Testing $U(1)$ flavour models

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Hierarchy of masses and mixings of fermions: One of the biggest puzzles of particle physics

• Flavour symmetries: A very useful tool which are now testable and give insight in other aspects of physics BSM (e.g. Leptogenesis)

Why $U(1)$ Abelian models?

• Although not-very interesting in comparison to non-Abelian flavour theories (e.g. $U(2)$, $SU(3)$, etc) they are easy to construct and well motivated by String Theory.

• Try to constrain them as much as possible in order to compare them with successful Non-Abelian models. Is a flavour theory Non-Abelian?

• Can we establish some criteria to determine the non-Abelianity or Abelianity of Flavour models?

. No if we just focus on the description of fermion masses and mixings

. With assumptions and tests in the supersymmetric sector we can test which flavour

. models survive, e.g. $B(\mu \to e\gamma)$

Bottom up approach

Phenomenological Constraints

Fermion Sector	Guidelines	SUSY sector	Guidelines
• Quark masses & mixings	GST relation	• SUSY CP problem	EDM & LFV
$V_{us}=\left\lvert\sqrt{\frac{m_d}{m_s}}-e^{-i\Phi_1}\sqrt{\frac{m_u}{m_c}}\right\rvert$			
• Lepton masses & mixings	ν_RDominance	• SUSY Flavour problem	bounds
	$\rightarrow\ l_2=l_3$		
. Available Experimental info.	*True* $O(1)$ coeff.		
$\rightarrow$ Precision fit			

Motivations for a flavour symmetry

$$\varepsilon\sim\lambda=0.224$$

$$M^d=m_b\begin{pmatrix}\varepsilon^6 & \varepsilon^3 & \varepsilon^3\\ & \varepsilon^2 & \varepsilon^2\\ & & 1\end{pmatrix},\quad M^u=m_t\begin{pmatrix}\varepsilon^6 & \varepsilon^6 & \varepsilon^6\\ & \varepsilon^4 & \varepsilon^2\\ & & 1\end{pmatrix},M^e=m_\tau\begin{pmatrix}\varepsilon^6 & \varepsilon^3 & \varepsilon^3\\ & \varepsilon^2 & \varepsilon^2\\ & & 1\end{pmatrix}$$

- Hierarchical description of Yukawa couplings
- Relations between quark & lepton sectors (independent of a possible GUT)

- A way to solve (or help solving) SUSY CP & Flavour problems

. ⋆ e.g. spontaneous breaking of CP only in the flavour sector

. ⋆ e.g. trilinears either aligned with Yukawa couplings or with non dangerous contributions

$U(1)'s$ quite easy to construct

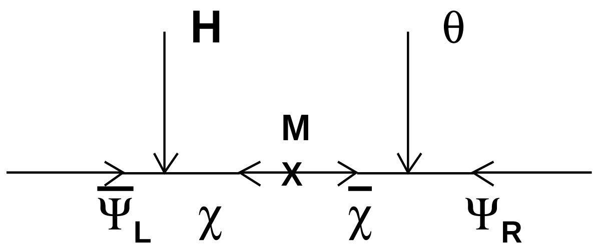

Figure 1: General Froggatt Nielsen supergraph generating fermion masses.
Nucl. Phys. B 147 p. 277, 1979 Froggatt, Nielsen

$$m_\Psi \approx h h' \frac{\langle H\rangle\langle\theta\rangle}{M}, \qquad \epsilon \equiv \frac{\langle\theta\rangle}{M}$$

$$(\mathbf{Y^u})_{\mathbf{ij}} = \mathbf{a^u_{ij}}\epsilon^{|\mathbf{Q^u_{ij}}|}, \qquad (\mathbf{Y^d})_{\mathbf{ij}} = \mathbf{a^d_{ij}}\epsilon^{|\mathbf{Q^d_{ij}}|}$$

$$\mathbf{Q^u_{ij}} = -(\alpha_\mathbf{i} - \alpha_\mathbf{3}) - (\beta_\mathbf{j} - \beta_\mathbf{3}), \qquad \mathbf{Q^d_{ij}} = -(\alpha_\mathbf{i} - \alpha_\mathbf{3}) - (\beta_\mathbf{j} - \beta'_\mathbf{3}),$$

coefficients of $O(1)$

Classification of Anomalous $U(1)$ (according to our criteria)

$(\mathbf{i})$ According the way in which anomaly cancellation is achieved (using the Green-Schwartz mechanism) $- - - \rightarrow$

. (a), (b) At M_F μ term —-GUT theories ($SU(5)$, $S0(10)$)

. (c) or Extended GUT-like theories with no μ term at M_F

$(\mathbf{ii})$ Constraints and predictions in the quark sector

$(\mathbf{iii})$ Constraints and predictions in the lepton sector

$(\mathbf{iv})$ Constraints and predictions in the susy sector

(i) Green-Schwartz anomaly conditions

Following the notation

	Q_i	u_i^c	d_i^c	L_i	e_i^c	ν_i^c	H_u	H_d
$U(1)_F$	q_i	u_i	d_i	l_i	e_i	n_i	h_u	h_d

⋆ In this case, the mixed $U(1)' - SU(3) - SU(3)$, $U(1)' - SU(2) - SU(2)$ and $U(1)' - U(1) - U(1)$ anomalies, A_3, A_2, and A_1 cancel if they appear in the ratio:

$A_3 : A_2 : A_1 = k_3 : k_2 : k_1$

k_i are the Kac-Moody levels of the gauge groups, defined by the GUT-scale relation:

$$g_3^2 k_3 = g_2^2 k_2 = g_1^2 k_1$$

$$A_3 = \frac{1}{2}\left[\sum_{i=1}^{3}(2q_i + u_i + d_i)\right]$$

$$A_2 = \frac{1}{2}\left[\sum_{i=1}^{3}(3q_i + l_i) + h_u + h_d\right]$$

$$\frac{3}{5}A_1 = \frac{1}{2}\left[\sum_{i=1}^{3}\left(\frac{q_i}{5} + \frac{8u_i}{5} + \frac{2}{5}d_i + \frac{3l_i}{5} + \frac{6e_i}{5}\right) + \frac{3}{5}(h_u + h_d)\right]$$

$$A_1' = \sum_{i=1}^{3}(-q_i^2 + 2u_i^2 - d_i^2 + l_i^2 - e_i^2) + (h_d^2 - h_u^2) = 0$$

⋆ We can parameterise the anomalies as follows

$$\sum_{i=1}^{3} q_i = x + u \qquad \sum_{i=1}^{3} u_i = x + 2u \qquad \sum_{i=1}^{3} d_i = y + v \sum_{i=1}^{3} l_i = y$$

$$\sum_{i=1}^{3} e_i = x \qquad \mathbf{h_u = -z, \quad h_d = z + (u + v)}$$

$$A_3 = A_2 = \frac{5}{3} A_1$$

hep-ph/9412367 Jain & Schrock

$\mathbf{(a)\ u = v = 0}$

⋆ We can decompose the $U(1)$ charges in flavour independent and flavour dependent parts

$$f_i = \frac{1}{3} f + f_i', \qquad \sum_{i=1}^{3} f_i' = 0$$

We can always find the x and y which satisfy $\sum_{i=1}^{3} f_i' = 0$

In this way A_1' can be expressed in flavour independent, flavour dependent terms

