Understanding the differences in neutrino and charged fermion flavour structures

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Fermion masses and mixings

Charged fermions	Neutrinos
• Masses $\mathcal{O}(M_W)$.	• Masses $\ll M_W \lesssim 10^{-11} M_W$.
• Strong intergenerational hierarchy.	• Mild intergenerational hierarchy.
• Small mixing angles.	• Mixing angles close to maximal.

Can be explained simultaneously with a common symmetry ??

$\Longrightarrow$ Seesaw mechanism can explain the different mass scales but mixings and mass hierarchy remain difficult.

Hierarchy

Dirac Yukawa couplings equal to up-quark Yukawas.

Strong Yukawa hierarchy: $y_3/y_2 \sim m_c/m_t \sim 10^{-2} - 10^{-3}$.

Mild neutrino hierarchy: $\Delta M_{\rm Sol}/\Delta M_{\rm Atm} \sim 1/6$.

Seesaw $\Rightarrow$ hierarchy in $M_{\rm R} \sim (Y_\nu)^2$.

Mixings

A.- Large mixings from charged leptons and down quarks.

but naturally maximal ???.

B.- Large ν mixings from $\mathcal{O}(V_{\rm CKM})$ mixings in Yukawas (?).

$\Downarrow$

Possible, but not naturally obtained with symmetry reasons

Seesaw mechanism

$$m_{\nu_L} = v_2^2\ Y_\nu \cdot \frac{1}{M_R} \cdot Y_\nu^T$$

If $M_R = m_X \times Y_\nu^T \cdot Y_\nu \Rightarrow m_{\nu_L} = m_X \times 1\!\!1$!!

Good..., but neutrinos not degenerate and large mixings.

However from the Seesaw formula $\Rightarrow M_R = v_2^2\ Y_\nu^T \cdot m_{\nu_L}^{-1} \cdot Y_\nu$

with this structure:

1.- Texture in M_R strongly hierarchical with small mixings $\sim Y_\nu$.

2.- Structure of $M_{\rm R}$ fixed by low energy matrix, $m_{\nu_{\rm L}}^{-1}$.

Start from bimaximal mixing and left handed Majorana masses m_1, m_2, m_3:

$$m_{\nu_{\rm L}}^{-1} = \frac{1}{m_3}\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} + \frac{1}{m_2}\begin{pmatrix} \frac{1}{2} & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{4} & \frac{1}{4} \end{pmatrix} + \frac{1}{m_1}\begin{pmatrix} \frac{1}{2} & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{2\sqrt{2}} & -\frac{1}{4} & \frac{1}{4} \end{pmatrix}.$$

3.- Structure of $m_{\nu_{\rm L}}$ basically decoupled from Yukawa Y_ν.

Can we reproduce this $M_{\rm R}$ structure through a two-step Seesaw ??

Double Seesaw and flavour symmetries

- $\nu^c_{\rm R}$ and $\nu_{\rm L}$ charged under flavour symmetry generating $(Y_\nu)^{ij}$.
- $\nu^c_{\rm R}$ must couple only through the combination $(\nu^c_{\rm R})_i (Y_\nu)^{ij}$.
- Scalar vev breaking lepton number charged under flavour, λ with $L = 1$
- One singlet (also flavour) field S per generation with Majorana mass M_S

$$\left((\nu_L)_i \quad (\nu^c_R)_j \quad S^a \right) \cdot \begin{pmatrix} 0 & Y_{ij} & 0 \\ Y^T_{ij} & 0 & Y^T_{kj} \cdot \langle \lambda^a \rangle_k \\ 0 & \langle \lambda^a \rangle^T_k \cdot Y_{kj} & M_{S_a} \end{pmatrix} \cdot \begin{pmatrix} (\nu_L)_i \\ (\nu^c_R)_j \\ S^a \end{pmatrix}$$

To obtain this matrix in the framework of a flavour theory:

1.- Same coupling Y_{ij} in (1,2) and (3,2) $\Rightarrow \lambda_i$ same flavour charges $(\nu_L)_i$

2.- Approximate zeros in (1,3) and (2,2) $\Rightarrow$ only λ_i with L=1

Neutrinos in a $SU(3)$ Flavour model

- $\psi_i \sim \mathbf{3}$, $\psi_i^c \sim \mathbf{3}$ and $\lambda_i \sim \mathbf{3}$; flavon fields: $\theta_3, \theta_{23} \sim \overline{\mathbf{3}}$, $\overline{\theta}_3, \overline{\theta}_{23} \sim \mathbf{3}$
- Family Symmetry breaking: $SU(3) \xrightarrow{\langle\theta_3\rangle} SU(2) \xrightarrow{\langle\theta_{23}\rangle} \emptyset$

$$\theta_3, \overline{\theta}_3 = \begin{pmatrix} 0 \\ 0 \\ a_3 \end{pmatrix}, \quad \theta_{23}, \overline{\theta}_{23} = \begin{pmatrix} 0 \\ b \\ b \end{pmatrix} \text{with} \quad \left(\frac{a_3}{M}\right) \sim \mathcal{O}(1), \left(\frac{b}{M_u}\right) \simeq \left(\frac{b}{M_d}\right)^2 = \varepsilon \sim 0.05.$$

