Lepton flavor violation as a probe of quark-lepton unification

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Current Oscillation Data From LP 2005

Atmospheric neutrino (best fit in physical region from SK and K2K):

$$\Delta m^2_{\rm atm} = 2.5 \times 10^{-3}\ {\rm eV}^2, \quad \sin^2 2\theta_{\rm atm} = 1.0 \quad (> 0.93 \text{ at } 90\% \text{ CL})$$

Solar neutrino (solar + KamLAND):

$$\Delta m^2_{\rm sol} = 8.0^{+0.6}_{-0.4} \times 10^{-5}\,{\rm eV}^2, \quad \tan^2\theta_{\rm sol} = 0.45^{+0.09}_{-0.07}$$

Reactor ν (CHOOZ):

$$\sin^2 2\theta_{13} < 0.2\ (90\%\, CL)$$

Neutrino Mixing Matrix (PMNS)

$$U_{\rm PMNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & e^{-i\delta_{CP}}\sin\theta_{13} \\ 0 & 1 & 0 \\ -e^{i\delta_{CP}}\sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix}$$

$$\times \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha/2} & 0 \\ 0 & 0 & e^{i\beta/2} \end{pmatrix}$$

$\theta_{23} \simeq 45^\circ$ $\theta_{13} = ?$ $\theta_{12} \simeq 33.8^{\circ\,+2.5^\circ}_{\;\;-2.2^\circ}$

CKM matrices

In Wolfenstein convention, and expanded in $\lambda \approx 0.22$

$$\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(1+\lambda^2/)(\bar\rho - i\bar\eta) \\ -\lambda - iA^2\lambda^5\bar\eta & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\bar\rho-\bar\eta) & -A\lambda^2 - iA\lambda^4\bar\eta & 1 \end{pmatrix}$$

$$\theta_C^{\rm CKM} \approx 12.7^\circ$$

Motivations for Quark Lepton Complimentarity

The solar ν angle and the Cabbibo angle add to $\pi/4$

$$\theta_{\rm sol}^{PMNS} + \theta_C^{\rm CKM} = 46.5^{\circ}{}^{+2.5^{\circ}}_{-2.2^{\circ}}$$

$$(\theta_{\rm sol}^{PMNS} + \theta_C^{\rm CKM} = 45.3^{\circ} \pm 1.6^{\circ}) \quad \text{(Raidal PRL 2004)}$$

Other observations

$$\theta_{\rm atm} + \theta_{23}^{\rm CKM} = \frac{\pi}{4}$$

$$\theta_{13}^{\rm MNS} \sim \theta_{13}^{\rm CKM} \sim O(\lambda^3)$$

These unrelated relations between the leptonic and quark sectors draw some speculation that the lepton and quark sectors may be unified at some scales.

Outline

- Relate the $U_{\rm PMNS}$ matrix to the CKM matrix
- How it leads to Lepton Flavor Violation (LFV)
- Predictions

Possible forms of PMNS mixing matrix

The PMNS mixing matrix can be described by a small perturbation with ($\lambda = \sin\theta_C$) from the bimaximal mixing matrix.

$$U_{\rm PMNS} = \begin{cases} U^\dagger(\lambda)\, U_{\rm bimax} \\ U_{\rm bimax}\, U^\dagger(\lambda) \\ U_{23}^m(\lambda)\, U^\dagger(\lambda)\, U_{12}^m \end{cases}$$

where

$$U_{23}^m = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad U_{12}^m = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

We show that $U^\dagger(\lambda)$ can be related to $U^\dagger_{\rm CKM}$ in GUT scenarios.

Case (1) $U_{\rm PMNS} = U^\dagger(\lambda)\, U_{\rm bimax}$

In see-saw scenario with heavy RH neutrinos

$$W_{\rm lepton} = Y_l \widehat{L} \widehat{l}^c_L \widehat{H}_d + Y_\nu \widehat{L} \widehat{N}^c_L \widehat{H}_u - \frac{1}{2} \widehat{N}^{cT}_L M_R \widehat{N}^c_L$$

Suppose Y_l and Y_ν can parametrized as

$$Y_l = U_l Y_l^{diag} V_l^\dagger, \quad Y_\nu = U_0 Y_\nu^{diag} V_0^\dagger$$

The see-saw generates the neutrino mass matrix

$$M_\nu \sim (Y_\nu L H)(Y_\nu L H)/M_R = \left(U_0\, Y_\nu^{diag} \frac{v_u}{\sqrt{2}}\, V_0^\dagger \right)\; M_R^{-1}\; \left(V_0^*\, Y_\nu^{diag} \frac{v_u}{\sqrt{2}}\, U_0^T \right)$$