$A_1' = A'_{1FI} + A'_{1FD}$

Following this, we have

$$A'_1 \quad = \quad \frac{1}{3}\left[-q^2 + 2u^2 - d^2 + l^2 - e^2\right] + \sum_{i=1}^{3}(-q'^{\,2}_i + 2u'^{\,2}_i - d'^{\,2}_i + l'^{\,2}_i - e'^{\,2}_i)$$

It turns out that the **first term** is automatically zero for

⋆ For $SU(5)$ cases where $\mathbf{q = u = e = x}$ and $\mathbf{l = d = y}$

⋆ For Pati-Salam cases where $\mathbf{u = e = d}$ and $\mathbf{l = q}$

Then we have to make the family dependent (**second term**) part to vanish

⋆ For $SU(5)$ cases where $\mathbf{q_i = u_i = e_i}$ and $\mathbf{l_i = d_i}$

⋆ For Pati-Salam cases where $\mathbf{u_i = e_i = d_i}$ and $\mathbf{l_i = q_i}$

$\mathbf{(b)\ u+v=0}$

Then we have

$$A_1' = \frac{1}{3}\left[6u^2+6xu+2yu\right] - \sum_{i=1}^{3}\left({q_i'}^2 - 2{u_i'}^2 + {d_i'}^2 - {l_i'}^2 + {e_i'}^2\right)$$

The family independent part will vanish if $\mathbf{u=-v=-\left(x+\frac{y}{3}\right)}$

Thus we have

$$\sum_{i=1}^{3} q_i = -\frac{y}{3} \qquad \sum_{i=1}^{3} u_i = -(x+\frac{2y}{3})$$

$$\sum_{i=1}^{3} d_i = x+\frac{4y}{3} \qquad \sum_{i=1}^{3} l_i = y \qquad \sum_{i=1}^{3} e_i = x$$

We can learn from the previous example, in the sense of looking for the most easily achievable solution, in this case we have

$$\mathbf{q_i = -\frac{l_i}{3}\ ,\ u_i = -(\frac{2l_i}{3}+e_i)\ ,\ d_i = \frac{4l_i}{3}+e_i}$$

The same equation holds for the primed charges, thus we have

$$\begin{aligned} A_1' &= \frac{1}{3}\left[x^2(6-6)+\frac{2}{3}y^2(1-1)+xy(4-2-2)\right] \\ &\quad - \sum_{i=1}^{3}\left({l_i'}^2\frac{1}{9}(-1+8-16+9)+{e_i'}^2(2-1-1)\right) = 0 \end{aligned}$$

$(\mathbf{c})\ \mathbf{u}+\mathbf{v}\neq \mathbf{0}$ (Forbidd the μ term in the superpotential)

⋆ In this case we have

$$A_1' = -2(4u^2 + u(v+3x+z) + v(z-y)) - 2\sum_{f=u,d,l,e,q} g_f(f_1(f_2+f_3)+f_2f_3)$$

where $g_f = 1, -2, 1, -1, 1$ respectively for $f = q, u, d, l, e$.

⋆ It is difficult to depart from here in order to find some ansatz which cancel the A_1' anomaly. Instead we can generalize the kind of relations which in the limit of $u = v = 0$ would give the $SU(5)$ cases or the Pati-Salam cases.

$SU(5)$ like cases

• In this case we can see from the parameterisation that we used to cancel anomalies, that if we have the linear relations

$$\mathbf{q_i} = \mathbf{u_i} + \alpha = \mathbf{e_i} + \gamma, \quad \mathbf{d_i} = \mathbf{l_i} + \beta,$$

in the limit of the $u = v = 0$ we recover the $SU(5)$ case. In agreement with the cancellation of anomalies then

$$\mathbf{q_i} = \mathbf{u_i} - \frac{\mathbf{u}}{\mathbf{3}} = \mathbf{e_i} + \frac{\mathbf{u}}{\mathbf{3}}, \quad \mathbf{d_i} = \mathbf{l_i} + \frac{\mathbf{v}}{\mathbf{3}}$$

• In the expression of the A_1' anomaly the sums of squared charges cancel and we can write it just in terms of sum of charges, which we have parameterised in terms of u, v, x, y,

$$A_1' = -10\frac{u^2}{3} - \frac{2}{3}v^2 + 2u(x+v) + 2y\frac{v}{3} - 2z(u+v) = 0$$

Pati-Salam -like cases

⋆ We look for solutions which in the $u = v = 0$ limit reproduce the Pati-Salam case, so we should have the relations

$$\mathbf{q_i} = \mathbf{l_i} + \alpha, \quad \mathbf{u_i} = \mathbf{d_i} + \beta$$

also e_i and n_i need to be related to u_i by a constant, as before. In these case in order to satisfy the G-S anomaly conditions we need

$$\mathbf{q_i} = \mathbf{l_i} + \frac{\mathbf{u} + (\mathbf{x} - \mathbf{y})}{\mathbf{3}}, \quad \mathbf{u_i} = \mathbf{e_i} + \frac{\mathbf{2u}}{\mathbf{3}}, \quad \mathbf{d_i} = \mathbf{e_i} + \frac{\mathbf{v} + (\mathbf{y} - \mathbf{x})}{\mathbf{3}}$$

Thus the expression for the A_1' anomaly is

$$A_1' = -\frac{2}{9}\left[8u^2 + 4v^2 + u(9v + 11x - 2y) + 2(x - y)^2 - v(2x + y)\right] - 2z(u + v)$$

Form of the Yukawa matrices in each case

$(\mathbf{a})\ \mathbf{u}=\mathbf{v}=\mathbf{0}$

$SU(5)$ case

$$y_t=\epsilon^{|2e_3+h_u|}=1 \quad \to h_u=-2e_3$$

$$Y^u \approx \begin{bmatrix} \epsilon^{|2(e_1-e_3)|} & \epsilon^{|e_1+e_2-2e_3|} & \epsilon^{|e_1-e_3|} \\ \epsilon^{|e_1+e_2-2e_3|} & \epsilon^{|2(e_2-e_3)|} & \epsilon^{|e_2-e_3|} \\ \epsilon^{|e_1-e_3|} & \epsilon^{|e_2-e_3|} & 1 \end{bmatrix},\ Y^d \approx \begin{bmatrix} \epsilon^{|l_1+e_1+h_d|} & \epsilon^{|l_2+e_1+h_d|} & \epsilon^{|l_3+e_1+h_d|} \\ \epsilon^{|l_1+e_2+h_d|} & \epsilon^{|l_2+e_2+h_d|} & \epsilon^{|l_3+e_2+h_d|} \\ \epsilon^{|l_1+e_3+h_d|} & \epsilon^{|l_2+e_3+h_d|} & \epsilon^{|l_3+e_3+h_d|} \end{bmatrix}$$

