careful operator methods?

Implication for LHC Electroweak Symmetry Breaking (EWSB)

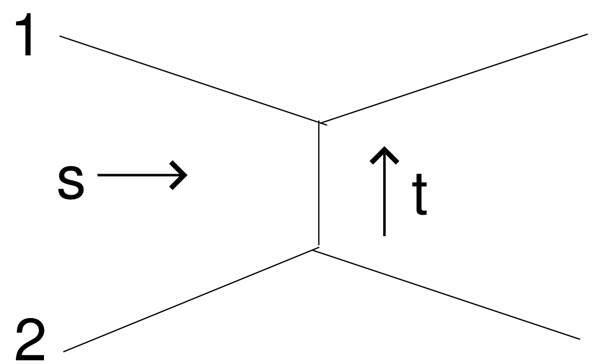
The measurement of the techni-ρ's couplings to W's will not distinguish $g^2N \ll 1$ and $g^2N \gg 1$ EWSB scenarios.

Must work harder to do so, but how?

A second example: BFKL

When objects are boosted to very high energy, how does they change?

Experimental version: in $2 \to 2$ scattering

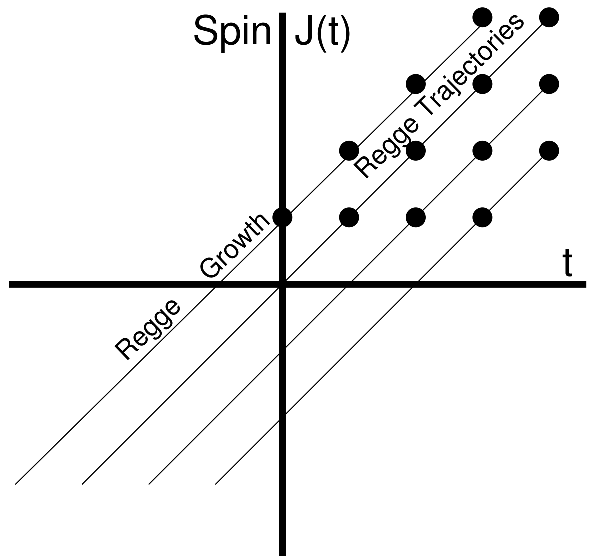


t fixed and small, $s \to \infty$ (large relative boost)

How do amplitudes grow with s (or, in DIS/diffraction, $1/x$)

Answer in flat-space string theory: strings become dense and grow

String amplitudes $\rightarrow$ Regge behavior $A \sim s^{J(t)}$ $\quad (J(t) \equiv \alpha(t))$

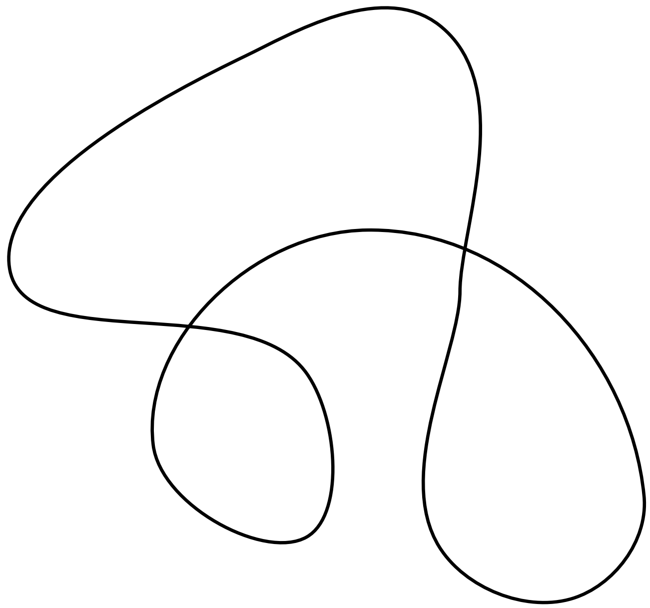


- t positive (timelike, unphysical) get massive states at $J(t) =$ integer
- $t = 0$: $J(0) = \alpha_0 = 2 > 1 \Rightarrow$ Strings become dense, "black"
- t negative; Take Fourier tr. $\vec{q} \rightarrow \vec{x}$; linear behavior of $J(t) \Rightarrow$

