High precision sparticle spectra and anomaly mediation

With Tim Jones, Richard Hodgson and Graham Ross

SUSY05, Durham

Outline

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Introduction

- Increasing precision of sparticle spectrum calculations is an important part of theoretical preparation for the LHC and the ILC.

- MSUGRA scenario (unified scalar, gaugino masses etc) not founded on compelling underlying theory.

- Here we focus on Anomaly Mediated Supersymmetry Breaking (AMSB)

- Single mass parameter determines soft parameters as renormalisation group (RG) invariant functions of the dimensionless couplings.

- Flavour-blind, FCNCs suppressed

- AMSB in its simplest form leads to tachyonic sleptons.

- Introduction of a Fayet-Iliopoulos (FI) D-term is a natural solution which retains the RG invariance (and hence the ultra-violet insensitivity) of the predictions.

- Here we present the most precise spectrum calculations to date in the AMSB scenario.

- Also show how the low energy theory employed can arise in a natural way from a theory with an additional anomaly-free U_1 broken at a high scale.

AMSB:

[Randall, Sundrum; Giudice, Luty, Murayama, Rattazzi; Kobayashi, Kubo, Zoupanos; Pomarol, Rattazzi; Ghergetta, Giudice, Wells; Luty, Rattazzi; Chacko, Luty, Maksymyk, Ponton; Katz, Shadmi, Shirman; Feng, Moroi; Kribs; Su; Bagger, Moroi, Poppitz; Rattazzi, Strumia, Wells; Paige, Wells; Allanach, Dedes]

D-term:

Pomarol, Rattazzi; IJ, Jones; Carena, Huitu, Kobayashi; Arkani-Hamed, Kaplan, Murayama and Nomura; Murakami, Wells; Kitano, Kribs and Murayama; Ibe, Kitano and Murayama

The Minimal Supersymmetric Standard Model

The MSSM is defined by the superpotential:

$$W = H_2 Q Y_t t^c + H_1 Q Y_b b^c + H_1 L Y_\tau \tau^c + \mu H_1 H_2$$

with soft breaking terms:

$$\begin{aligned} L_{\mathrm{SOFT}} &= \sum_\phi m_\phi^2 \phi^* \phi \\ &+ \left[m_3^2 H_1 H_2 + \sum_{i=1}^{3} \frac{1}{2} M_i \lambda_i \lambda_i + \mathrm{h.c.} \right] \\ &+ \left[H_2 Q h_t t^c + H_1 Q h_b b^c + H_1 L h_\tau \tau^c + \mathrm{h.c.} \right] \end{aligned}$$

where in general $Y_{t,b,\tau}$ and $h_{t,b,\tau}$ are 3×3 matrices. We work throughout in the approximation that the Yukawa matrices are diagonal, and neglect the Yukawa couplings of the first two generations.

The AMSB Solution

Remarkably the following results are RG invariant:

$$
\begin{aligned}
M &= m_0 \beta_g / g \\
h &= -m_0 \beta_Y \\
(\overline{m}^2)^i{}_j &= \frac{1}{2} m_0^2 \mu \frac{d}{d\mu} \gamma^i{}_j + k(Y)^i{}_j = (m^2)^i{}_j + k(Y)^i{}_j \\
m_3^2 &= -m_0 \beta_\mu
\end{aligned}
$$

- k is a constant, Y is a set of charges corresponding to a U_1 symmetry of the theory with no mixed anomalies with the gauge group. (I.e. $\mathrm{Tr}[C(R)Y] = 0$, $C(R) = R^a R^a$.)

- The kY term corresponds in form to a FI D-term.

- The m_0-dependent parts of the expressions for M, h and m^2 are obtained if the only source of breaking is a vev in the supergravity multiplet itself: the AMSB scenario (m_0 is then the gravitino mass).

Anomaly-free U_1 symmetries

The MSSM admits two independent generation-blind mixed-anomaly-free U_1 symmetries. Possible charge assignments:

Q	u^c	d^c	H_1	H_2
$-\frac{1}{3}L$	$-e-\frac{2}{3}L$	$e+\frac{4}{3}L$	$-e-L$	$e+L$

- The SM gauged U_1^{SM} is $L=1, e=-2$.

- An anomaly-free, flavour-blind U_1' implies equal and opposite charges for $H_{1,2}$ and hence an allowed Higgs μ-term.

- One of the attractive features of AMSB is that squark/slepton mediated flavour changing neutral currents are naturally small; this feature is preserved by a flavour-blind U_1'.

