Predicting neutrino parameters from SO(3) family symmetry and quark-lepton unification

1. Neutrino phenomenology
2. Tri-bimaximal mixing from constrained sequential dominance
3. (Tri)Bimaximal quark-lepton complementarity
4. SO(3) family symmetry and Pati-Salam
5. Predictions for neutrino parameters

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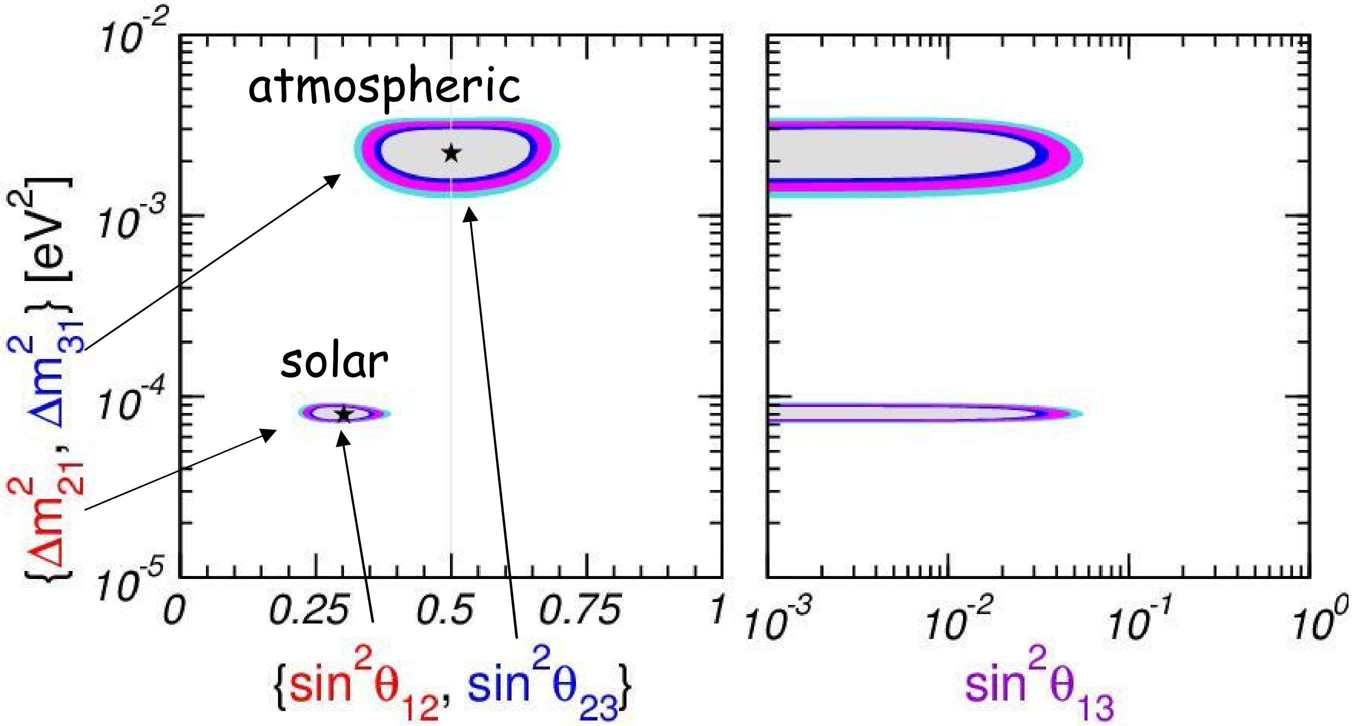
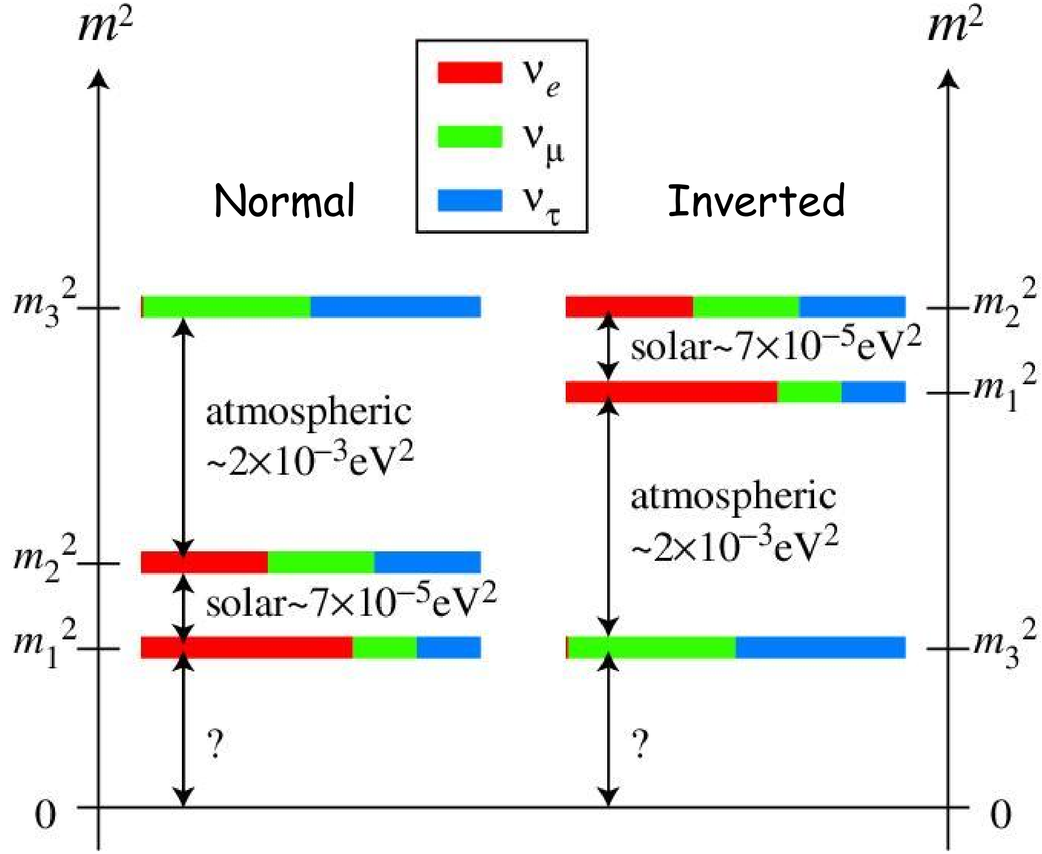
Neutrino Oscillations