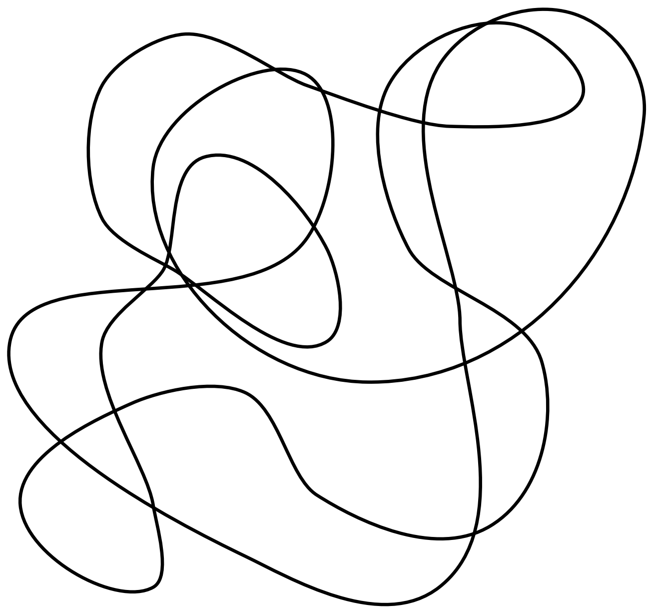
$$\mathcal{A} \sim s^{\alpha_0} \frac{\exp\left[-|\vec{x}|^2/\alpha' \ln s\right]}{\sqrt{\ln s}}$$

Strings grow in size: $\langle \vec{x}^2 \rangle \sim \ln s$

Diffusion Effect



Diffusion Effect



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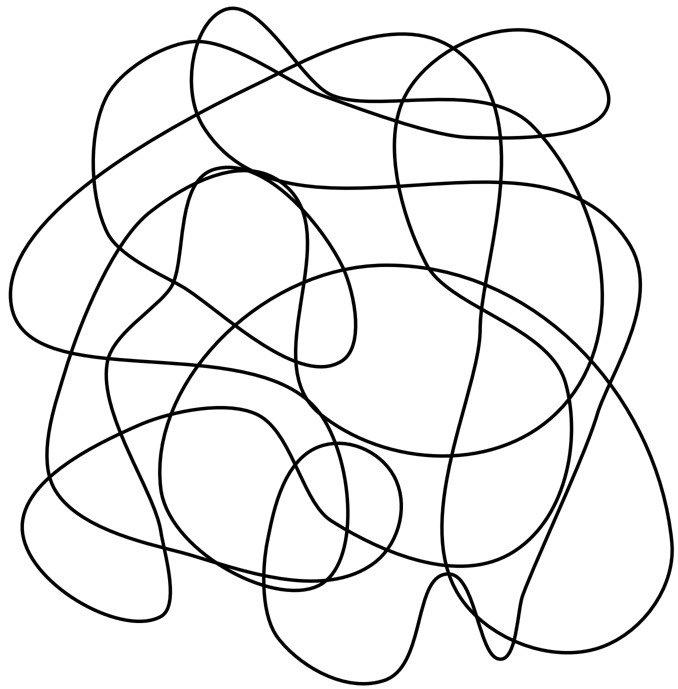