$$Y^d = Y^{e\ T}$$

Pati-Salam case

$$Y^f = \begin{pmatrix} \epsilon^{|l_1+e_1+h_f|} & \epsilon^{|l_1+e_2+h_f|} & \epsilon^{|l_1+e_3+h_f|} \\ \epsilon^{|l_2+e_1+h_f|} & \epsilon^{|l_2+e_2+h_f|} & \epsilon^{|l_2+e_3+h_f|} \\ \epsilon^{|l_3+e_1+h_f|} & \epsilon^{|l_3+e_2+h_f|} & \epsilon^{|l_3+e_3+h_f|} \end{pmatrix}$$

for $h_f=h_u,\ h_d$. In this case we always need to satisfy $x=y$ and the restriction

$$e_1=x-(e_2+e_3), \quad l_1=x-(l_2+l_3)$$

$\mathbf{(b)\ u + v = 0}$

$$Y^u \approx \begin{bmatrix} \epsilon^{|l_1+e_1|} & \epsilon^{|\frac{1}{3}(l_1+2l_2)+e_2|} & \epsilon^{|\frac{1}{3}(l_1+2l_3)+e_3|} \\ \epsilon^{|\frac{1}{3}(l_2+2l_1)+e_1|} & \epsilon^{|l_2+e_2|} & \epsilon^{|\frac{1}{3}(l_2+2l_3)+e_3|} \\ \epsilon^{|\frac{1}{3}(l_3+2l_1)+e_1|} & \epsilon^{|\frac{1}{3}(l_3+2l_2)+e_2|} & \epsilon^{|l_3+e_3|} \end{bmatrix}, \quad Y^e \approx \begin{bmatrix} \epsilon^{|l_1+e_1|} & \epsilon^{|l_2+e_1|} & \epsilon^{|l_3+e_1|} \\ \epsilon^{|l_1+e_2|} & \epsilon^{|l_2+e_2|} & \epsilon^{|l_3+e_2|} \\ \epsilon^{|l_1+e_3|} & \epsilon^{|l_2+e_3|} & \epsilon^{|l_3+e_3|} \end{bmatrix}$$

$$Y^d \approx \begin{bmatrix} \epsilon^{|l_1+e_1|} & \epsilon^{|\frac{1}{3}(-l_1+4l_2)+e_2|} & \epsilon^{|\frac{1}{3}(-l_1+4l_3)+e_3|} \\ \epsilon^{|\frac{1}{3}(-l_2+4l_1)+e_1|} & \epsilon^{|l_2+e_2|} & \epsilon^{|\frac{1}{3}(-l_2+4l_3)+e_3|} \\ \epsilon^{|\frac{1}{3}(-l_3+4l_1)+e_1|} & \epsilon^{|\frac{1}{3}(-l_3+4l_2)+e_2|} & \epsilon^{|l_3+e_3|} \end{bmatrix}$$

$\mathbf{(c)\ u + v \neq 0}$

Extended $SU(5)$-like case

$$h_u = -z = \frac{u}{3} - 2u_3$$

$$h_d = 2u + v + 2e_3$$

$$\mathcal{C}(Y^u_{ij}) = |e_i + e_j - 2e_3|$$

$$\mathcal{C}(Y^d_{ij}) = |e_i + l_j + 2e_3 + \frac{7u}{3} + \frac{4v}{3}|$$

$$\mathcal{C}(Y^e_{ij}) = |l_i + e_j + 2e_3 + 2u + v|$$

Extended Pati-Salam-like case

$$h_u = -z = -(l_3 + e_3 + u + \frac{x-y}{3})$$

$$h_d = l_3 + e_3 + 2u + v + \frac{x-y}{3}$$

$$\mathcal{C}(Y^u_{ij}) = |l_i - l_3 + e_j - e_3|$$

$$\mathcal{C}(Y^d_{ij}) = |l_i + e_j + l_3 + e_3 + \frac{4v+7u+(x-y)}{3} + \frac{4v}{3}|$$

$$\mathcal{C}(Y^e_{ij}) = |l_i + e_j + 2e_3 + 2u + v|$$

(ii) Constraints and predictions in the quark sector

$\longrightarrow$Precise determination from experimental Information

$$M^u_{\rm diag} = L^{u\dagger} M^u R^u, \qquad M^d_{\rm diag} = L^{d\dagger} M^d R^d$$

$$V_{\rm CKM} = L^{u\dagger} L^d \qquad U_{\rm MNS} = L^{l\dagger} L^\nu$$

(a) Semileptonic decays of B mesons $\rightarrow |V_{ub}/V_{cb}|$ $\surd$

(b) CP violation in the K system $\rightarrow |\epsilon_K|$ $\surd$

(c) $B^0_{d,s} - \bar{B}^0_{d,s}$ oscillations $\rightarrow \Delta m_{B_d}$ $\surd$

. Δm_{B_s} [u.l.]

$A, \; \lambda \approx 0.224, \; \rho, \; \eta$

(d) CP asymmetries in B decays, $\rightarrow \sin(2\beta)$

$B \rightarrow J/\Psi K_s$ $\surd$ SM

$B \rightarrow \Phi K_s$??

$*$

$*$

$$\frac{|V_{ub}|}{|V_{cb}|} \quad \frac{|V_{td}|}{|V_{ts}|} \quad |V_{us}| \quad {\rm Im}\{J\}$$

$$\frac{m_u}{m_c} \quad \frac{m_c}{m_t} \quad \frac{m_d}{m_s} \quad \frac{m_s}{m_b}$$

Constraints for the Yukawa matrices in each case

$(\mathbf{a})\ \mathbf{u} = \mathbf{v} = \mathbf{0}$

$$r_f = f_2 - f_3, \quad s = (q_2 - q_3), \qquad r'_f = f_1 - f_3 = x^f - (f_2 + 2f_3)$$
$$s' \;=\; (q_1 - q_3) = x - (q_2 + 2q_3)$$

$SU(5)$ case

Such that we have

$$Y^u = \begin{pmatrix} \varepsilon^{|2s'|} & \varepsilon^{|s'+s|} & \varepsilon^{|s'|} \\ \varepsilon^{|s'+s|} & \varepsilon^{|2s|} & \varepsilon^{|s|} \\ \varepsilon^{|s'|} & \varepsilon^{|s|} & 1 \end{pmatrix}, \; Y^d = Y^{lT} = \begin{pmatrix} \varepsilon^{|s'+r'_d+k_d|} & \varepsilon^{|s'+r_d+k_d|} & \varepsilon^{|s'+k_d|} \\ \varepsilon^{|s+r'_d+k_d|} & \varepsilon^{|s+r_d+k_d|} & \varepsilon^{|s+k_d|} \\ \varepsilon^{|r'_d+k_d|} & \varepsilon^{|r_d+k_d|} & \varepsilon^{|k_d|} \end{pmatrix}$$

Solutions satisfying the SNRHD (ν_R dominance, $l_2 = l_3$) and the **GST relation**

$$\nu_{\mathbf{R}}\textbf{dominance}, \quad k_d + s = k_d \qquad .$$

$$\begin{aligned} \textbf{GST} \qquad |k_d + r'_d + s| \;&=\; |k_d + s'| \\ |k_d + r'_d + s'| - |k_d| \;&>\; |k_d + r'_d + s| + |k_d + s'| - |k_d + s| \\ |r'_d + k_d| \;&>\; |k_d| \end{aligned}$$