from: Maltoni, Schwetz, Tortola, Valle ('04)

Neutrino Masses

parameter	best fit	2σ	3σ	4σ
$\Delta m^2_{21}\ [10^{-5}\mathrm{eV}^2]$	7.9	7.3–8.5	7.1–8.9	6.8–9.3
$\Delta m^2_{31}\ [10^{-3}\mathrm{eV}^2]$	2.2	1.7–2.9	1.4–3.3	1.1–3.7



Lepton Mixing Angles

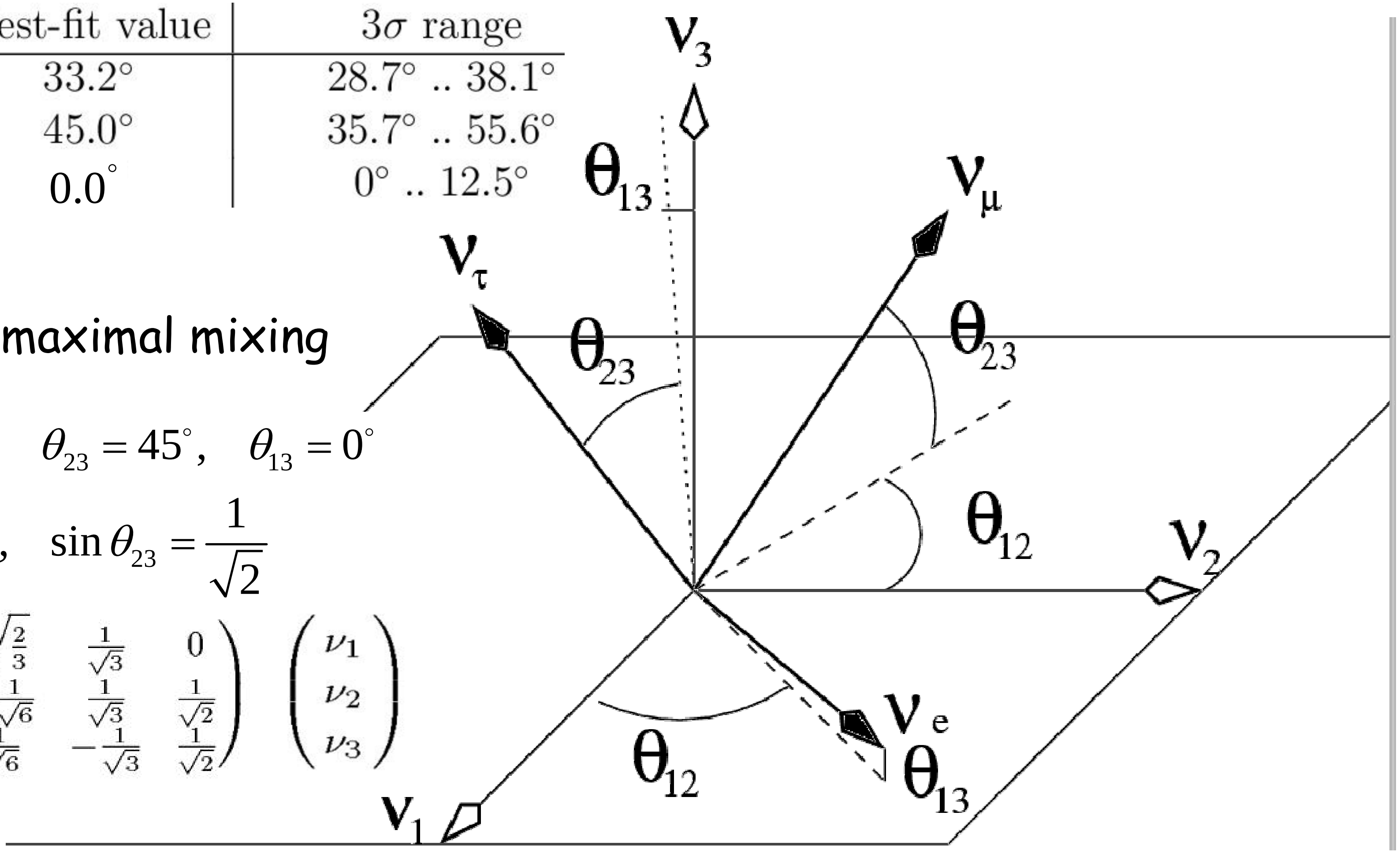
Parameter	Best-fit value	3σ range
θ_{12}	$33.2°$	$28.7°$ .. $38.1°$
θ_{23}	$45.0°$	$35.7°$ .. $55.6°$
θ_{13}	$0.0°$	$0°$ .. $12.5°$