Write

$$M_\nu = U_0\, \left(M^{diag}_{dirac}\, V_0^\dagger\, M_R^{-1}\, V_0^*\, M^{diag}_{dirac} \right)\, U_0^T$$

where

$$M^{diag}_{dirac} = Y_\nu^{diag} \frac{v_u}{\sqrt{2}}$$

Suppose V_M diagonalizes $\left(M_{dirac}^{diag}\, V_0^\dagger\, M_R^{-1}\, V_0^*\, M_{dirac}^{diag}\right)$, we can write

$$M_\nu = U_0\, V_M\, M_\nu^{diag}\, V_M^T\, U_0^T = U_\nu\, M_\nu^{diag}\, U_\nu^T$$

Therefore, the PMNS matrix

$$U_{\rm PMNS} = U_l^\dagger\, U_\nu = U_l^\dagger U_0 V_M$$

Impose quark-lepton unification in the simple SU(5):

$$Y_e = Y_d^T \quad \Rightarrow \quad U_l Y_l^{diag} V_l^\dagger = (U_d Y_d^{diag} V_d^\dagger)^T \quad \Rightarrow \quad U_l^\dagger = V_d^T$$

Then we can write

$$U_{\rm PMNS} = V_d^T\, U_0\, V_M$$

We can involve the CKM mixing matrix by relating the Dirac neutrino Yukawa coupling to the u-type Yukawa coupling

$$SO(10): \qquad Y_\nu = Y_u \qquad \Rightarrow \qquad U_0\, Y_\nu^{diag}\, V_0^\dagger = U_u\, Y_u^{diag}\, V_u^\dagger$$

Therefore,

$$U_{\rm PMNS} = V_d^T\, U_u\, V_M = V_d^T\, U_d\, U_{\rm CKM}^\dagger\, V_M$$

If we further assumer the d-type Yukawa to be symmetric, then $U_d = V_d$, and

$$U_{\rm PMNS} = U_{\rm CKM}^\dagger\, V_M$$

Here V_M represents the bimaximal mixing matrix. Thus, we derive the first form:

$$U_{\rm PMNS} = U_{\rm CKM}^\dagger\, U_{\rm bimax}$$

Other Forms (2) and (3)

$$U_{\mathrm{PMNS}} = V_d^T\, U_d\, U_{\mathrm{CKM}}^\dagger\, V_M$$

(2) $U_{\mathrm{PMNS}} = U_{\mathrm{bimax}}\, U_{\mathrm{CKM}}^\dagger$:
Take

$$V_d^T\, U_d = U_{\mathrm{bimax}}\,, \qquad\qquad V_M = I$$

(3) $U_{\mathrm{PMNS}} = U_{23}^m\, U_{\mathrm{CKM}}^\dagger\, U_{12}^m$:
Take

$$V_d^T\, U_d = U_{23}^m\,, \qquad\qquad V_M = U_{12}^m$$

In these ways, the U_{PMNS} is related to U_{CKM} through quark-lepton unification.

How it leads to LFV

Embed into the supersymmetric GUT scenarios.

Renormalization group running from the GUT scale to the RH neutrino scale induces off-diagonal terms in slepton mass matrix:

$$m^2_{\tilde{l}_{ij}} \simeq -\frac{1}{8\pi^2}(3m_0^2 + A_0^2)(Y'_\nu Y'^\dagger_\nu)_{ij} \log\frac{M_G}{M_X},$$

m_0, A_0 are universal soft mass and A parameter.

Y'_ν is the Dirac neutrino Yukawa matrix in the basis that the charged leptons and RH neutrinos are real and diagonal.

- In this basis:

$$Y'_\nu\, Y'^\dagger_\nu = \left(U_0 Y^{\rm diag}_\nu V_0^\dagger\right)\left(V_0 Y^{\rm diag}_\nu U_0^\dagger\right)$$

where $Y^{\rm diag}_\nu$ is the diagonalized Dirac neutrino Yukawa matrix.