Spontaneously broken U_1'

With the MSSM augmented by an additional U_1', it is natural to ask at what scale this U_1' is broken. We concentrate on the idea that it is broken at very high energies and that the only low energy remnant of it is the set of FI-type terms that we require.

Natural to think that if a U_1' is broken at some high scale M then, by the decoupling theorem, all its effects would be suppressed at energies $E << M$ by powers of $1/M$. Shall see that with a FI term this is not the case and can be $O(M_{\text{SUSY}})$ scalar mass contributions from FI term.

Toy model with gauged U_1' whose only effect on the low-energy theory is the appearance of required FI terms:

Introduce fields $\phi, \overline{\phi}$ with opposite charges and singlet s, with a superpotential $W = \lambda_1 \phi \overline{\phi} s$. The scalar potential takes the form:

$$V = m_\phi^2(\phi^* \phi + \overline{\phi}^* \overline{\phi}) + \cdots$$

$$+ \quad \frac{1}{2}\left[\xi - q_\phi(\phi^*\phi - \overline{\phi}^*\overline{\phi}) - \sum_{\text{matter}} e_i\chi_i^*\chi_i\right]^2 + \cdots ..$$

where χ_i stands for all the MSSM scalars, and we have introduced a FI term for U_1'. Take

$$\xi > 0, \quad q_\phi > 0, \quad \xi >> m_0^2.$$

Scalar masses are anomaly-mediation contributions. Include U_1' contributions in the anomalous dimensions of the fields.

$$\lambda_1 < g' \Rightarrow m_\phi^2 < 0.$$

Minimum with only $\langle\phi\rangle$ nonzero:

$$\langle\phi^*\phi\rangle = \frac{1}{2}v_\phi^2 = \frac{q_\phi\xi - m_\phi^2}{q_\phi^2} \tag{1}$$

so $\langle\phi\rangle = O(\sqrt{\xi})$ for large ξ and $V \sim m_\phi^2\xi/q_\phi$. Expanding about the minimum, ie with $\phi = (v_\phi + H)/\sqrt{2}$, we find

$$V = \frac{m_\phi^2\xi}{q_\phi} - \frac{m_\phi^4}{2q_\phi^2} + (m_\phi^2 + m_{\overline{\phi}}^2 + \frac{1}{2}v_\phi^2\lambda_1^2)\overline{\phi}^*\overline{\phi} - \frac{e_i}{q_\phi}m_\phi^2\chi_i^*\chi_i$$

$$+ \quad \frac{1}{2}v_\phi^2\lambda_1^2 s^* s + \frac{1}{2}\left(v_\phi q_\phi H - q_\phi \overline{\phi}^* \overline{\phi} + e_i \chi_i^* \chi_i\right)^2 \cdots$$

- For large ξ (i.e. large v_ϕ) all trace of the U_1' disappears except for $O(m_\phi^2)$ contributions to the scalar masses.

- Can see this by treating the heavy H-field as non-propagating and eliminating it via its equation of motion.

- In the large ξ limit the breaking of U_1' preserves supersymmetry; thus the U_1' gauge boson, its gaugino, ψ_H and H form a massive supermultiplet which decouples from the theory.

- The fact that supersymmetry is good at large ξ protects the light χ fields from obtaining masses of $O(\sqrt{\xi})$ from loop corrections.

- Now pick charges $L, e > 0$ so that the contributions to their slepton masses are positive.

- It is easy to show that (modulo electroweak breaking) this represents the absolute minimum of the potential.

- λ_1 plays a crucial role here in that for $\lambda_1 = 0$ the D-flat direction $\langle \phi \rangle = \langle \overline{\phi} \rangle >> \sqrt{\xi}$ would lead to an potential unbounded from below.

The sparticle spectrum

We turn now to the effective low energy theory. Have decoupling of the U_1' at low energies so that the anomalous dimensions of the fields are as in the MSSM; thus for Higgses and 3rd generation matter fields we have (at one loop):