e.g. Tri-bimaximal mixing

$$\theta_{12} = 35.26^\circ, \quad \theta_{23} = 45^\circ, \quad \theta_{13} = 0^\circ$$

$$\sin\theta_{12} = \frac{1}{\sqrt{3}}, \quad \sin\theta_{23} = \frac{1}{\sqrt{2}}$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



Lepton mixing matrix U

Neutrino mass matrix (Majorana)

$$V^{E_L} m_{LR}^E V^{E_R \dagger} = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \qquad V^{\nu_L} m_{LL}^\nu V^{\nu_L T} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

Defined as $U = V^{E_L} V^{\nu_L \dagger}$

Can be parametrised as $U = U_{23} U_{13} U_{12}$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} e^{-i\delta_{23}} \\ 0 & -s_{23} e^{-i\delta_{23}} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} e^{-i\delta_{12}} & 0 \\ -s_{12} e^{-i\delta_{12}} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Atmospheric — Reactor — Solar

Oscillation phase $\delta = \delta_{13} - \delta_{23} - \delta_{12}$

Effect of small charged lepton mixing angles

SFK hep-ph/0204360

$$s_{23}e^{-i\delta_{23}} \approx s_{23}^{\nu}e^{-i\delta_{23}^{\nu}} - \theta_{23}^{E}c_{23}^{\nu}e^{-i\delta_{23}^{E}}$$

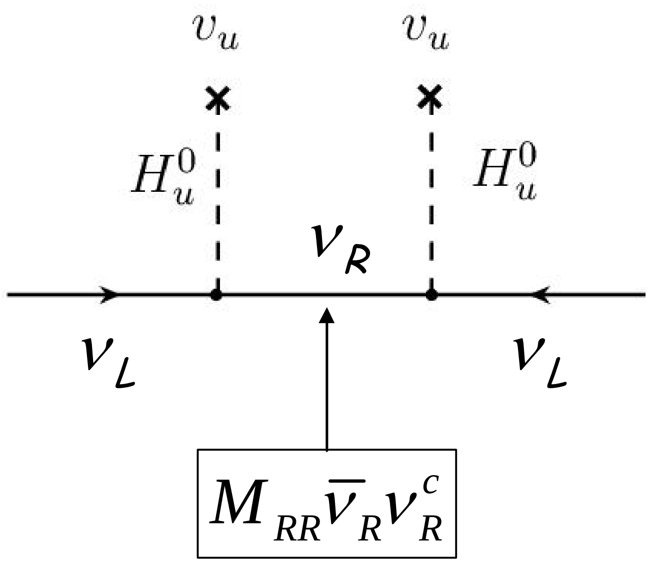
$$\theta_{13}e^{-i\delta_{13}} \approx \theta_{13}^{\nu}e^{-i\delta_{13}^{\nu}} - \theta_{13}^{E}c_{23}^{\nu}e^{-i\delta_{13}^{E}} - \theta_{12}^{E}s_{23}^{\nu}e^{-i(\delta_{12}^{E}+\delta_{23}^{\nu})}$$

$$s_{12}e^{-i\delta_{12}} \approx s_{12}^{\nu}e^{-i\delta_{12}^{\nu}} + \theta_{13}^{E}c_{12}^{\nu}s_{23}^{\nu}e^{i(\delta_{23}^{\nu}-\delta_{13}^{E})} - \theta_{12}^{E}c_{23}^{\nu}c_{12}^{\nu}e^{-i\delta_{12}^{E}}$$