We look for solutions (in terms of just one expansion parameter $\varepsilon = O(\lambda)$), such as

$$Y^u = \begin{pmatrix} ... & \varepsilon^6 & ... \\ \varepsilon^6 & \varepsilon^4 & \varepsilon^2 \\ ... & \varepsilon^2 & 1 \end{pmatrix}, \quad Y^d = \begin{pmatrix} ... & \varepsilon^5 & \varepsilon^5 \\ \varepsilon^5 & \varepsilon^4 & \varepsilon^4 \\ ... & \varepsilon^2 & \varepsilon^2 \end{pmatrix}$$

for which we have

$$s^u_{12} \approx \varepsilon^2 \quad s^d_{12} \approx \varepsilon, \quad s^d_{23} \approx \varepsilon^2, \quad s^d_{13} \approx \varepsilon^3$$
$$\frac{m_c}{m_t} \approx \varepsilon^4, \quad \frac{m_s}{m_b} \approx \varepsilon^2, \quad \frac{m_b}{m_t} \approx \varepsilon^2$$

In this case we see that we can have plausible solutions in the up sector by allowing half integer solutions

$$|s' + s| = 13/2,\ 6,\ 11/2$$

- $|s + s'| = 13/2 \quad Y^u = \begin{pmatrix} \varepsilon^{35/2} & \varepsilon^{13/2} & \varepsilon^{35/4} \\ \varepsilon^{13/2} & \varepsilon^{9/2} & \varepsilon^{9/4} \\ \varepsilon^{35/4} & \varepsilon^{9/4} & 1 \end{pmatrix}, \quad Y^d = \begin{pmatrix} \varepsilon^{69/4} & \varepsilon^{25/4} & \varepsilon^{25/4} \\ \varepsilon^{25/4} & \varepsilon^{19/4} & \varepsilon^{19/4} \\ \varepsilon^{17/2} & \varepsilon^{5/2} & \varepsilon^{5/2} \end{pmatrix}$

$$\text{for} \qquad s = -\frac{9}{4},\ s' = \frac{35}{4},\ k_d = -\frac{5}{2},\ r'_d = l_1 - l_3 = 11 \quad \text{or}$$

$$s = \frac{9}{4},\ s' = -\frac{35}{4},\ k_d = \frac{5}{2},\ r'_d = l_1 - l_3 = -11$$

- $|s + s'| = 6 \quad Y^u = \begin{pmatrix} \varepsilon^{16} & \varepsilon^{6} & \varepsilon^{8} \\ \varepsilon^{6} & \varepsilon^{4} & \varepsilon^{2} \\ \varepsilon^{8} & \varepsilon^{2} & 1 \end{pmatrix}, \quad Y^d = \begin{pmatrix} \varepsilon^{31/2} & \varepsilon^{11/2} & \varepsilon^{11/2} \\ \varepsilon^{11/2} & \varepsilon^{9/2} & \varepsilon^{9/2} \\ \varepsilon^{15/2} & \varepsilon^{5/2} & \varepsilon^{5/2} \end{pmatrix}$

$$\text{for} \qquad s = -2,\ s' = 8,\ k_d = -\frac{5}{2},\ r'_d = l_1 - l_3 = 10 \quad \text{or}$$

$$s = 2,\ s' = -8,\ k_d = \frac{5}{2},\ r'_d = l_1 - l_3 = -10$$

$$\bullet |s+s'| = 11/2 \quad Y^u = \begin{pmatrix} \varepsilon^{29/2} & \varepsilon^{11/2} & \varepsilon^{29/4} \\ \varepsilon^{11/2} & \varepsilon^{7/2} & \varepsilon^{7/4} \\ \varepsilon^{29/4} & \varepsilon^{7/4} & 1 \end{pmatrix}, \quad Y^d = \begin{pmatrix} \varepsilon^{31/2} & \varepsilon^{21/4} & \varepsilon^{21/4} \\ \varepsilon^{21/4} & \varepsilon^{15/4} & \varepsilon^{15/4} \\ \varepsilon^{33/4} & \varepsilon^2 & \varepsilon^2 \end{pmatrix}$$

for $s = -\frac{7}{4},\ s' = \frac{29}{4},\ k_d = -2,\ r'_d = l_1 - l_3 = 41/4$ or

$s = \frac{7}{4},\ s' = -\frac{29}{4},\ k_d = 2,\ r'_d = l_1 - l_3 = -41/4$

Pati-Salam case

$$
\begin{aligned}
Y^{u,\nu} &= \begin{pmatrix} \varepsilon^{|l_1-l_3+e_1-e_3|} & \varepsilon^{|l_1-l_3+e_2-e_3|} & \varepsilon^{|l_1-l_3|} \\ \varepsilon^{|e_1-e_3|} & \varepsilon^{|e_2-e_3|} & 0 \\ \varepsilon^{|e_1-e_3|} & \varepsilon^{|e_2-e_3|} & 0 \end{pmatrix} \\
C(Y^{d,l}) &= C(Y^{u,\nu}) + (2(l_3+e_3)) \otimes 1
\end{aligned}
$$

$U(1)$ symmetry does not give an appropriate description of fermion masses and mixings but also the non-renormalizable operators of the Pati-Salam group can play a role (hep-ph/ S.King and Oliveira)

$$
\begin{pmatrix} \varepsilon^{|e_2-e_3|} & \delta \\ \varepsilon^{|e_2-e_3|} & \delta \end{pmatrix}
$$

(iii) Constraints and predictions in the lepton sector

Majorana neutrinos & Neutrino right handed dominance (NRHD)

hep-ph/9904210 S.King

$$m_{LL} = v_u^2 Y^\nu M_R^{-1} Y^{\nu T} =$$

$$\frac{v_u^2}{M_3}\begin{pmatrix} Y^{\nu 2}_{13} & Y^\nu_{13}Y^\nu_{23} & Y^\nu_{13}Y^\nu_{23} \\ Y^\nu_{13}Y^\nu_{23} & Y^{\nu 2}_{23} & Y^\nu_{23}Y^\nu_{33} \\ Y^\nu_{13}Y^\nu_{33} & Y^\nu_{33}Y^\nu_{23} & Y^{\nu 2}_{23} \end{pmatrix} + \frac{v_u^2}{M_2}\begin{pmatrix} Y^{\nu 2}_{12} & Y^\nu_{12}Y^\nu_{22} & Y^\nu_{12}Y^\nu_{32} \\ Y^\nu_{12}Y^\nu_{22} & Y^{\nu 2}_{22} & Y^\nu_{22}Y^\nu_{32} \\ Y^\nu_{12}Y^\nu_{32} & Y^\nu_{22}Y^\nu_{32} & Y^{\nu 2}_{32} \end{pmatrix} + \frac{v_u^2}{M_1}\begin{pmatrix} Y^{\nu 2}_{11} & Y^\nu_{11}Y^\nu_{21} & Y^\nu_{11}Y^\nu_{31} \\ Y^\nu_{11}Y^\nu_{21} & Y^{\nu 2}_{21} & Y^\nu_{21}Y^\nu_{31} \\ Y^\nu_{11}Y^\nu_{31} & Y^\nu_{21}Y^\nu_{31} & Y^{\nu 2}_{31} \end{pmatrix}$$