- Yukawa superpotential: $W_Y = H\psi_i\psi_j^c \left[\theta_3^i\theta_3^j + \theta_{23}^i\theta_{23}^j\Sigma + \epsilon^{ikl}\overline{\theta}_{23,k}\overline{\theta}_{3,l}\theta_{23}^j\left(\theta_{23}\overline{\theta}_3\right)\right]$

$$Y^f = \begin{pmatrix} 0 & a\ \varepsilon^3 & b\ \varepsilon^3 \\ a\ \varepsilon^3 & \varepsilon^2 \frac{\Sigma}{|a_3|^2} & c\ \varepsilon^2 \frac{\Sigma}{|a_3|^2} \\ b\ \varepsilon^3 & c\ \varepsilon^2 \frac{\Sigma}{|a_3|^2} & 1 \end{pmatrix} \frac{|a_3|^2}{M^2},$$

- (B–L) breaking vev, $\langle\lambda\rangle = v_{B-L}\begin{pmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{2}\\ \frac{1}{2}\end{pmatrix} \Rightarrow W_1 = S\ \lambda_i\ \nu_j^c\ Y^{ij}$

  effective coupling $\frac{1}{M_S}(\nu_R^c)_i Y^{ki}\langle\lambda_k\rangle\langle\lambda_l\rangle Y^{lj}(\nu_R^c)_i.$

- Using only the flavon vevs avaliable $\Rightarrow W_2 = S'\ (\overline{\theta}_{23})_i\ \nu_j^c\ (\theta_3\lambda)\ Y^{ij}$

  effective coupling $\frac{1}{M_{S'}}(\theta_3\lambda)^2\,(\nu_R^c)_i Y^{ki}\langle(\overline{\theta}_{23})_k\rangle\langle(\overline{\theta}_{23})_l\rangle Y^{lj}(\nu_R^c)_i.$

- no other flavon vev (triplet) orthogonal to λ and $\overline{\theta}_{23}$, but $(\epsilon_{ikl}\theta_{23}^k\theta_3^l) \propto \begin{pmatrix}1\\0\\0\end{pmatrix}$

  $\Rightarrow W_3 = \hat{S}\ (\epsilon_{ikl}\theta_{23}^k\theta_3^l)\ \nu_j^c\ (\theta_3\lambda)\ Y^{ij} \Rightarrow M_R^{(3)} = \frac{(\theta_3\lambda)^2}{M_{\hat{S}}} Y^T\begin{pmatrix}1&0&0\\0&0&0\\0&0&0\end{pmatrix}Y$

$$m_\nu^{-1} = \frac{1}{\tilde{m}_1}\begin{pmatrix} \frac{1}{2} & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{2\sqrt{2}} & -\frac{1}{4} & \frac{1}{4} \end{pmatrix} + \frac{1}{\tilde{m}_2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{1}{\tilde{m}_3}\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

Rotation (2,3) still maximal, $R_{23}(\pi/4)$, ✓

undoing this rotation: $R_{23}^T(\pi/4) \cdot m_\nu^{-1} \cdot R_{23}(\pi/4) = \frac{1}{\tilde{m}_1}\begin{pmatrix} \frac{1}{2}+x & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & y \end{pmatrix}$

$\Rightarrow \sin\theta_{12} \simeq \frac{1}{\sqrt{2}}\left(1 - \frac{x}{2} - \frac{x^2}{4}\right)$ and (without charged lepton) $\sin\theta_{13} = 0$

mass eigenstates: $m_1 \simeq \tilde{m}_1$, $m_2 \simeq 2\,\tilde{m}_2$ and $m_3 = \tilde{m}_3$.

$x = \frac{2\,m_1}{m_2} \ll 1 \Rightarrow$ hierarchical spectrum. $\frac{m_2}{m_3} \simeq \frac{1}{6} \Rightarrow y = \frac{m_1}{m_3} \simeq \frac{x}{12}$

m_1 fixed by x. ✓

Vacuum alignment . . .

Flavon Superpotential:

$$W = V\left((\theta_{23}\bar\theta_{23}) + P^2\right) + X\left((\theta_3\bar\theta_3)^4 + P\bar P\right) + Y(\theta_3\bar\theta_2) + Z\left((\theta_{23}\bar\theta_3)(\theta_{23}\bar\theta_2) + \bar P^2\right)$$

(B–L) Superpotential:

$$W_{B-L} = R\left((\theta_3\lambda) + M^2\right) + T(\theta_{23}\lambda) + \bar T(\bar\lambda\bar\theta_{23}) + U(\lambda\bar\lambda)$$

Global charges:

Field	θ_3	θ_{23}	$\overline{\theta}_3$	$\overline{\theta}_3$	Σ	S	S'	$\hat S$
U(1)	2	−4	7	−2	12	0	0	0
Z_2	0	−1	1	0	2	0	0	1
Z_4	0	0	1	−2	4	0	2	0