The see-saw mechanism

Type I see-saw mechanism

P. Minkowski (1977), Gell-Mann, Glashow, Mohapatra, Ramond, Senjanovic, Slanski, Yanagida (1979/1980)

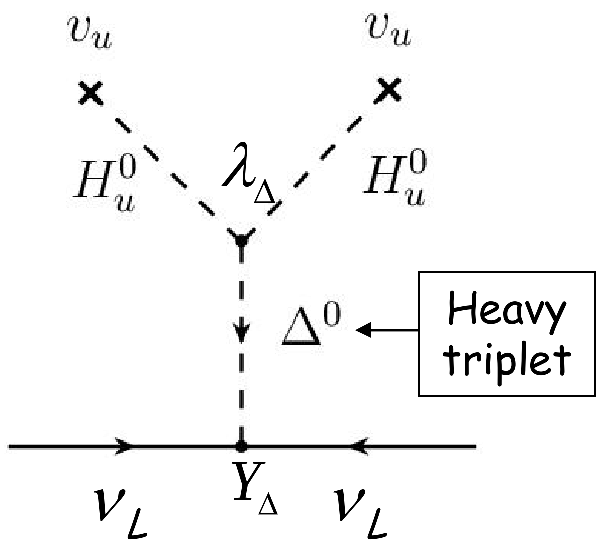


$$m_{LL}^{I} \approx -m_{LR} M_{RR}^{-1} m_{LR}^{T}$$

Type I

Type II see-saw mechanism (SUSY)

Lazarides, Magg, Mohapatra, Senjanovic, Shafi, Wetterich (1981)



$$m_{LL}^{II} \approx \lambda_\Delta Y_\Delta \frac{v_u^2}{M_\Delta}$$

Type II

The See-Saw Matrix

Type II contribution (frequently ignored) → m_{LL}^{II}

Dirac matrix → m_{LR}

Heavy Majorana matrix → M_{RR}

$$\begin{pmatrix} \overline{\nu_L} & \overline{\nu_R^c} \end{pmatrix} \begin{pmatrix} m_{LL}^{II} & m_{LR} \\ m_{LR}^T & M_{RR} \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}$$

Diagonalise to give effective mass $\rightarrow m_{LL}\overline{\nu}_L \nu_L^c$

Light Majorana matrix ⟶

$$m_{LL} = m_{LL}^{II} - m_{LR} M_{RR}^{-1} m_{LR}^T$$

Sequential dominance: a technically natural way of achieving a neutrino mass hierarchy with large mixing angles from type I see-saw mechanism

SFK 98-

$$M_{\rm RR} \approx \begin{pmatrix} Y & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X' \end{pmatrix} \qquad Y^{\nu}_{\rm LR} = \begin{pmatrix} d & a & a' \\ e & b & b' \\ f & c & c' \end{pmatrix}$$

sequential dominance of right-handed neutrinos
$(d=0)$

$$\frac{|e^2|,|f^2|,|ef|}{Y} \gg \frac{|xy|}{X} \gg \frac{|x'y'|}{X'} \longrightarrow m_{LL} \approx \begin{pmatrix} \left(\frac{a^2}{X}\right) & \left(\frac{ab}{X}\right) & \left(\frac{ac}{X}\right) \\ . & \left(\frac{b^2}{X}+\frac{e^2}{Y}\right) & \left(\frac{bc}{X}+\frac{ef}{Y}\right) \\ . & . & \left(\frac{c^2}{X}+\frac{f^2}{Y}\right) \end{pmatrix}$$

$$\begin{aligned} m_1 &\sim O\left(\frac{x'y'}{X'}\right)v_u^2 \\ m_2 &\approx \frac{|a|^2}{X(s_{12}^{\nu})^2}v_u^2 \\ m_3 &\approx \frac{(|e|^2+|f|^2)}{Y}v_u^2 \end{aligned}$$

$$\begin{aligned} \tan\theta_{23}^{\nu} &\approx \frac{|e|}{|f|} \\ \tan\theta_{12}^{\nu} &\approx \frac{|a|}{c_{23}^{\nu}|b|\cos(\phi_b') - s_{23}^{\nu}|c|\cos(\phi_c')} \\ \theta_{13}^{\nu} &\approx e^{i(\phi_2^{\nu}+\phi_a-\phi_e)}\frac{|a|(e^*b+f^*c)}{[|e|^2+|f|^2]^{3/2}}\frac{Y}{X} \end{aligned}$$

Note that the large mixing angles are given by ratios of Yukawa couplings and are independent of the neutrino masses