In the basis that the charged leptons and RH neutrinos are real and diagonal.

$$U_{\rm PMNS} = U_l^\dagger \, U_\nu = U_l^\dagger \, U_0 \, V_M = U_0 \, V_M \,; \qquad U_{\rm PMNS} = U_{\rm CKM}^\dagger \, V_M$$

Therefore

$$U_0 = U_{\rm CKM}^\dagger$$

We can then write

$$\begin{aligned} Y'_\nu \, Y'^\dagger_\nu &= \left(U_0 Y_\nu^{\rm diag} V_0^\dagger \right) \left(V_0^\dagger Y_\nu^{\rm diag} U_0 \right) \\ &= U_{\rm CKM}^\dagger \left(Y_\nu^{\rm diag} \right)^2 U_{\rm CKM} \end{aligned}$$

LFV prediction for $l_i \to l_j \gamma$ in Case (1)

The one-loop contribution to the BR of the LFV decay $l_i \to l_j \gamma$ is

$$B(l_i \to l_j \gamma) \simeq \frac{\alpha^2}{G_F^2} \tan^2 \beta \left| \frac{m^2_{\tilde{l}_{ij}}}{M^4_{\rm susy}} \right|^2$$

Therefore,

$$B(l_i \to l_j \gamma) \sim \left(Y'_\nu \, Y'^\dagger_\nu \right)^2 = \left(U^\dagger_{\rm CKM} \left(Y^{\rm diag}_\nu \right)^2 U_{\rm CKM} \right)^2$$

Assume some hierachical form of

$$Y_\nu^{\rm diag} \equiv Y_3 \begin{pmatrix} \lambda^{n_1} & & \\ & \lambda^{n_2} & \\ & & 1 \end{pmatrix} .$$

For quark-lepton unification $Y_3 = m_t/v_u$ and $n_1 = 8, n_2 = 4$.

The term $(Y'_\nu Y'^\dagger_\nu)$ is given to leading order as

$$Y'_\nu Y'^\dagger_\nu \sim \left(\frac{m_t}{v_u}\right)^2 \times \begin{pmatrix} \lambda^{2n_1} + \lambda^{2n_2+2} + \lambda^6 & \lambda^{2n_1+1} - \lambda^{2n_2+1} - \lambda^5 & \lambda^3 \\ \lambda^{2n_1+1} - \lambda^{2n_2+1} - \lambda^5 & \lambda^{2n_1+2} + \lambda^{2n_2} + \lambda^4 & -\lambda^2 \\ \lambda^3 & -\lambda^2 & 1 \end{pmatrix}$$

In the case $n_1, n_2 > 2$,

$$\begin{aligned} B(\mu \to e\gamma) : B(\tau \to e\gamma) : B(\tau \to \mu\gamma) &\simeq (-\lambda^{2n_1-1} + \lambda^{2n_2-1} + \lambda^3)^2 : \lambda^2 : 1 \\ &\simeq \lambda^6 \; : \; \lambda^2 \; : \; 1 \end{aligned}$$

Realistic Quark-Lepton Unification

So far the relation $Y_e = Y_d^T$ is not realistic. From the well-known relation

$$|V_{us}| \simeq \sqrt{\frac{m_d}{m_s}} \simeq 3\sqrt{\frac{m_e}{m_\mu}}$$

the $U(\lambda)$ should have the same form as $U_{\rm CKM}$ but with

$$\lambda \longrightarrow \lambda/3$$

which can be obtained by introducing the Higgs sector transforming under the representation **45** of $SU(5)$ or **126** of $SO(10)$.

$$
\begin{aligned}
Y'_\nu \, Y'^\dagger_\nu &= U^\dagger_{\rm CKM} \left(Y^{\rm diag}_\nu\right)^2 U_{\rm CKM} \\
(Y'_\nu Y'^\dagger_\nu)_{21} &\simeq \frac{\lambda^5}{6} + \frac{\lambda^{1+2n_1}}{3} - \frac{\lambda^{1+2n_2}}{3} \\
(Y'_\nu Y'^\dagger_\nu)_{31} &\simeq \frac{\lambda^3}{6} - \frac{\lambda^{3+2n_1}}{2} + \frac{\lambda^{3+2n_2}}{3} \\
(Y'_\nu Y'^\dagger_\nu)_{32} &\simeq \lambda^2 - \frac{\lambda^{4+2n_1}}{6} - \lambda^{2+2n_2}
\end{aligned}
$$

Then the prediction for LFV:

$$B(\mu \to e\gamma) : B(\tau \to e\gamma) : B(\tau \to \mu\gamma) \simeq (\lambda^{2n_1} - \lambda^{2n_2} + \lambda^4)^2 : \lambda^4 : 1$$

$$\simeq \lambda^8 : \lambda^4 : 1 \qquad \text{for } n_1, n_2 > 2$$

LFV prediction for $l_i \to l_j \gamma$ for Case (2)