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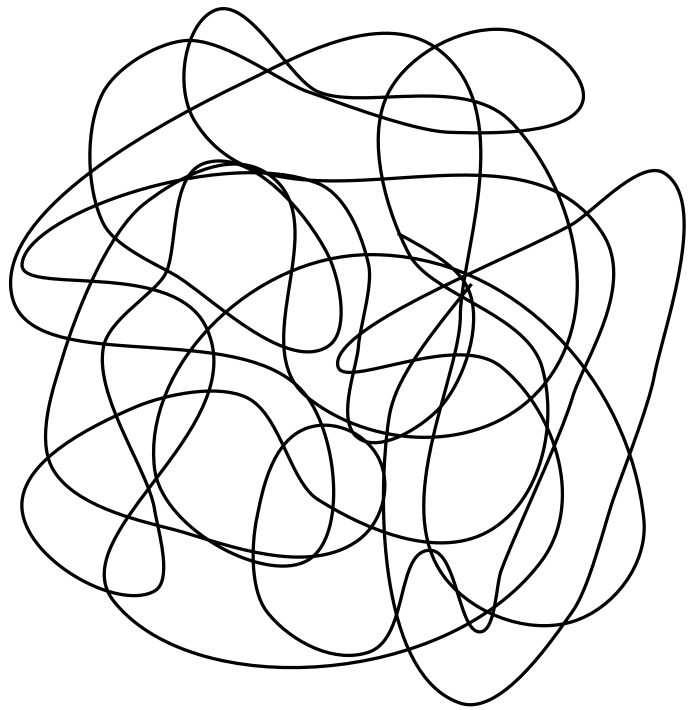

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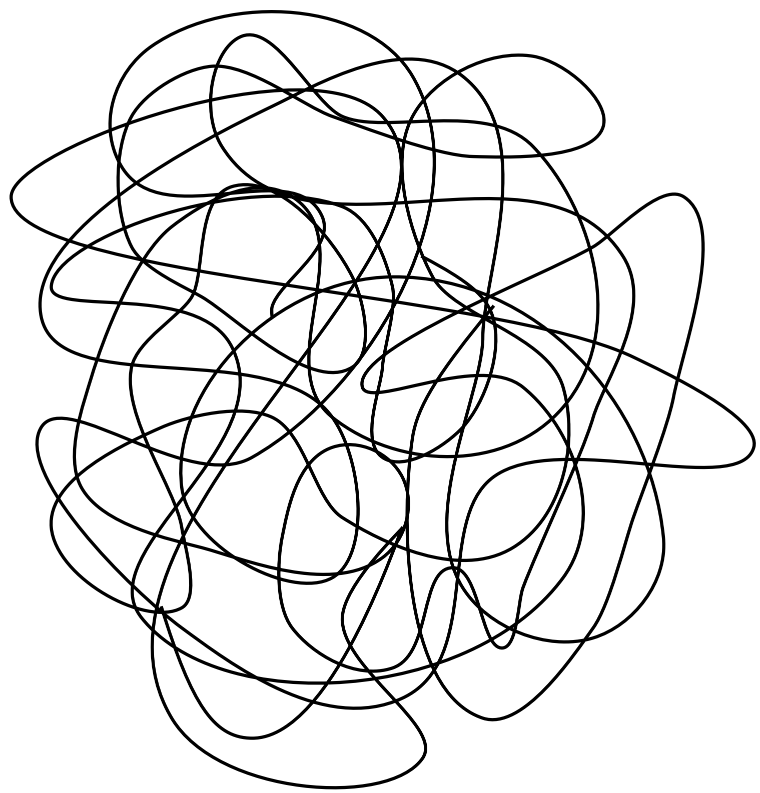