Tri-bimaximal neutrino mixing from constrained sequential dominance

SFK hep-ph/0506297

$$\begin{aligned}
|a| &= |b| = |c| \,, \\
|d| &= 0 \,, \\
|e| &= |f| \,, \\
\phi_b' &= 0 \,, \\
\phi_c' &= \pi \,.
\end{aligned}$$

$$\phi_c - \phi_b + \phi_e - \phi_f = \pi$$

$$e^* b + f^* c = 0$$

Constrained sequential dominance

(plus previous) conditions

$\longrightarrow$

$$\begin{aligned}
\tan \theta_{23}^{\nu} &\approx 1 \\
\tan \theta_{12}^{\nu} &\approx \frac{1}{\sqrt{2}} \\
\theta_{13}^{\nu} &\approx 0
\end{aligned}$$

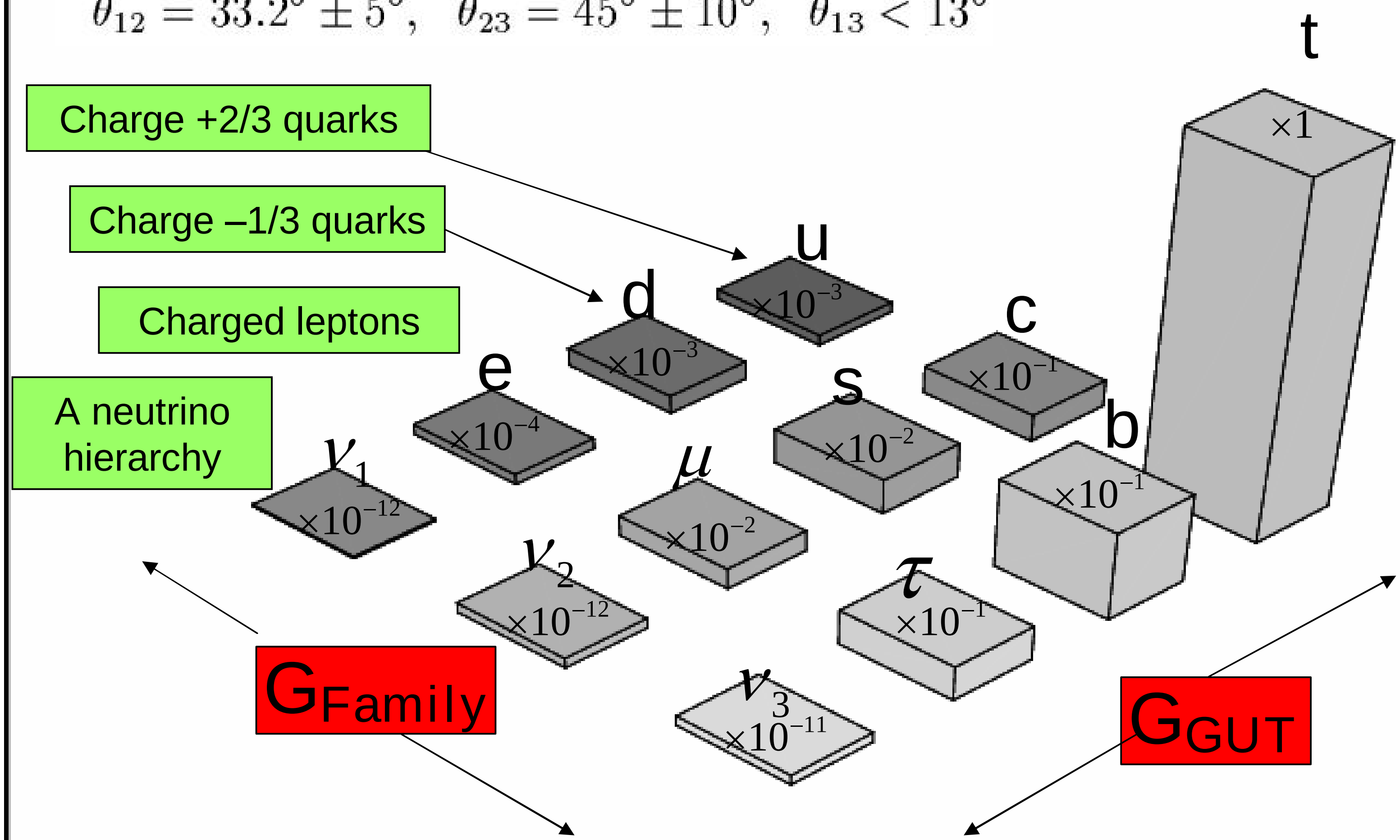
$$U_{\nu_L}^{\dagger} \approx \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Tri-bimaximal neutrino mixing

Quark and lepton masses and mixings

$$\theta_{12}^{\rm CKM} = 13.0^\circ \pm 0.1^\circ, \quad \theta_{23}^{\rm CKM} = 2.4^\circ \pm 0.1^\circ, \quad \theta_{13}^{\rm CKM} = 0.2^\circ \pm 0.1^\circ, \quad \delta^{\rm CKM} = 60^\circ \pm 14^\circ$$

$$\theta_{12} = 33.2^\circ \pm 5^\circ, \quad \theta_{23} = 45^\circ \pm 10^\circ, \quad \theta_{13} < 13^\circ$$



