$SU(3)$ family symmetry and ν bi-tri-maximal mixing

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Outline

The data

Mixing angles values

- $s_{12}^2 = 0.30 \pm 0.08$
- $s_{23}^2 = 0.50 \pm 0.18$
- $s_{13}^2 < 0.047$

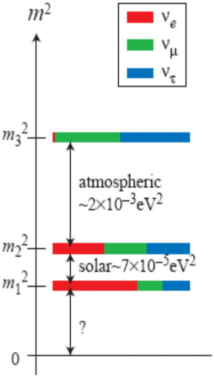

Figure: ν mixing

Bi-tri-maximal ansätz

The Harrison-Perkins-Scott ansätz

$$V_{PMNS} = \begin{pmatrix} -\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix}$$

- P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B **530**, 167 (2002) [arXiv:hep-ph/0202074].

The symmetries of the model

- Underlying $SO(10) \otimes SU(3)_F$
- $SO(10) \rightarrow G_{PS} = SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R$
- $U(1) \otimes U(1)'$ to constrain the allowed terms
- Consider only the supersymmetric case

Field content

Fields under $SU(3)_F$

Field	$SU(3)_F$
ψ	$\mathbf{3}$
ψ^c	$\mathbf{3}$
θ	$\mathbf{3}$
$\bar{\theta}$	$\bar{\mathbf{3}}$
H	$\mathbf{1}$
H_{45}	$\mathbf{1}$
ϕ_X	$\mathbf{3}$
$\bar{\phi}_X$	$\bar{\mathbf{3}}$

- ψ & ψ^c: fermion fields
- θ & $\bar{\theta}$: their vevs break $SU(4)_{PS}$ and lepton number
- H & H_{45}: doublet Higgs & **45** of $SO(10)$
- ϕ_X & $\bar{\phi}_X$: their vevs break $SU(3)_F$

Vacuum expectation values for 3

$\bar{\phi}_3$ vev

$$\langle\bar{\phi}_3\rangle = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} a_u & 0 \\ 0 & a_d \end{pmatrix}$$

where the $SU(3)_F \otimes SU(2)_R$ structure is displayed

ϕ_3 vev

$$\langle\phi_3\rangle = \begin{pmatrix} 0 \\ \delta \\ \sqrt{a_u^2 + a_d^2} \end{pmatrix}$$

$\bar{\theta}$ vev

$$\langle\bar{\theta}\rangle \propto \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$$

θ vev

$$\langle\theta\rangle \propto \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

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θ vev

$$\langle \theta \rangle \propto \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Vevs for 23 and 123

$\bar{\phi}_{23}$ vev

$$\langle \bar{\phi}_{23} \rangle = \begin{pmatrix} 0 & b & -b \end{pmatrix}$$

ϕ_{23} vev

$$\langle \phi_{23} \rangle = \begin{pmatrix} 0 \\ b \\ b \end{pmatrix} e^{i\beta}$$

$\bar{\phi}_{123}$ vev

$$\langle \bar{\phi}_{123} \rangle = \begin{pmatrix} \bar{c} & \bar{c} & \bar{c} \end{pmatrix}$$

ϕ_{123} vev

$$\langle \phi_{123} \rangle = \begin{pmatrix} c \\ c \\ c \end{pmatrix}$$

where $c = \bar{c} e^{i\gamma}$

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Vev for H_{45}

- $\langle H_{45} \rangle$ preserves G_{PS}
- $\langle H_{45} \rangle \propto Y = T_{3_R} + (B - L)/2$
- $\frac{Y(e_R)}{Y(d_R)} = 3$ will generate the Georgi-Jarlskog factor between charged leptons and down quarks
- H. Georgi and C. Jarlskog, Phys. Lett. B **86** (1979) 297.