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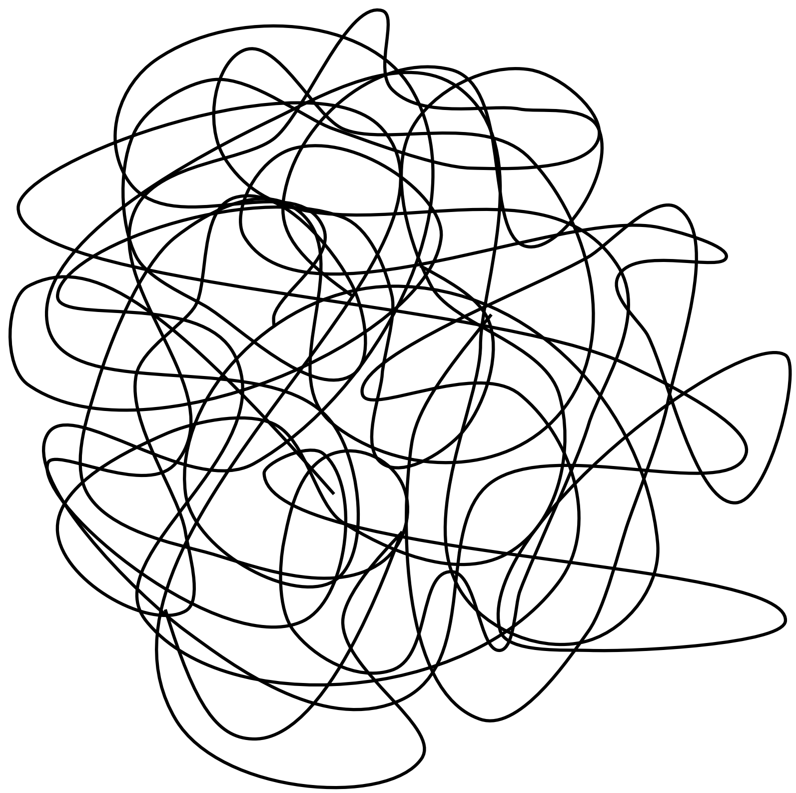
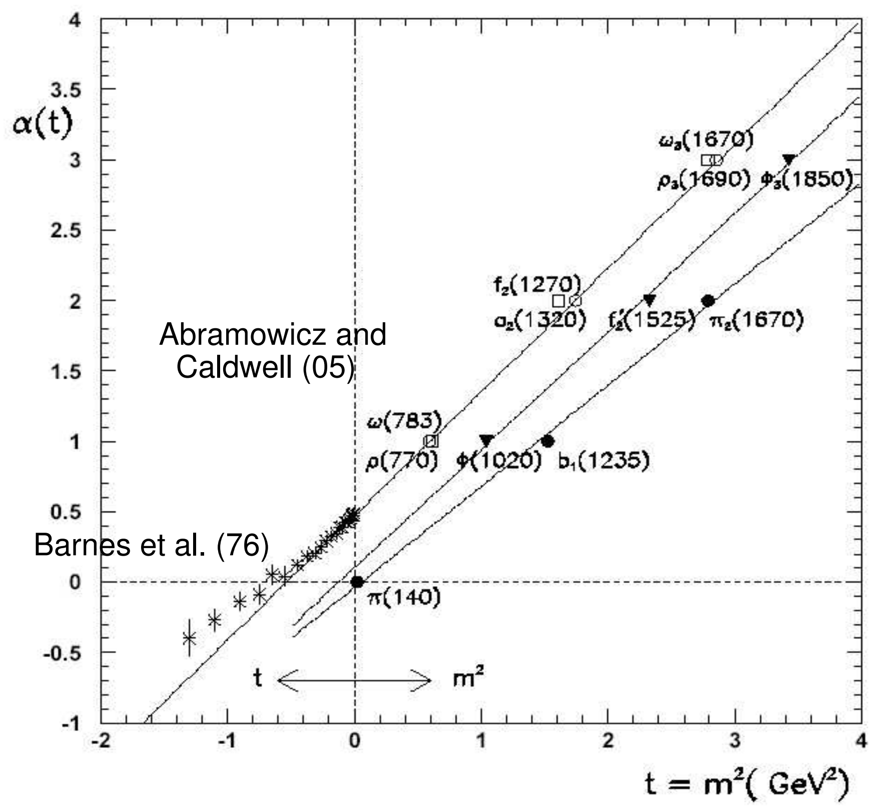


Data shows the string answer is also true of hadrons: (ρ channel)



Hadrons become dense, grow; due to "wee partons" at small x.