Quark-Lepton Complementarity

Lepton mixings

Parameter	Best-fit value	3σ range
θ_{12}	33.2°	28.7° .. 38.1°
θ_{23}	45.0°	35.7° .. 55.6°
θ_{13}	2.6°	0° .. 12.5°

Quark mixings

Parameter	Best-fit value	2σ range
θ_C	12.88°	12.75° .. 13.01°
θ_{13}^{CKM}	0.21°	0.17° .. 0.25°
θ_{23}^{CKM}	2.36°	2.25° .. 2.48°

Present experimental data allows for relations like the bimaximal complementarity relation:

$$\theta_{12} + \theta_C \approx 45^\circ$$

Is such quark-lepton complementarity a hint of an underlying quark-lepton unification?

Two possibilities:

1. Bimaximal complementarity — Raidal ('04) Smirnov, Minakata ('04)
2. Tri-bimaximal complementarity — SFK ('05)

Bimaximal complementarity

Antusch, SFK, Mohapatra (05).

Ingredients:

1 $\theta_{12}^{\nu} = \pi/4, \quad \theta_{13}^{\nu} = 0, \quad \theta_{23}^{\nu} \approx \pi/4$

Bimaximal neutrino mixing from a pseudo-Dirac inverted neutrino mass hierarchy

2
$$
\begin{aligned}
s_{23}e^{-i\delta_{23}} &\approx e^{-i\delta_{23}^{\nu_L}}\left[s_{23}^{\nu_L} - \theta_{23}^{E_L} c_{23}^{\nu_L} e^{-i(\delta_{23}^{E_L} - \delta_{23}^{\nu_L})}\right] \\
\theta_{13}e^{-i\delta_{13}} &\approx -\theta_{12}^{E_L} s_{23}^{\nu_L} e^{-i(\delta_{23}^{\nu_L} + \delta_{12}^{E_L})} \\
s_{12}e^{-i\delta_{12}} &\approx e^{-i\delta_{12}^{\nu_L}}\left[s_{12}^{\nu_L} - \theta_{12}^{E_L} c_{23}^{\nu_L} c_{12}^{\nu_L} e^{-i(\delta_{12}^{E_L} - \delta_{12}^{\nu_L})}\right]
\end{aligned}
$$

Charged lepton corrections to neutrino mixing angles

3 $\theta_{12}^{E} \approx \frac{3}{2}\theta_C$

Quark-lepton unification relates charged lepton angle to Cabibbo (note unconventional factor of 3/2)

Leads to: $\left(\frac{3}{2} \times \frac{1}{\sqrt{2}} \approx 1.06\right)$

$$\theta_{13} \approx 1.06\,\theta_C$$

Prediction (on the edge!)

$$\theta_{12} + 1.06\,\theta_C \approx \pi/4$$

Bimaximal complementarity (assuming zero phases)

Tri-bimaximal complementarity

SFK(05).

Ingredients:

1 $\theta_{12}^{\nu} = 35.26^{\circ}, \quad \theta_{13}^{\nu} = 0, \quad \theta_{23}^{\nu} \approx \pi/4$ — Tri-Bimaximal neutrino mixing from constrained sequential dominance

2 Charged lepton corrections to neutrino mixing angles

$$\begin{aligned} s_{23}e^{-i\delta_{23}} &\approx e^{-i\delta_{23}^{\nu_L}}\left[s_{23}^{\nu_L} - \theta_{23}^{E_L}c_{23}^{\nu_L}e^{-i(\delta_{23}^{E_L}-\delta_{23}^{\nu_L})}\right] \\ \theta_{13}e^{-i\delta_{13}} &\approx -\theta_{12}^{E_L}s_{23}^{\nu_L}e^{-i(\delta_{23}^{\nu_L}+\delta_{12}^{E_L})} \\ s_{12}e^{-i\delta_{12}} &\approx e^{-i\delta_{12}^{\nu_L}}\left[s_{12}^{\nu_L} - \theta_{12}^{E_L}c_{23}^{\nu_L}c_{12}^{\nu_L}e^{-i(\delta_{12}^{E_L}-\delta_{12}^{\nu_L})}\right] \end{aligned}$$

3 $\theta_{12}^{E} \approx \dfrac{\theta_C}{3}$ — Quark-lepton unification relates charged lepton angle to Cabibbo (note conventional Georgi-Jarlskog factor of 1/3)

Leads to:

$$\theta_{13} \approx \frac{\theta_C}{3\sqrt{2}}$$

Prediction

$$\theta_{12} + \frac{1}{\sqrt{2}}\frac{\theta_C}{3}\cos(\delta - \pi) \approx 35.26^{\circ}$$

Tri-bimaximal complementarity (where δ is the oscillation phase)