Mixings (M_3)

$$t_{23} = \frac{Y^\nu_{23}}{Y^\nu_{33}}, \qquad t_{13} = \frac{Y^\nu_{13}}{\sqrt{Y^\nu_{23} + Y^\nu_{33}}} \qquad t_{12} = \frac{Y^\nu_{12}(Y^{\nu 2}_{33} + Y^{\nu 2}_{23}) - Y^\nu_{13}(Y^\nu_{33}Y^\nu_{32} - Y^\nu_{22}Y^\nu_{23})}{(Y^\nu_{33}Y^\nu_{33} - Y^\nu_{32}Y^\nu_{23})\sqrt{Y^{\nu 2}_{33} + Y^{\nu 2}_{23} + Y^{\nu 2}_{13}}}$$

(Interesting relation among U_{MNS} phases, leptogenesis & neutrinoless double beta decay)

hep-ph/0211228 S.King, hep-ph/0307071 L. V-S

The heaviest low neutrino masses are given by

$$m_{\nu_3} = \frac{a_3^{\nu 2} \varepsilon^{2|l_2' + n_3|} v^2}{\varepsilon^{|2n_3+\sigma|} < \Sigma >}, \qquad m_{\nu_2} = \frac{a_2^{\nu 2} \varepsilon^{2|l_2' + n_2|} v^2}{\varepsilon^{|2n_2+\sigma|} < \Sigma >}$$

Examples

1. $SU(5)$ GUT & GST relation

$$Y^e \sim Y^{d\,T} = Y^d \approx \begin{bmatrix} \epsilon^{|l_1+e_1+h_d|} & \epsilon^{|l_2+e_1+h_d|} & \epsilon^{|l_3+e_1+h_d|} \\ \epsilon^{|l_1+e_2+h_d|} & \epsilon^{|l_2+e_2+h_d|} & \epsilon^{|l_3+e_2+h_d|} \\ \epsilon^{|l_1+e_3+h_d|} & \epsilon^{|l_2+e_3+h_d|} & \epsilon^{|l_3+e_3+h_d|} \end{bmatrix}, \quad l_2 = l_3$$

$$Y^u = \begin{pmatrix} \varepsilon^{16} & \varepsilon^{6} & \varepsilon^{8} \\ \varepsilon^{6} & \varepsilon^{4} & \varepsilon^{2} \\ \varepsilon^{8} & \varepsilon^{2} & 1 \end{pmatrix}, \quad Y^d = \begin{pmatrix} \varepsilon^{31/2} & \varepsilon^{11/2} & \varepsilon^{11/2} \\ \varepsilon^{11/2} & \varepsilon^{9/2} & \varepsilon^{9/2} \\ \varepsilon^{15/2} & \varepsilon^{5/2} & \varepsilon^{5/2} \end{pmatrix}$$

$$Y^\nu = \begin{pmatrix} \epsilon^{|n_1+5|} & \epsilon^{\frac{41}{8}} & \epsilon^{\frac{11}{2}} \\ \epsilon^{|n_1-5|} & \epsilon^{\frac{39}{8}} & \epsilon^{\frac{9}{2}} \\ \epsilon^{|n_1-5|} & \epsilon^{\frac{39}{8}} & \epsilon^{\frac{9}{2}} \end{pmatrix}$$

$$M_{RR} = \begin{pmatrix} \epsilon^{|2n_1+\sigma|} & \epsilon^{|1/8+n_1+\sigma|} & \epsilon^{|1/2+n_1+\sigma|} \\ . & a^N_{22}\epsilon^{|1/4+\sigma|} & \epsilon^{|5/8+\sigma|} \\ . & . & \epsilon^{|1+\sigma|} \end{pmatrix} \langle \Sigma \rangle$$

Mixings and masses

$$t^\nu_{23} = \frac{a^\nu_{23}}{a^\nu_{33}}, \quad t^\nu_{13} = \frac{a^\nu_{13}\varepsilon^{|2n_3|}}{\sqrt{a^{\nu 2}_{33} + a^{\nu 2}_{23}}}, \quad t^\nu_{12} = \frac{a^\nu_{12}\varepsilon^{|2n_2|}}{(c_{23}a^\nu_{22} - s_{23}a^\nu_{32})},$$

$$m_{\nu_3} = \frac{v^2}{<\Sigma>}(a^{\nu 2}_{33} + a^{\nu 2}_{23})\varepsilon^{|p_3|}, \quad \frac{m_{\nu_2}}{m_{\nu_3}} = \frac{c^{\nu 2}_{23}}{c^{\nu 2}_{12}} \frac{(a^\nu_{22} - a^\nu_{32}t^\nu_{23})^2}{a^{\nu 2}_{33} + a^{\nu 2}_{23}}\varepsilon^{|4(n_3-n_2)|}$$

p_{12}	p_{13}	$p_2 - p_3$	n_2	n_3
$1/4$	1	$3/2$	$1/8$	$1/2$
$1/2$	1	1	$1/4$	$1/2$

Quark Fitted Parameters			
GST sol. 1		GST sol. 1'a, $u, v \neq 0$	GST sol. 1'b $u, v \neq 0$
Parameter	BFP Value	BFP Value	BFP Value
a^u_{12}	2.74 ± 0.61	1.04 ± 0.19	2.74 ± 0.71
a^u_{23}	1.68 ± 0.17	1.34 ± 0.13	1.41 ± 0.18
a^d_{22}	1.08 ± 0.18	1.05 ± 0.11	0.70 ± 0.23
a^d_{12}	0.93 ± 0.15	0.55 ± 0.20	0.74 ± 0.13
a^d_{13}	0.29 ± 0.21	0.30 ± 0.14	0.74 ± 0.17
a^d_{23}	0.79 ± 0.10	0.70 ± 0.13	0.66 ± 0.35
a^d_{32}	0.48 ± 0.17	1.28 ± 0.32	1.28 ± 0.58
$\cos(\Phi_2)$	0.454 ± 0.041	0.456 ± 0.041	0.547 ± 0.424
ε	0.183	0.217	0.154
χ^2	1.47	2.41	4.32

Table 1: Quark fitted parameters. The second column corresponds to the Solution 1 in the $SU(5)$ ($u = v = 0$) case, the third column to the Solution 1 in the $u \neq -v \neq 0$ case. The fourth column presents the fit to the Solution 3 in the $u \neq -v \neq 0$ case.