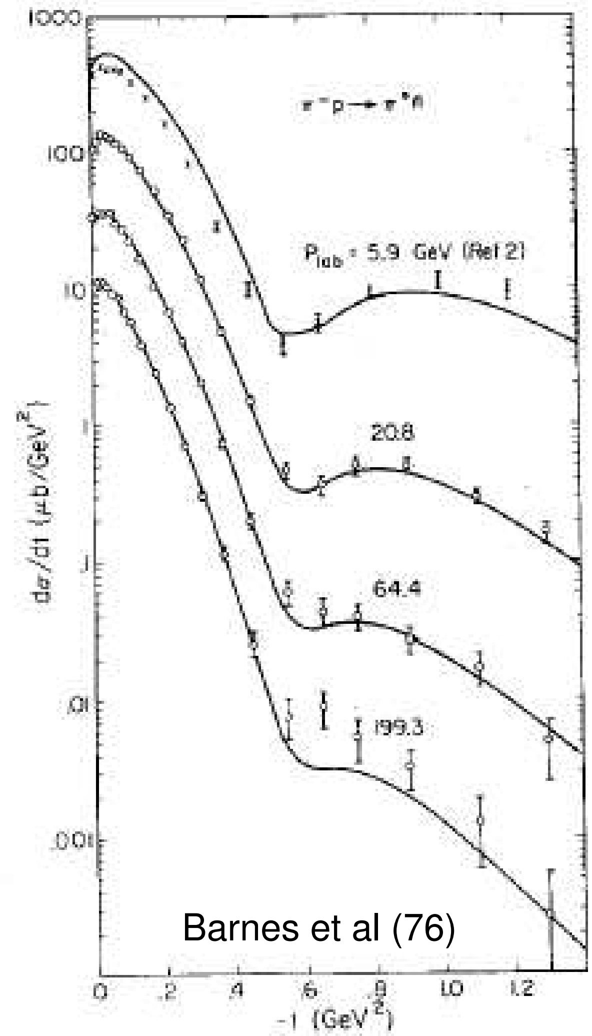


Fig. 1

Shows Gaussian falloff $d\sigma/dt \sim e^{-\alpha'|t|\ln s}$

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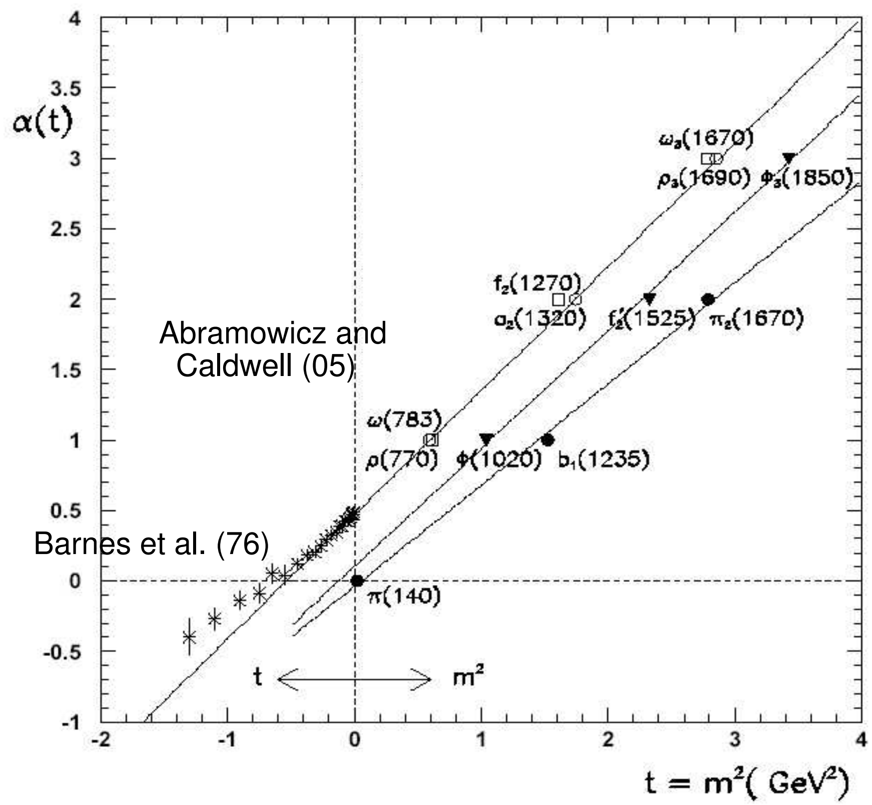


Hadrons become dense, grow; due to "wee partons" at small x.