$$
\begin{aligned}
16\pi^2\gamma_{H_1} &= 3\lambda_b^2 + \lambda_\tau^2 - \tfrac{3}{2}g_2^2 - \tfrac{3}{10}g_1^2, \\
16\pi^2\gamma_{H_2} &= 3\lambda_t^2 - \tfrac{3}{2}g_2^2 - \tfrac{3}{10}g_1^2, \\
16\pi^2\gamma_L &= \lambda_\tau^2 - \tfrac{3}{2}g_2^2 - \tfrac{3}{10}g_1^2, \\
16\pi^2\gamma_Q &= \lambda_b^2 + \lambda_t^2 - \tfrac{8}{3}g_3^2 - \tfrac{3}{2}g_2^2 - \tfrac{1}{30}g_1^2, \\
16\pi^2\gamma_{t^c} &= 2\lambda_t^2 - \tfrac{8}{3}g_3^2 - \tfrac{8}{15}g_1^2, \\
16\pi^2\gamma_{b^c} &= 2\lambda_b^2 - \tfrac{8}{3}g_3^2 - \tfrac{2}{15}g_1^2, \\
16\pi^2\gamma_{\tau^c} &= 2\lambda_\tau^2 - \tfrac{6}{5}g_1^2,
\end{aligned}
$$

where $\lambda_{t,b,\tau}$ are the third generation Yukawa couplings. For the first two generations we use the same expressions but without the Yukawa contributions.

The soft scalar masses are given by

$$
\begin{aligned}
\overline{m}_Q^2 &= m_Q^2 - \tfrac{1}{3}L\xi' \quad \overline{m}_{t^c}^2 = m_{t^c}^2 - (\tfrac{2}{3}L + e)\xi', \\
\overline{m}_{b^c}^2 &= m_{b^c}^2 + (\tfrac{4}{3}L + e)\xi', \quad \overline{m}_L^2 = m_L^2 + L\xi', \\
\overline{m}_{\tau^c}^2 &= m_{\tau^c}^2 + e\xi',
\end{aligned}
$$

where

$$
m_Q^2 = \tfrac{1}{2}|m_0|^2 \mu \frac{d}{d\mu}\gamma_Q = \tfrac{1}{2}|m_0|^2 \beta_i \frac{\partial}{\partial\lambda_i}\gamma_Q \tag{2}
$$

and so on, and we have written the effective FI parameter as

$$
\xi' = -\frac{m_\phi^2}{q_\phi}.
$$

The 3rd generation A-parameters are given by

$$
\begin{aligned}
A_t &= -m_0(\gamma_Q + \gamma_{t^c} + \gamma_{H_2}), \\
A_b &= -m_0(\gamma_Q + \gamma_{b^c} + \gamma_{H_1}), \\
A_\tau &= -m_0(\gamma_L + \gamma_{\tau^c} + \gamma_{H_1})
\end{aligned}
$$

and we set the corresponding first and second generation quantities to zero. The gaugino masses are given by

$$M_i = m_0 |\frac{\beta_{g_i}}{g_i}|.$$

Without loss of generality set $\xi' = 1(\text{TeV})^2$.

- Choose input values for m_0, $\tan\beta$, L, e and signμ

- Calculate the appropriate dimensionless coupling input values at the scale M_Z by an iterative procedure involving the sparticle spectrum, and the loop corrections to $\alpha_{1\ldots 3}$, m_t, m_b and m_τ.

- Determine a given sparticle mass at its own scale by running the dimensionless couplings up and then using AMSB results including full one-loop corrections. Bagger, Matchev, Pierce, Zhang

- We will present results for $m_0 = 40\text{TeV}$, for which value the gluino mass is around 900GeV.