2. $SU(5)$ GUT & NO GST relation

$$Y^d = \varepsilon^{|k_d|}\begin{pmatrix} \varepsilon^{|s'+l_1-l_3|} & \varepsilon^{|s'|} & \varepsilon^{|s'|} \\ \varepsilon^{|s+l_1-l_3|} & \varepsilon^{|s|} & \varepsilon^{|s|} \\ \varepsilon^{|l_1-l_3|} & 1 & 1 \end{pmatrix}, \quad |s|=2,\ |s'|=3,\ r'_d = l_1 - l_3$$

$$Y^u = \begin{pmatrix} \epsilon^6 & \epsilon^5 & \epsilon^3 \\ \epsilon^5 & \epsilon^4 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & a^u_{33} \end{pmatrix}, \quad Y^d = \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & a^d_{32} & a^d_{33} \end{pmatrix}\epsilon^{|k_d|}$$

$$Y^e = \varepsilon^{|k_d|}\begin{pmatrix} \varepsilon^4 & \varepsilon^3 & \varepsilon \\ \varepsilon^3 & \varepsilon^2 & 1 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{pmatrix}, \quad Y^\nu = \begin{pmatrix} \varepsilon^{|n_1+1|} & \varepsilon^{5/8} & \varepsilon \\ \varepsilon^{|n_1-3/8|} & \varepsilon^{3/8} & a^\nu_{23} \\ \varepsilon^{|n_1|} & \varepsilon^{3/8} & a^\nu_{33} \end{pmatrix}$$

$$M_R = <\Sigma> \begin{pmatrix} \varepsilon^{|2n_1+5|} & \varepsilon^{|n_1+37/8|} & \varepsilon^{|n_1+5|} \\ \varepsilon^{|n_1+37/8|} & \varepsilon^{17/4} & \varepsilon^{37/8} \\ \varepsilon^{|n_1+5|} & \varepsilon^{37/8} & \varepsilon^5 \end{pmatrix}$$

Masses and Mixings

$$t^\nu_{13} = \frac{a^\nu_{13}\varepsilon^{|r'_d+n_3|-|n_3|}}{\sqrt{a^{\nu 2} + a^{\nu 2}_{23}}}, \quad t^\nu_{12} = \frac{a^\nu_{12}\varepsilon^{|r'_d+n_2|-|n_2|}}{(c_{23}a^\nu_{22} - s_{23}a^\nu_{32})}$$

$$a^u = \begin{bmatrix} 0.5 & 0.6e^{-i\pi/2} & 0.5 \\ 0.6e^{-i\pi/2} & e^{-i\pi} & 0.43e^{-i\pi/2} \\ 0.5 & 0.43e^{-i\pi/2} & 1 \end{bmatrix},$$

$$a^d = \begin{bmatrix} 1 & 0.72 & 0.29e^{i0.49} \\ 1.82e^{-i2.28} & 0.76e^{-i1.12} & 0.55e^{-i0.71} \\ e^{-i1.57} & 0.4e^{-i0.41} & e^{-i2.951} \end{bmatrix}.$$

For this fit we have $\chi^2 = 2.10$.

3. Extended $SU(5)$-like case

• $Y^e_{ij} \neq Y^{dT}_{ij} \rightarrow Y^e_{ij} = Y^d_{ji} + f(u,v)$

$$\begin{aligned}
\mathcal{C}(Y^d_{ji}) &= |l_i + e_j + 2e_3 + \frac{7u}{3} + \frac{4v}{3}| \\
\mathcal{C}(Y^e_{ij}) &= |l_i + e_j + 2e_3 + 2u + v|
\end{aligned}$$

$$\frac{7u}{3} + \frac{4v}{3} \sim 2u + v$$

Comparison to a generic $SU(3)$ case

$$Y^f = \begin{bmatrix} \varepsilon_f^8 & \varepsilon_f^3 & \varepsilon_f^3 \\ \varepsilon_f^3 & \varepsilon_f^2 & \varepsilon_f^2 \\ \varepsilon_f^3 & \varepsilon_f^2 & 1 \end{bmatrix},$$

Quark fitted Parameters, SU(3)-like case		
Parameter	BFP Value $\pm\sigma$	BFP Value $\pm\sigma$
$a'u_{22}$	1.11 ± 0.55	1.11 ± 0.07
a^d_{12}	0.66 ± 0.32	2.45 ± 0.20
a^d_{13}	0.10 ± 0.12	0.91 ± 0.15
a^d_{22}	0.74 ± 0.10	1.77 ± 0.09
a^d_{23}	0.45 ± 0.29	1.18 ± 0.12
ϵ^u	0.05 ± 0.007	0.05 ± 0.007
ϵ^d	0.25 ± 0.03	0.16 ± 0.02
$\cos(\Phi_2)$	0.516 ± 0.1	0.450 ± 0.045
Quark Fixed Parameters, SU(3)-like case		
Φ^*_1	$-1.25 \approx = -0.8\pi/2$	$1.120 \approx 0.7\pi/2$
χ^2		
χ^2	0.972	0.974

Criteria of Comparison in Fermion Sector			
	$U(1)$ (GST)	$U(1)$ (Non-GST)	$SU(3)$-like
# of expansion pars.	1	1	2
# of free pars.(quark sector)	12	$>$18	10
GST relation	yes	no	yes
prediction for $\tan\beta$	small	small	no
lepton sector	o.k.	o.k	o.k
simple flavour charges	no	yes	yes

Table 2: Some criteria of comparison. Here the number of free parameters corresponds to the number of coefficients, phases and parameter expansions that need to be adjusted or determined in the fits.

(iv). Constraints and predictions in the supersymmetric sector

SUSY Flavour Problem

Many more flavour structures than in the SM

SUSY CP Problem

Too many phases appearing in the new flavour structures

Flavour structures

⋆At $\mathrm{M}_{\mathrm{U}(1)}$

$$M^2_{\tilde{L}}, \quad M^2_{\tilde{U}}, \quad M^2_{\tilde{D}}, \quad (Y_d^A)_{ij} = A_{ij}Y^d_{ij}, \quad (Y_u^A)_{ij} = A_{ij}Y^u_{ij}$$

$$(Y^l)_{ij}, \quad (Y^u)_{ij}, \quad (Y^d)_{ij}$$

⋆ **Canonical normalisation of fields**

⋆ **Effects of the RGE**

⋆**Low energies SCKM basis**

Quark matrices diagonalised

$$Y^u_{\rm di} = L^{u\dagger} Y^u R^u \qquad Y^d_{\rm di} = L^{d\dagger} Y^d R^d$$

$$(M^2)'_{\tilde{U}} = L^{u\dagger} M^2_{\tilde{U}} R^u \qquad (M^2)'_{\tilde{D}} = L^{d\dagger} M^2_{\tilde{D}} R^d$$

s-Quark matrices rotated

(Analogously for leptons)

Effective supergravity theory

- Superpotential consistent with $U(1)_A$ + extra symmetries

$$Y^f_{ij} = \varepsilon^{(q_i - q_3)+(d_j - d_3)}$$

- Kähler potential

$$K = K_o(T_\alpha, T^{\bar{\alpha}}) - ln(S + \bar{S} - f_{GS}) + \Pi_t \bar{\Phi}\Phi + \sum_{\Phi_i(MSSM)} K_{ij*}\Phi^i\bar{\Phi}^{j*}$$

$$K_{ij} = \delta_{ij}\Pi_t + Z_{ij*}\left[\theta(\phi_i - \phi_j)\Pi_t(n_j, \bar{n}_{\Phi,ij*})\frac{\phi}{M_P}^{\phi_i-\phi_j} + \theta(\phi_i - \phi_j)\Pi_t(n_j, n_{\Phi,ij*})\frac{\bar{\phi}}{M_P}^{\phi_j-\phi_i}\right]$$

$$+ \quad f\left(\frac{\phi'_S}{M'_S}\right), \quad \Pi_t = \Pi^p_{\alpha=1} t_\alpha^{n_\phi^{(\alpha)}}$$

hep-ph/9606383 E. Dudas, C. Grojean and C. Savoy

Could they give a different structure in Y^f than before canonical normalisation?