<u>Calculate This?</u>

- Perturbation Theory? $R_{proton} \sim \Lambda^{-1}$, coupling too large.
- Lattice? boosts are Minkowskian.

BFKL

BFKL: attempt to solve the relevant problem of amplitudes at large s by resumming perturbation theory in analogy with RG

Sum all $(\alpha_s \ln s)^n$ *terms at leading order in* α_s.

But is the formalism correct?

Difficulties when hadrons of size of order Λ are involved.

A tractable example

Toy problem (Mueller et al?) helps clarify what is calculable:

Suppose QCD had stable heavy quarks, long-lived quarkonium

The quarkonium state is *small*

$$r_{Q\bar{Q}} \sim \frac{1}{\alpha_s M_Q} \ll \frac{1}{\Lambda}$$

where $\alpha_s = \alpha_s(\alpha_s M_Q)$

Then growth of size and density of the state with boosts *can* be calculated using perturbation theory, properly resummed

This allows rigorous computation of quarkonium-quarkonium scattering at large s

– or such is the claim. Strongly criticized in some quarters [cf. R. Jaffe]

Enter string theory and SUSY

At very small constant [not running] α_s the BFKL calculation is universal: independent of N, matter content, etc.

So can use $\mathcal{N} = 4$ SUSY, if desired.

At small $g^2 N$, Regge behavior from BFKL resummation.

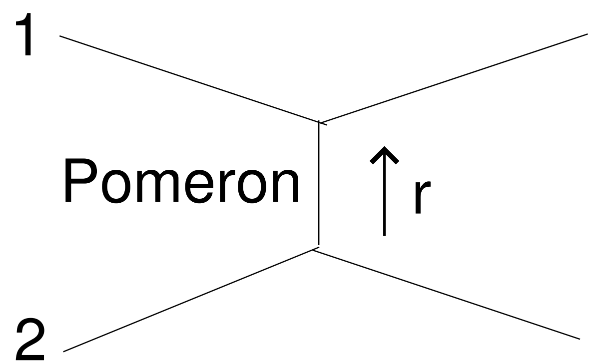
At large $g^2 N$, can describe theory using strings; Regge behavior easily obtained, not from a resummation calculation: but from a simple perturbative string theory calculation on curved space.

Essential idea:

need the $2 \rightarrow 2$ Veneziano amplitude (or Virasoro-Shapiro)

but now in curved $4 + 1$ $(+5)$ *–dimensional space*

Exchange of graviton dominates, but in Regge regime need entire graviton trajectory — the Pomeron — propagating in 5th dimension!



- Amplitude exactly of BFKL *form*, with $k_\perp \to r$.
- Coefficients differ (as expected; g^2N is different)
- BFKL result is just Regge behavior in 5 curved dimensions
- in short, BFKL diffusion is Regge diffusion in the variable $\ln r$.

Experts: 5d Pomeron = continuous set of 4d Pomerons = “BFKL cut”, which begins at $2 - \sqrt{16\pi/g^2N}$

Cut location, diffusion constant depend on g^2N.

Comments

- Supports spirit of BFKL computation
- Form of the answer could have been predicted in advance.
- Very easy to extend the calculation to nonzero $t < 0$ [spacelike].

Logarithmically-running coupling:

- converts cut to dense set of poles
- leading pole evolves slowly with t, for large negative t.

Accords with small g^2N results

Confinement and hadrons:

- In these theories, we can obtain the entire Regge behavior, *at all* t, including BFKL behavior, hadronic resonances, and the transition between them.
- Only requires studying second-order differential equations.