Effective superpotential terms for Dirac masses

Yukawa leading order terms

$$P_Y = \frac{1}{M^2}\bar{\phi}_3^i \psi_i \bar{\phi}_3^j \psi_j^c H$$

$$+\frac{1}{M^3}\bar{\phi}_{23}^i \psi_i \bar{\phi}_{23}^j \psi_j^c H H_{45}$$

$$+\frac{1}{M^2}\bar{\phi}_{23}^i \psi_i \bar{\phi}_{123}^j \psi_j^c H$$

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Effective superpotential terms for Dirac masses

Yukawa leading order terms

$$P_Y = \frac{1}{M^2}\bar{\phi}^i_3\psi_i\bar{\phi}^j_3\psi^c_j H$$

$$+\frac{1}{M^3}\bar{\phi}^i_{23}\psi_i\bar{\phi}^j_{23}\psi^c_j H H_{45}$$

$$+\frac{1}{M^2}\bar{\phi}^i_{23}\psi_i\bar{\phi}^j_{123}\psi^c_j H$$

$$+\frac{1}{M^2}\bar{\phi}^i_{123}\psi_i\bar{\phi}^j_{23}\psi^c_j H$$

Effective superpotential terms for Majorana masses

Majorana leading order terms

$$P_M = \frac{1}{M}\bar{\theta}^i \psi_i^c \bar{\theta}^j \psi_j^c$$

$$+\frac{1}{M^5}\bar{\phi}_{23}^i \psi_i^c \bar{\phi}_{23}^j \psi_j^c \bar{\theta}^k \phi_{123_k} \bar{\theta}^l \phi_{3_l}$$

$$+\frac{1}{M^5}\bar{\theta}^i \psi_i^c \bar{\phi}_{23}^j \psi_j^c \bar{\theta}^k \phi_{123_k} \bar{\phi}_{23}^l \phi_{3_l}$$

$$+\frac{1}{M^5}\bar{\phi}_{123}^i \psi_i^c \bar{\phi}_{123}^j \psi_j^c \bar{\theta}^k \phi_{23_k} \bar{\theta}^l \phi_{3_l}$$

$$+\frac{1}{M^5}\bar{\theta}^i \psi_i^c \bar{\phi}_{123}^j \psi_j^c \bar{\phi}_{123}^k \phi_{23_k} \bar{\theta}^l \phi_{3_l} + \frac{1}{M^5}\bar{\theta}^i \psi_i^c \bar{\phi}_{123}^j \psi_j^c \bar{\theta}^k \phi_{23_k} \bar{\phi}_{123}^l \phi_{3_l}$$

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$$+\frac{1}{M^5}\bar{\phi}^i_{123}\psi_i^c\bar{\phi}^j_{123}\psi_j^c\bar{\theta}^k\phi_{23_k}\bar{\theta}^l\phi_{3_l}$$

$$+\frac{1}{M^5}\bar{\theta}^i\psi_i^c\bar{\phi}^j_{123}\psi_j^c\bar{\phi}^k_{123}\phi_{23_k}\bar{\theta}^l\phi_{3_l}+\frac{1}{M^5}\bar{\theta}^i\psi_i^c\bar{\phi}^j_{123}\psi_j^c\bar{\theta}^k\phi_{23_k}\bar{\phi}^l_{123}\phi_{3_l}$$

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The messenger sector

Quark messenger mass scale relations

$$M_{d_R} \simeq \frac{1}{3} M_{u_R} \ll M_{Q_L}$$

Lepton messenger mass scale relations

$$M_{e_R} \simeq M_{d_R} \ll M_{L_L} \ll M_{\nu_R}$$

Quark expansion parameters

$$\epsilon_{u,d} \simeq \frac{b}{M_{u_R,d_R}}$$

Lepton expansion parameters

$$\epsilon_{\nu_R,\nu_L,e_R} \simeq \frac{b}{M_{\nu_R,L_L,e_R}}$$

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Lepton expansion parameters

$$\epsilon_{\nu_R,\nu_L,e_R} \simeq \frac{b}{M_{\nu_R,L_L,e_R}}$$

Quarks

Up quark Dirac matrix

$$Y_u \propto \begin{pmatrix} 0 & \epsilon_u^2 \epsilon_d & -\epsilon_u^2 \epsilon_d \\ \epsilon_u^2 \epsilon_d & -2\epsilon_u^2 \frac{\epsilon_u}{\epsilon_d} & 2\epsilon_u^2 \frac{\epsilon_u}{\epsilon_d} \\ -\epsilon_u^2 \epsilon_d & 2\epsilon_u^2 \frac{\epsilon_u}{\epsilon_d} & 1 \end{pmatrix}$$