Antusch, SFK 04
Barbieri, Hall, Kane, Ross 99

SO(3) family symmetry

Left handed quarks and leptons are triplets under SO(3) family symmetry
Right handed quarks and leptons are singlets under SO(3) family symmetry

$$F_i \sim \mathbf{3}, \qquad F_j^c \sim \mathbf{1}.$$

$$SO(3) \longrightarrow SO(2) \longrightarrow \text{Nothing}$$

Real vacuum alignment
(a,b,c,e,f,h real)

$$<\phi_{23}>=\begin{pmatrix}0\\e\\f\end{pmatrix} \quad <\phi_{123}>=\begin{pmatrix}a\\b\\c\end{pmatrix} \quad <\phi_3>=\begin{pmatrix}0\\0\\h\end{pmatrix}$$

$$\frac{y^a}{M}(L.\phi_a)\nu_R^a h \longrightarrow Y_{LR}^{\nu} \sim \begin{pmatrix} 0 & ae^{i\delta_2} & 0 \\ ee^{i\delta_1} & be^{i\delta_2} & 0 \\ fe^{i\delta_1} & ce^{i\delta_2} & he^{i\delta_3} \end{pmatrix}$$

The Model:

SFK hep-ph/0506297

$$SO(3)\times SU(4)_{PS}\times SU(2)_L\times SU(2)_R$$

$$F_L^i=(3,4,2,1)\ =\begin{pmatrix} u & u & u & \nu \\ d & d & d & e^- \end{pmatrix}_L^i \qquad F_R^i=(1,4,1,2)^i=\begin{pmatrix} u & u & u & \nu \\ d & d & d & e^- \end{pmatrix}_R^i$$

Field	SO(3)	$SU(4)_{PS}$	$SU(2)_L$	$SU(2)_R$	R	U(1)	Z_4^I	Z_4^{II}	Z_3^I	Z_3^{II}
F_i	3	4	2	1	1	0	0	0	0	0
F_1^c	1	$\overline{4}$	1	2	1	−3	α	0	0	0
F_2^c	1	$\overline{4}$	1	2	1	−3	0	β	0	0
F_3^c	1	$\overline{4}$	1	2	1	0	0	0	0	0
h	1	1	2	2	0	0	0	0	0	0
H	1	4	1	2	0	0	0	0	0	0
$\overline{H}$	1	$\overline{4}$	1	2	0	0	0	0	0	0
Σ	1	15	1	3	0	2	α^3	β^3	0	0
ϕ_1	3	1	1	1	0	0	0	0	γ	0
ϕ_2	3	1	1	1	0	0	0	0	0	δ
ϕ_3	3	1	1	$3\oplus 1$	0	0	0	0	0	0
ϕ_{23}	3	1	1	1	0	1	α	0	0	0
ϕ_{123}	3	1	1	1	0	1	0	β	0	0

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Allowed Operators

$$
\begin{aligned}
W_{\rm Yuk} &= \frac{y_1}{M^3}(F.\phi_{23})\phi_{23}^2 F_1^c h \\
&+ \frac{y_2}{M^3}(F.\phi_{123})\phi_{123}^2 F_2^c h + \frac{y_2'\Sigma}{M^2}(F.\phi_{23})F_2^c h \\
&+ \frac{y_3}{M}(F.\phi_3)F_3^c h + \frac{y_3'}{M^3}(F.\phi_2)\phi_2^2 F_3^c h + \frac{y_3''}{M^3}(F.\phi_1)\phi_1^2 F_3^c h \\
W_{\rm Maj} &\sim \frac{1}{M}(F_3^c H)^2 \\
&+ \frac{1}{M^7}(F_2^c H)^2(\phi_{123}^2\phi_{23}^4 + \phi_{123}^6) \\
&+ \frac{1}{M^7}(F_1^c H)^2(\phi_{23}^6 + \phi_{23}^2\phi_{123}^4) \\
&+ \frac{1}{M^5}(F_2^c H)(F_3^c H)\phi_{123}^3\phi_3 \\
&+ \frac{1}{M^5}(F_1^c H)(F_3^c H)\phi_{23}^3\phi_3 \\
&+ \frac{1}{M^7}(F_1^c H)(F_2^c H)\phi_{23}^3\phi_{123}^3,
\end{aligned}
$$

"little flavour magnets"