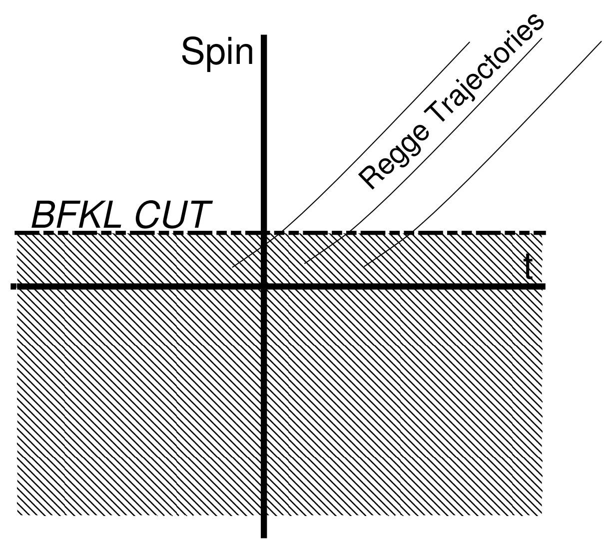
Confining theories

Answer unchanged for $t \ll -\Lambda^2$.

At timelike $t > 0$, see the poles corresponding to the physical Regge trajectories (leading and daughter)

For $|t| \sim \Lambda^2$, model-dependent.

BFKL cut present at all t. (UV constant coupling)



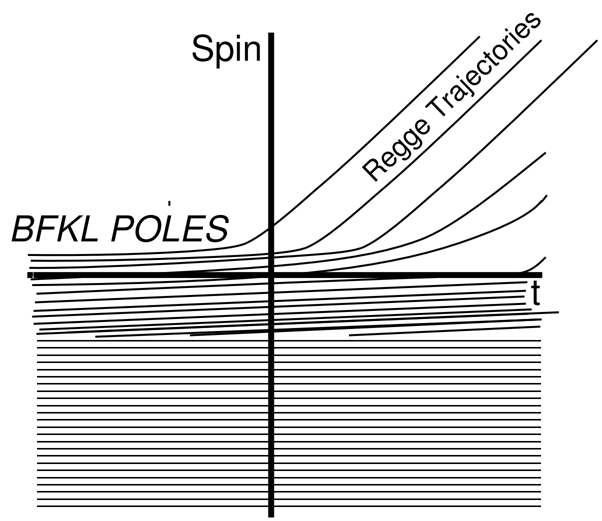
Confining theories

Answer unchanged for $t \ll -\Lambda^2$.

At timelike $t > 0$, see the poles corresponding to the physical Regge trajectories (leading and daughter)

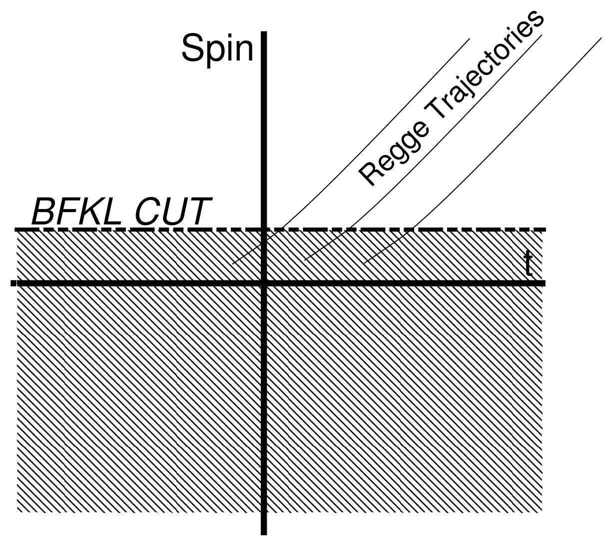
For $|t| \sim \Lambda^2$, model-dependent.

Poles at $t < 0$ join those at $t > 0$ (UV running coupling)



This global picture is inaccessible in QCD.