⋆ For the hierarchical cases and some cases with mild hierarchy there is a change only in $O(1)$ coeffiecients (hep-ph/0407012 S.King, I. Peddie, G.Ross, O. Vives & L.V-S. hep-ph/0405095 J.R. Espinosa & A. Ibarra,)

Could RGE effects destroy the $U(1)$ structure of masses and/or give dangerous FCNC effects?

⋆Only in some degenerate cases (neutrino CP conserving cases) (Lindner et al.)

⋆ Controllable (hep-ph/0307091 S.King and I. Peddie)

Other effects

⋆ If before the $U(1)_A$ is broken

$$K_{ij} = \delta_{ij}\Pi_t$$

then the flavour violation effects would come only from the $U(1)_A$ and other possible extra-symmetries

⋆ If we have trilinear terms, $(Y^A_\beta)_{ij} H_\beta Q_i f^c_j$, such that

$$(\mathbf{Y^A_\beta})_{ij} = Y^\beta_{ij} F^a \partial_a \left(\tilde{K} + \ln(K^\beta_\beta K^i_i K^j_j)\right) + F^a \partial_a \hat{Y}^\beta_{ij} = \underbrace{(\mathbf{A_i} + \mathbf{A_j})\mathbf{Y_{ij}}}_{\text{real}} + \mathbf{F^a} \partial_\mathbf{a} \hat{\mathbf{Y}}^\beta_\mathbf{ij}$$

and

$$\mathbf{W} \neq \mu \mathbf{H_u H_d} \quad \text{but instead} \quad \mathbf{K} = \mu \mathbf{H_u H_d} \quad \underbrace{\mu = \mathbf{O(m_{3/2})}}_{\text{real}}$$

(Giudice-masiero mechanism) then the CP violation effects would come only from the $U(1)_A$ and other possible extra-symmetries

Why the previous assumptions are important?

⋆ If $K_{ij}\delta_{ij}$ before $U(1)_A$ is broken, the moduli fields responsible for these non-proportionality tend to overproduce the contribution to FCNC

⋆ If μ is present in the superpotential, then in order to address this problem we would need to introduce and extra-symmetry

⋆ If we do not have $(\mathbf{Y}^{\mathbf{A}}_{\beta})_{ij} = (A_i + A_j)Y_{ij} + \mathbf{F^a}\partial_{\mathbf{a}}\hat{\mathbf{Y}}^{\beta}_{\mathbf{ij}}$ then also the contributions from the trilinears are excesive in some cases (hep-ph/0307091 S.King and I. Peddie)

⋆
. If we only have mainly the effects of the flavour symmetry

$$\delta^f_{ij} \propto (f_i - f_j)$$

In the cases for ν_R dominance, we have $l_2 = l_3 = d_2 = d_3$, e.g.
$(\delta^d_{LR})^{\rm ind}_{23} = (\delta^d_{LL})_{23}\frac{m_b(A_b - \mu\tan\beta)}{\tilde{m}^2}$
thus $(\delta^d_{LR})^{\rm ind}_{23}$ would only be given by RGE effects

⋆ The only suspicious contribution to the trilinear terms would be $\mathbf{F^a}\partial_{\mathbf{a}}\hat{\mathbf{Y}}^{\beta}_{\mathbf{ij}}$

but (hep-ph/0211279 G.G. Ross and O. Vives) if Y^{β}_{ij} is just function of the flavon field, FCNC ??

•

$$F_\theta \sim m_{3/2}\theta, \to \Delta A_{ij} \sim n_{ij} m_{3/2}$$

(Most relevant for $\mu \to e\gamma$)

• EDM's strongly constraint elements below the diagonal, a large right-handed contribution may be dangerous

Flavon Sector

$$\varepsilon = \frac{<\theta>}{M}, \quad \bar{\varepsilon} = \frac{<\overline{\theta}>}{M}$$

$$\begin{aligned}
W_\theta &= S(\theta\overline{\theta} - M_\theta^2), \\
V &= |\theta\overline{\theta} - M_\theta^2|^2 + \frac{g^2}{2}(|\theta|^2 + |\bar{\theta}|^2 - q_x|X|^2| + \xi^2)^2 + m^2(\theta^2 + \bar{\theta}^2), \quad <X> \neq 0 \\
<\theta> &= \xi\sqrt{1 + \frac{m_\theta^2}{\xi^2}}
\end{aligned}$$

Testing symmetries with $B(\mu \to e\,\gamma)$ & minimal sugra

$$m_0^2 = \frac{1}{4} m_{3/2}^2 \;, \quad A^0 = \sqrt{\frac{3}{4}} m_{3/2} \quad M_{1/2} = \sqrt{\frac{3}{4}} m_{3/2}.$$

1. The Fit 1 has $\mathrm{BR}(\mu \to e\gamma) <= 10^{-30}$ which is unattainably low, thus this fit is plausible within the context of the minimal sugra conditions that have been specified. The smallness of the branching ratio for fit 2 comes about because with no RG running, in mSUGRA this rate would be exactly zero. The RG flavour violation will come from terms proportional to $Y^{\nu\dagger} Y^{\nu}$, whose elements are tiny (the largest is $O(10^{-14})$).