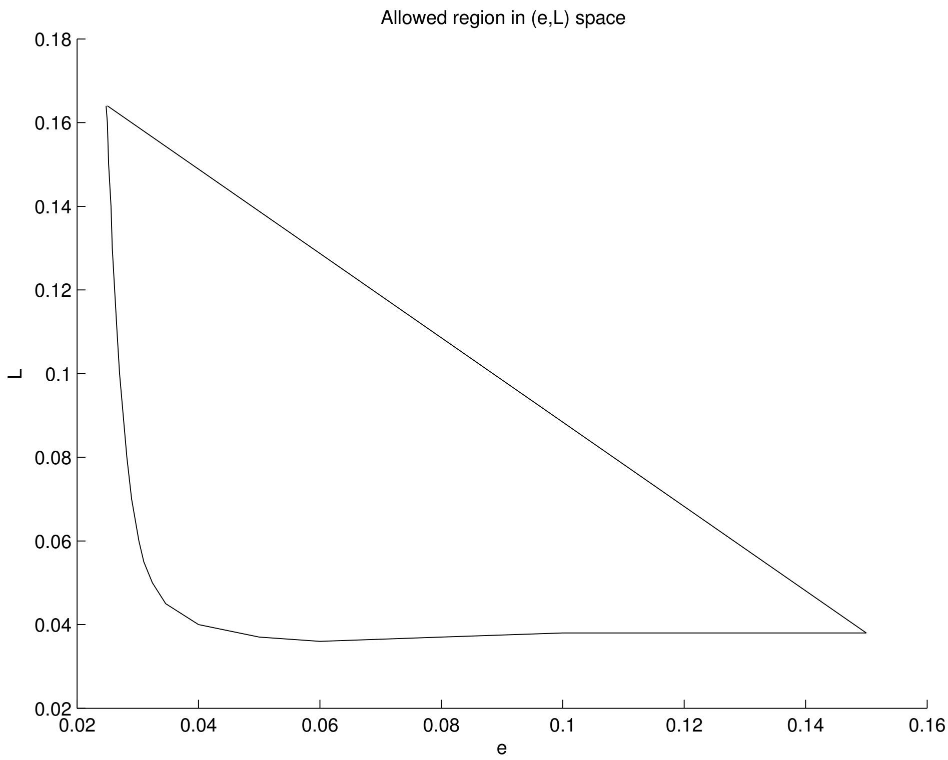


Fig. 1: The region of (e, L) space corresponding to an acceptable electroweak vacuum, for $m_0 = 40\,TeV$ and $\tan\beta = 10$.

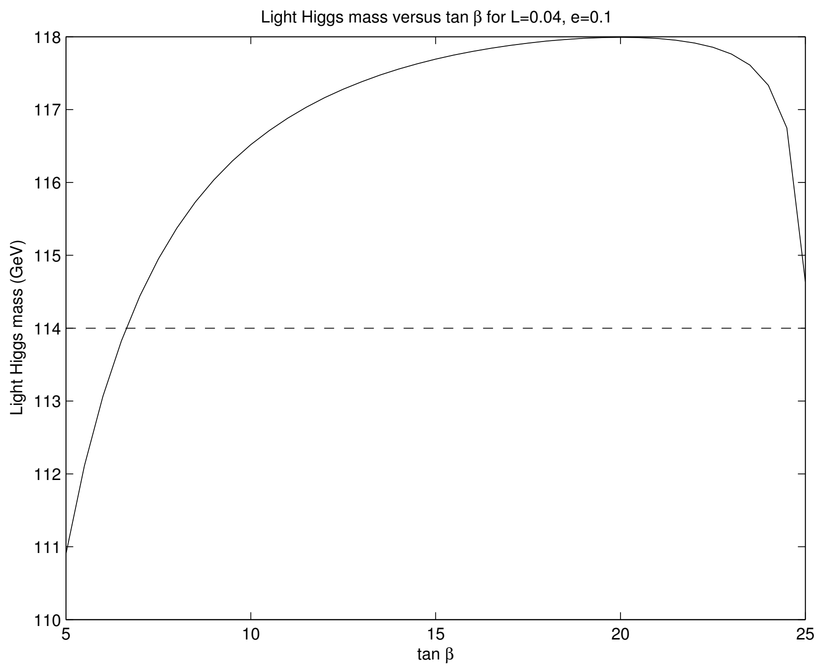
mass (GeV)	1loop	2loops	3loops
$\tilde{g}$	914	891	888
$\tilde{t}_1$	543	540	529
$\tilde{t}_2$	766	757	747
$\tilde{u}_L$	768	758	746
$\tilde{u}_R$	826	812	801
$\tilde{b}_1$	728	719	709
$\tilde{b}_2$	940	932	923
$\tilde{d}_L$	830	816	805
$\tilde{d}_R$	949	941	932
$\tilde{\tau}_1$	211	207	208
$\tilde{\tau}_2$	251	247	247
$\tilde{e}_L$	228	228	228
$\tilde{e}_R$	241	235	235
$\tilde{\nu}_e$	227	220	220
$\tilde{\nu}_\tau$	225	217	218
χ_1	105.677	129.499	129.618
χ_2	353	361	360
χ_3	530	554	545
χ_4	543	566	558
$\chi_1^\pm$	105.917	129.749	129.871
$\chi_2^\pm$	539	563	554
h	117	117	117
H	276	318	301
A	275	317	301
$H^\pm$	288	328	312
$\chi_1^\pm - \chi_1$ (MeV)	240	250	250