Here it is all encoded in a single one-dimensionsal Schrodinger-like equation with effective potential $V_t(r)$



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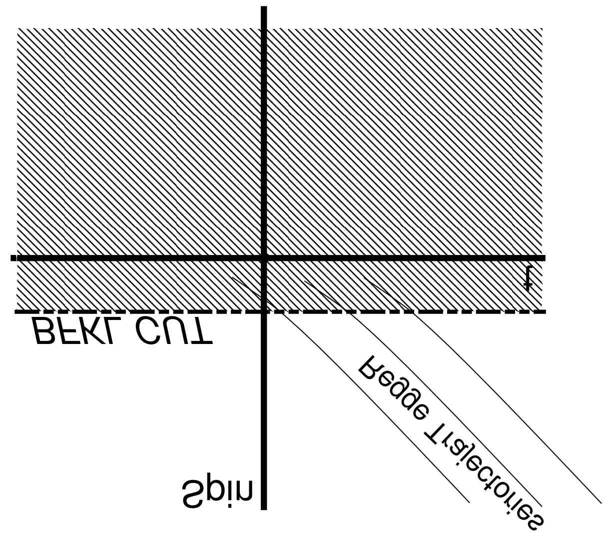

Plot of eigenvalues (as function of t) of eigenstates of V_t

Trajectories = Discrete Bound States

Cut = Continuum States

Constant Coupling $\Rightarrow V_t(r)$ bounded as $r \to \infty$

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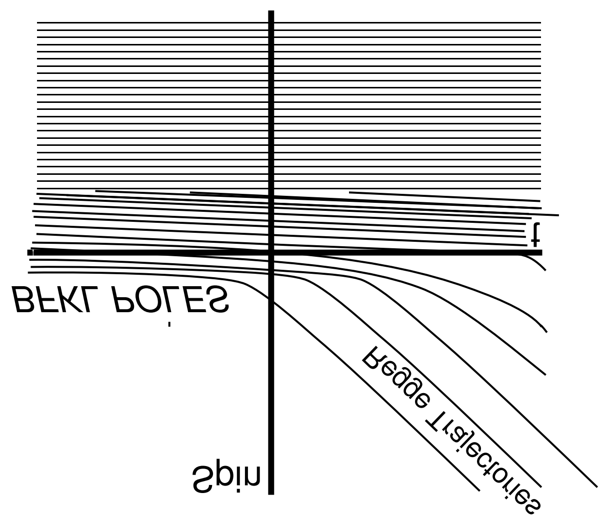

Plot of eigenvalues (as function of t) of eigenstates of V_t

Trajectories = Discrete Bound States

Cut = Continuum States

Running Coupling $\Rightarrow V_t(r)$ unbounded as $r \to \infty$

Our results suggest

- BFKL approach is probably sensible and the criticisms of its general structure are probably wrong
- but its model-independent hadron-independent confinement-independent form need not apply near $t = 0$
- in particular (at large g^2N) it will not apply directly to DIS on protons, no matter how large Q^2 is

Perhaps consistent with failure of BFKL to explain the HERA data

(though there are already perfectly good arguments in this regard concerning the connection between DGLAP evolution and BFKL and we do not claim to supersede these *e.g* Ciafaloni et al.)

Large-Q^2/small x behavior at large g^2N can be studied; at small g^2N is much more subtle but above conclusion still seems justified

Our results certainly resolve nothing at present but do shed some light on the problem and provide an additional source of possible insights

For the Future

The string theory calculation of BFKL was easy

(coeffs different but form could be identified and validity explored)

Unfortunately applicability to experiment is limited and the problem remains mostly of formal interest

- *Are there other challenging BFKL-like questions for which there is no existing formalism, but for which the string theory calculation is still relatively easy?*

- *Are there other pressing questions for which the string theory calculations are much harder but for which the payoff is much greater?*

e.g. Jets at LHC. [What's the question?!]

Summary

Large–g^2N gauge theories are coming under increasing theoretical control using supergravity and superstring theory

They provide

- toy models for QCD
- alternative viewpoints on QCD
- new model-building possibilities

• We are now seeing the first fruits of this development in the application of this formalism to issues of general theoretical importance in QCD

• We hope that we are on the verge of seeing additional developments in the near future that will be of interest in experimental physics beyond the context of RHIC.