Down quark Dirac matrix

$$Y_d \propto \begin{pmatrix} 0 & \epsilon_d^3 & -\epsilon_d^3 \\ \epsilon_d^3 & \epsilon_d^2 & -\epsilon_d^2 \\ -\epsilon_d^3 & -\epsilon_d^2 & 1 \end{pmatrix}$$

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Leptons

Charged lepton Dirac matrix

$$Y_l \propto \begin{pmatrix} 0 & \epsilon_d^3 & -\epsilon_d^3 \\ \epsilon_d^3 & 3\epsilon_d^2 & -3\epsilon_d^2 \\ -\epsilon_d^3 & -3\epsilon_d^2 & 1 \end{pmatrix}$$

ν Dirac matrix

$$Y_\nu \propto \begin{pmatrix} 0 & \epsilon_{\nu_L}^2 \epsilon_d & -\epsilon_{\nu_L}^2 \epsilon_d \\ \epsilon_{\nu_L}^2 \epsilon_d & (1+1)\epsilon_{\nu_L}^2 \epsilon_d & (1-1)\epsilon_{\nu_L}^2 \epsilon_d \\ -\epsilon_{\nu_L}^2 \epsilon_d & (-1+1)\epsilon_{\nu_L}^2 \epsilon_d & \frac{\epsilon_{\nu_L}^2}{\epsilon_d^2} \end{pmatrix}$$

Leptons

Charged lepton Dirac matrix

$$Y_l \propto \begin{pmatrix} 0 & \epsilon_d^3 & -\epsilon_d^3 \\ \epsilon_d^3 & 3\epsilon_d^2 & -3\epsilon_d^2 \\ -\epsilon_d^3 & -3\epsilon_d^2 & 1 \end{pmatrix}$$

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Right-handed Majorana ν

N_R Majorana matrix

$$M_{N_R} \simeq M_3 \begin{pmatrix} \lambda_1 \left(\frac{\epsilon_{\nu_R}}{\epsilon_d}\right)^4 \epsilon_d^5 & \lambda_1 \left(\frac{\epsilon_{\nu_R}}{\epsilon_d}\right)^4 \epsilon_d^5 & \lambda_3 \left(\frac{\epsilon_{\nu_R}}{\epsilon_d}\right)^4 \epsilon_d^5 \\ \lambda_1 \left(\frac{\epsilon_{\nu_R}}{\epsilon_d}\right)^4 \epsilon_d^5 & \lambda_2 \left(\frac{\epsilon_{\nu_R}}{\epsilon_d}\right)^4 \epsilon_d^4 & \lambda_4 \left(\frac{\epsilon_{\nu_R}}{\epsilon_d}\right)^4 \epsilon_d^4 \\ \lambda_3 \left(\frac{\epsilon_{\nu_R}}{\epsilon_d}\right)^4 \epsilon_d^5 & \lambda_4 \left(\frac{\epsilon_{\nu_R}}{\epsilon_d}\right)^4 \epsilon_d^4 & 1 \end{pmatrix}$$

Seesaw mechanism

$$m_\nu = \left(M_\nu^D\right) \left(M_{N_R}\right)^{-1} \left(M_\nu^D\right)^T$$

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Seesaw mechanism

$$m_\nu = \left(M_\nu^D\right) \left(M_{N_R}\right)^{-1} \left(M_\nu^D\right)^T$$

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Light ν

ν mixing angles

- $s^2_{12_\nu} = \frac{1}{3}$
- $s^2_{23_\nu} = \frac{1}{2}$
- $s^2_{13_\nu} = 0$

- Perfect ν bi-tri-maximal mixing
- PMNS mixing angles get contributions from charged lepton mixing
- Charged lepton mixing angles are small

The predictions

Mixing angles values predicted

- $s_{12}^2 \approx \frac{1}{3}{}^{+0.047}_{-0.045}$
- $s_{23}^2 \approx \frac{1}{2}{}^{+0.061}_{-0.058}$
- $s_{13}^2 \approx 0.024$

Mixing angles values measured experimentally

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Summary

- The model is based on a unified spontaneously broken $SU(3)_{Family}$ symmetry and is phenomenologically viable.
- Seesaw mechanism and misalignment of vevs play very important roles.
- Model is fairly complicated (existence proof). Hopefully more elegant versions exist.

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Acknowledgement

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