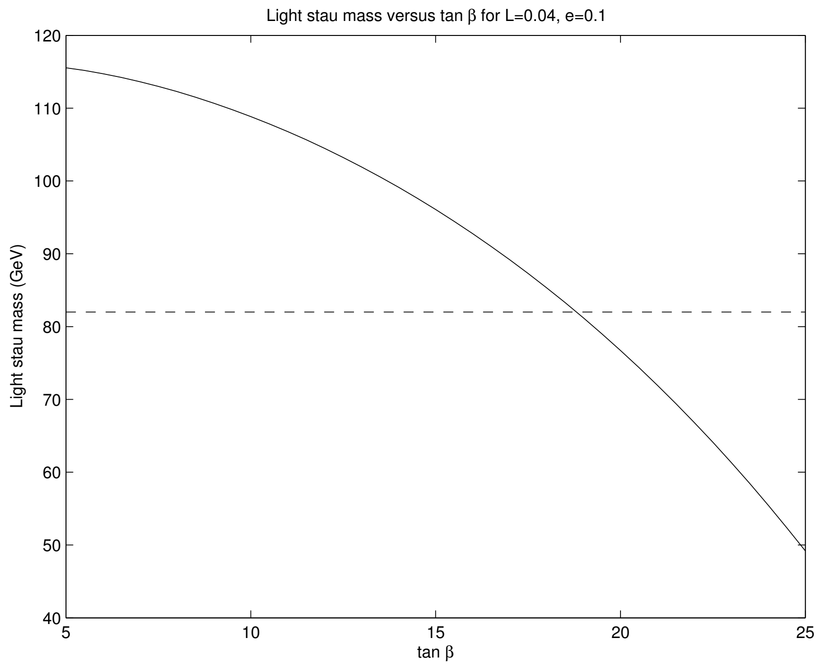
Table 1: Mass spectrum for $m_0 = 40$TeV, $\tan\beta = 10$,

- Large 3-loop effects for squarks

- Characteristic feature of AMSB: $M_2 < M_1$, where $M_{1,2}$ are the bino and wino masses respectively. As a result the lightest neutralino (often the LSP) is predominately the neutral wino and the lighter chargino (often the NLSP) is almost degenerate with it.

- The lightest neutralino is the LSP.





We see that acceptable values of m_h are obtained for $7 < \tan\beta < 25$, and of the stau mass for $\tan\beta < 19$.

Mass sum rules

The following sum rules for the physical masses are essentially independent of L, e (due to cancellation of L, e terms in tree masses).

$$
\begin{aligned}
m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2} + m^2_{\tilde{b}_1} + m^2_{\tilde{b}_2} - 2m_t^2 &= 2.76\,(m_{\tilde{g}})^2 \\
m^2_{\tilde{\tau}_1} + m^2_{\tilde{\tau}_2} + m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2} - 2m_t^2 &= 1.14\,(m_{\tilde{g}})^2\,, \\
m^2_{\tilde{e}_L} + 2m^2_{\tilde{u}_L} + m^2_{\tilde{d}_L} &= 2.60\,(m_{\tilde{g}})^2\,, \\
m^2_{\tilde{u}_R} + m^2_{\tilde{d}_R} + m^2_{\tilde{u}_L} + m^2_{\tilde{d}_L} &= 3.51\,(m_{\tilde{g}})^2\,, \\
m^2_{\tilde{u}_L} + m^2_{\tilde{d}_L} - m^2_{\tilde{u}_R} - m^2_{\tilde{e}_R} &= 0.88\,(m_{\tilde{g}})^2\,, \\
m_A^2 - 2\sec 2\beta \left(m^2_{\tilde{e}_L} + m^2_{\tilde{e}_R}\right) &= 0.4\,(m_{\tilde{g}})^2\,, \\
m_A^2 - 2\sec 2\beta \left(m^2_{\tilde{\tau}_1} + m^2_{\tilde{\tau}_2} - 2m_\tau^2\right) &= = 0.39\,(m_{\tilde{g}})^2\,.
\end{aligned}
$$

- The numerical coefficients on the RHS are slowly varying functions of $\tan\beta$; the results above are for $\tan\beta = 5$.
- The existence of these sum rules will be a useful distinguishing feature of the AMSB scenario.

Conclusions

- The AMSB scenario is an attractive alternative to (and easily distinguished from) MSUGRA.

- Have shown how a U_1' gauge symmetry broken at high energies can lead in a natural way to the FI-solution to the tachyonic slepton problem in the context of anomaly mediation.

- Gives sparticle spectrum described by the parameter set $m_0, L, e, \tan\beta, \mathsf{sign}(\mu)$.

- Allowed set of L, e comparatively restricted.

- Scenario immediately testable should sparticles be discovered in experiments at the LHC.