2. Fits 2 produce regions below and above the experimental limits on $B(\mu \to e\gamma)$:

3. Fits 3 not possible in this context although may be realizable in non minimal sugra scenarios

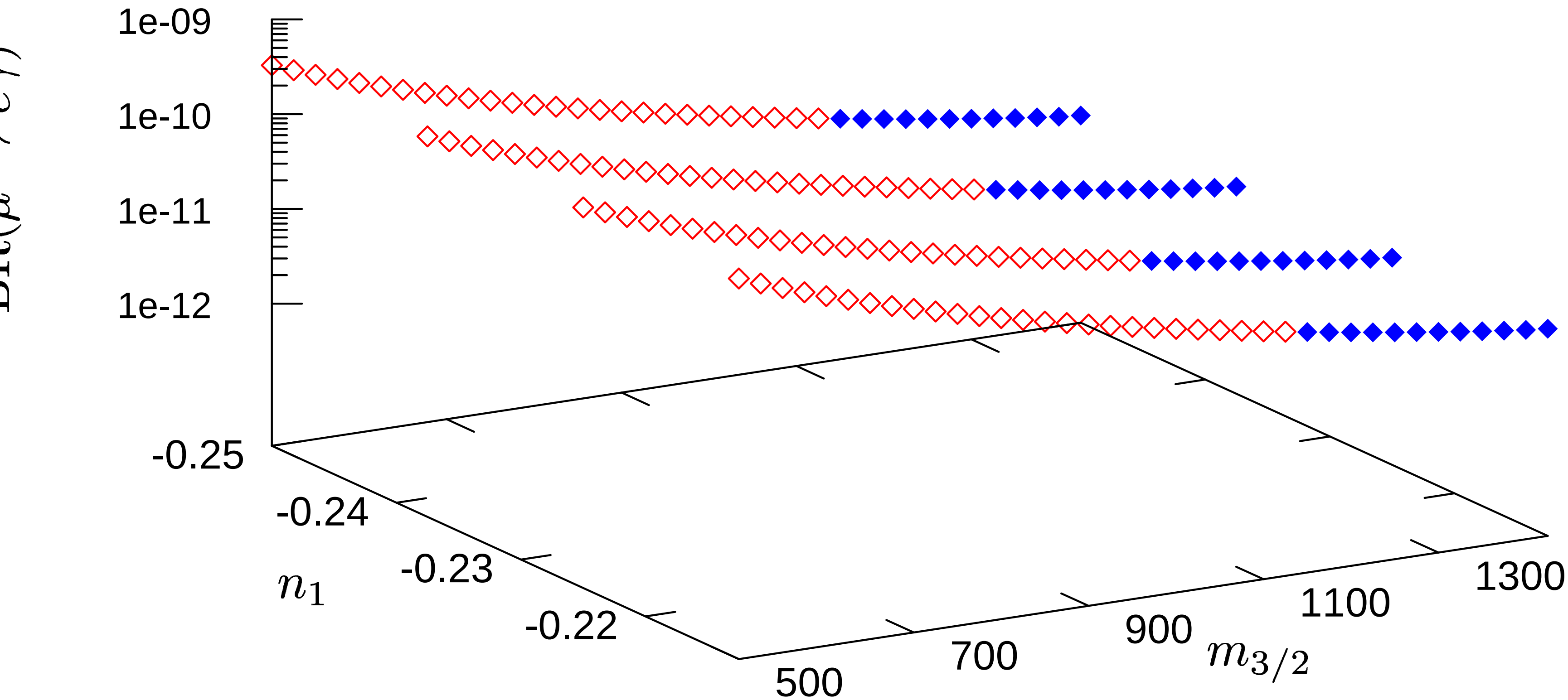

Figure 2: $\mathrm{BR}(\mu \to e\gamma)$ for fit 4, with $\langle\Sigma\rangle = O(M_G)$. The solid points are below the experimental limit of $1.1 \cdot 10^{-11}$, and the hollow points are above.

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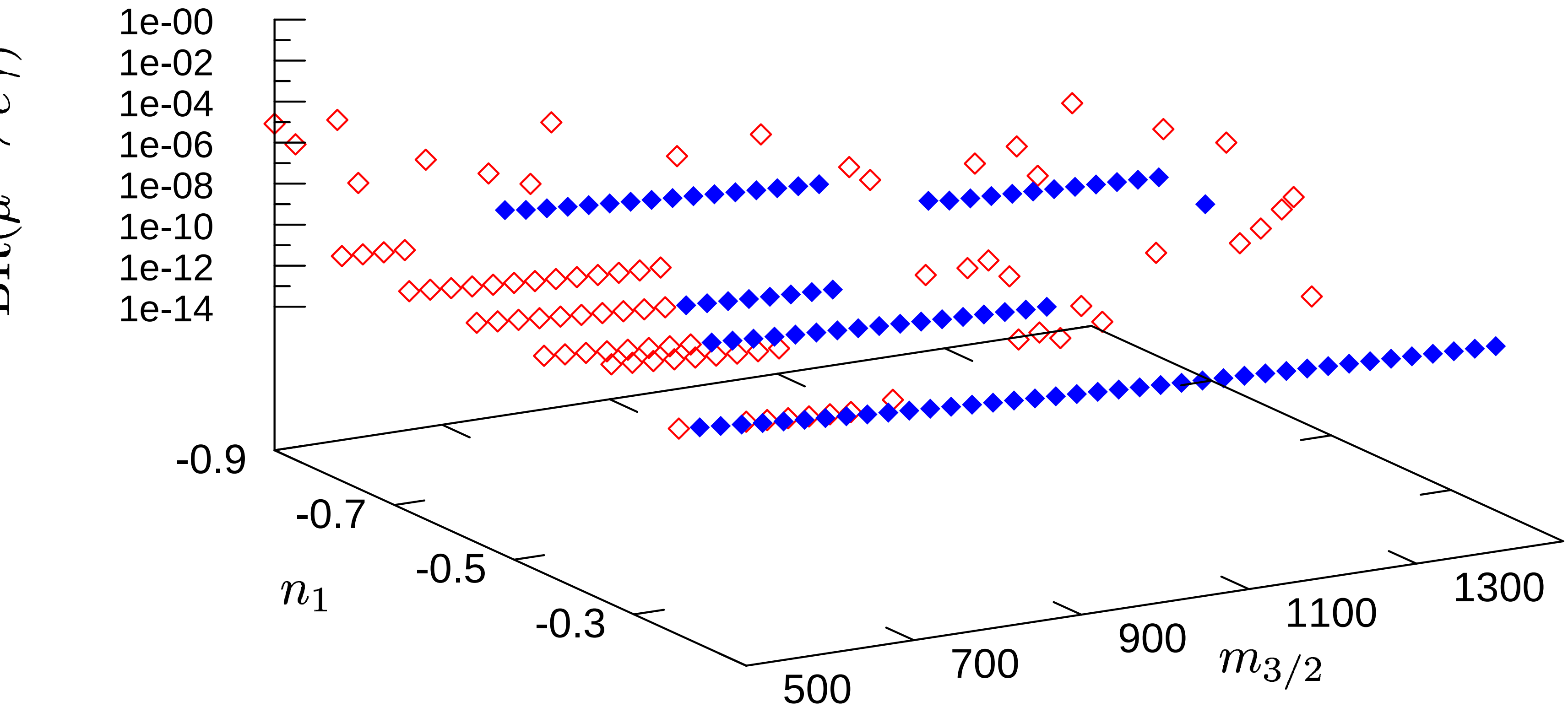

Figure 3: $\mathrm{BR}(\mu \to e\gamma)$ for fit 4, with $\langle\Sigma\rangle = O(M_P)$. The solid points are below the experimental limit of $1.1 \cdot 10^{-11}$, and the hollow points are above.

Summary and Conclusions

- Exhaust $U(1)$ theories that obey our bottom up approach

. GST relation (ckm elements to mass ratios), no complicated charges ($\rightarrow$ non GST
. solutions), minimal content of flavon fields (2), etc.

- Establish criteria of comparison between these models and non-Abelian models (e.g. $SU(3)$)

. At the level of precision in the quark sector is important to fit accurately masses
. and mixings, not just assume $O(1)$ coefficients are possible

. Predictions in the neutrino sector? Need to assume see-saw mechanism for example

- Should the flavour theory be non-Abelian?

. We cannot conclude anything just considering the fermion sector, more precision in
. neutrino experimental information may help to further discern.

- Need to make assumptions in the supersymmetric sector and test the models

. $B(\mu \rightarrow e\,\gamma)$ & specific point in minimal SUGRA already can rule out some
. models, which however may be still realizable in other contexts