$U_{\rm PMNS} = U_{\rm bimax}\, U^\dagger_{\rm CKM}$:

Take

$$V_d^T\, U_d = U_{\rm bimax}\,, \qquad\qquad V_M = I$$

So

$$U_{\rm PMNS} = U_l^\dagger\, U_0\, V_M = U_0 = U_{\rm bimax}\, U^\dagger_{\rm CKM}$$

Therefore,

$$Y'_\nu\, Y'^\dagger_\nu = U_{\rm bimax}\, U^\dagger_{\rm CKM} \left(Y_\nu^{\rm diag}\right)^2 U_{\rm CKM}\, U^\dagger_{\rm bimax}$$

and prediction:

$$B(\mu \to e\gamma) : B(\tau \to e\gamma) : B(\tau \to \mu\gamma) \simeq \lambda^4 \;:\; \lambda^4 \;:\; 1$$

LFV prediction for $l_i \to l_j \gamma$ for Case (3)

$U_{\rm PMNS} = U_{23}^m \, U_{\rm CKM}^\dagger \, U_{12}^m$:

Take

$$V_d^T \, U_d = U_{23}^m \, , \qquad\qquad V_M = U_{12}^m$$

So

$$U_{\rm PMNS} = U_l^\dagger \, U_0 \, V_M = U_0 U_{12}^m$$
$$\Rightarrow U_0 = U_{23}^m U_{\rm CKM}^\dagger$$

Therefore,

$$Y_\nu' \, Y_\nu'^\dagger = U_{23}^m \, U_{\rm CKM}^\dagger \left(Y_\nu^{\rm diag} \right)^2 U_{\rm CKM} \, U_{23}^{m\dagger}$$

and prediction:

$$B(\mu \to e\gamma) : B(\tau \to e\gamma) : B(\tau \to \mu\gamma) \simeq \lambda^6 \;:\; \lambda^6 \;:\; 1$$

Experimental Tests

Note that in various scenarios, $B(\mu \to e\gamma)$ is the smallest. Currently,

$$\begin{aligned} B(\mu- > e\gamma) &< 1.2 \times 10^{-11} \\ B(\tau \to e\gamma) &< 2.7 \times 10^{-6} \\ B(\tau \to \mu\gamma) &< 1.1 \times 10^{-6} \end{aligned}$$

Suppose $\mu \to e\gamma$ would be observed soon, say

$$B(\mu \to e\gamma) \simeq 10^{-12} - 10^{-11}$$

The predictions for cases (2) and (3) give

$$\begin{aligned} B(\tau \to e\gamma) &< 10^{-12} - 10^{-11} \\ B(\tau \to \mu\gamma) &< 4 \times 10^{-10} - 9 \times 10^{-8} \end{aligned}$$

But both are still 3 orders of magnitude below the current limit. Hopeless!

Case (1) predicts

$$
\begin{aligned}
B(\tau \to e\gamma) &< 4 \times 10^{-10} - 4 \times 10^{-9} \\
B(\tau \to \mu\gamma) &< 2 \times 10^{-7} - 2 \times 10^{-6}
\end{aligned}
$$

$\tau \to e\gamma$ is still hopeless. But $\tau \to \mu\gamma$ is right at the order of the current limit.

• Therefore, if $\tau \to \mu\gamma$ is observed with $\mu \to e\gamma$ in near future, the quark-lepton unification scenarios (2) and (3) can be ruled out.

• If $\mu \to e\gamma$ is observed but not $\tau \to \mu\gamma$, then the scenario (1) may be ruled out.

Conclusions

- $\theta_{\rm sol} + \theta_C = \pi/4$ may just be an accident, otherwise it may be realized in quark-lepton unification scenarios.
- $U_{\rm PMNS}$ can be linked to $U_{\rm CKM}$ by expressing $U_{\rm PMNS}$ as a perturbation (in $\lambda = \sin\theta_C$) deviating from the $U_{\rm bimax}$. We studied 3 different parameterizations.
- When embedded into the SUSY GUT framework, LFV in slepton mass matrix can be induced by RG running. The 3 parameterizations of $U_{\rm PMNS}$ give different LFV.
- Among the 3 cases that we considered, the one

$$U_{\rm PMNS} = U^\dagger_{\rm CKM}\, U_{\rm bimax}$$

predicts testable $B(\mu \to e\gamma)$ and $B(\tau \to \mu\gamma)$ in near future.