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Real vacuum alignment of SO(3) and Pati-Salam

$$\phi_1 \sim \begin{pmatrix}1\\0\\0\end{pmatrix}, \quad \phi_2 \sim \begin{pmatrix}0\\1\\0\end{pmatrix}, \quad \phi_3 \sim \begin{pmatrix}0\\0\\1\end{pmatrix}, \quad \phi_{23} \sim \begin{pmatrix}0\\1\\-1\end{pmatrix}, \quad \phi_{123} \sim \begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$Y^U_{LR} \approx \begin{pmatrix} 0 & y_2\epsilon^3 & y_3''\epsilon^3 \\ y_1\epsilon^3 & y_2\epsilon^3 - 2y_2'\epsilon^2 & 0.34y_3'\epsilon^2 \\ -y_1\epsilon^3 & y_2\epsilon^3 + 2y_2'\epsilon^2 & y_3\bar{\epsilon}^{\frac{1}{2}} \end{pmatrix},$$

$$Y^D_{LR} \approx \begin{pmatrix} 0 & y_2\bar{\epsilon}^3 & y_3''\bar{\epsilon}^3 \\ y_1\bar{\epsilon}^3 & y_2\bar{\epsilon}^3 + y_2'\bar{\epsilon}^2 & y_3'\bar{\epsilon}^2 \\ -y_1\bar{\epsilon}^3 & y_2\bar{\epsilon}^3 - y_2'\bar{\epsilon}^2 & y_3\bar{\epsilon}^{\frac{1}{2}} \end{pmatrix},$$

$$Y^E_{LR} \approx \begin{pmatrix} 0 & y_2\bar{\epsilon}^3 & y_3''\bar{\epsilon}^3 \\ y_1\bar{\epsilon}^3 & y_2\bar{\epsilon}^3 + 3y_2'\bar{\epsilon}^2 & y_3'\bar{\epsilon}^2 \\ -y_1\bar{\epsilon}^3 & y_2\bar{\epsilon}^3 - 3y_2'\bar{\epsilon}^2 & y_3\bar{\epsilon}^{\frac{1}{2}} \end{pmatrix},$$

$$Y^\nu_{LR} \approx \begin{pmatrix} 0 & y_2\epsilon^3 & y_3''\epsilon^3 \\ y_1\epsilon^3 & y_2\epsilon^3 & 0.34y_3'\epsilon^2 \\ -y_1\epsilon^3 & y_2\epsilon^3 & y_3\bar{\epsilon}^{\frac{1}{2}} \end{pmatrix}.$$

$$M_{RR} = \begin{pmatrix} p\epsilon^6 & 0 & 0 \\ 0 & q\epsilon^6 & 0 \\ 0 & 0 & 1 \end{pmatrix} M_3$$

$$\varepsilon \approx 0.05, \quad \overline{\varepsilon} \approx 0.15$$

→ Satisfies conditions of constrained sequential dominance providing:

$$\frac{|y_1^2|}{p} \gg \frac{|y_2^2|}{q} \gg |y_3^2|\bar{\epsilon}.$$

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Predictions for neutrino parameters

Model predicts tri-bimaximal mixing with charged lepton corrections, where the charged lepton mixing angles are related to CKM angles:

$$\theta_{13} \approx \frac{\theta_C}{3\sqrt{2}} \longrightarrow \theta_{13} \approx 3.06^\circ, \quad \sin\theta_{13} \approx 0.052,$$

$$s_{12} \approx \frac{1}{\sqrt{3}}\left(1 - \frac{1}{3}\theta_C \cos(\delta - \pi)\right) \longrightarrow \theta_{12} + \frac{1}{\sqrt{2}}\frac{\theta_C}{3}\cos(\delta - \pi) \approx 35.26^\circ$$

$$s_{23} \approx \frac{1}{\sqrt{2}}(1 + \theta_{23}^{\mathrm{CKM}} \cos(\delta_3 - \delta_3')) \longrightarrow \theta_{23} = 45^\circ \pm 2.4^\circ.$$

A prediction for the CP phase:

$$\cos(\delta - \pi) \approx \frac{35.26^\circ - \theta_{12}^\circ}{3.06^\circ}$$

RG corrections (using REAP package of Antusch et al):

$$\theta_{12}(M_Z) - \theta_{12}(M_X) \sim 1^\circ$$
$$\theta_{13}(M_Z) - \theta_{13}(M_X) \sim -0.5^\circ$$
$$\theta_{23}(M_Z) - \theta_{23}(M_X) \sim 2^\circ$$

Conclusions

- Neutrino data may provide new clues in the search for a theory of quark and lepton masses and mixing angles
- We have discussed corrections to tri-bimaximal mixing from charged lepton mixing angles related to quark mixing angles via quark-lepton unification
- Tri-bimaximal neutrino mixing can naturally originate from the see-saw mechanism via constrained sequential dominance
- Constrained sequential dominance can result from the vacuum alignment of a non-Abelian family symmetry such as SO(3)
- SO(3) models can be up-graded to a type II see-saw model with approximately degenerate neutrinos and improved prospects for leptogenesis
- We have constructed a realistic model of quark and lepton masses and mixings based on SO(3) family symmetry with quark-lepton unification based on the Pati-Salam gauge group
- The tri-bimaximal complementarity relation: $\theta_{12} + \frac{1}{\sqrt{2}} \frac{\theta_C}{3} \cos(\delta - \pi) \approx 35.26°$

should apply to a more general class of models