



SO(10) Model for Fermion Masses

SUSY 05

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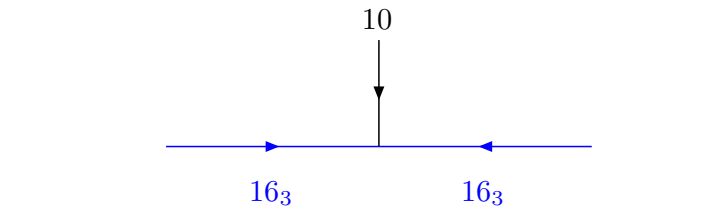
R.D. and S. Raby, hep-ph/0507045

Outline

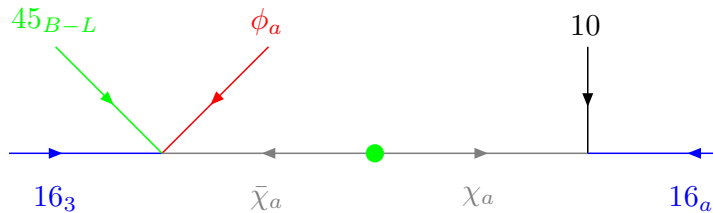
- SO(10) model with D_3 family symmetry
 - ▷ brief description of the model
 - ▷ mechanism for bi-large lepton mixing
 - ▷ fit for fermion masses and mixing
- Predictions:
 - ▷ 1-3 lepton mixing
 - ▷ neutrinoless double beta decay, J , ...
 - ▷ leptogenesis
 - ▷ SUSY & Higgs, $\Omega_\chi h^2$, $B_s \rightarrow \mu^+ \mu^-$, ...
- Conclusions

SO(10) × D₃ × [U(1) × Z₂ × Z₃] model

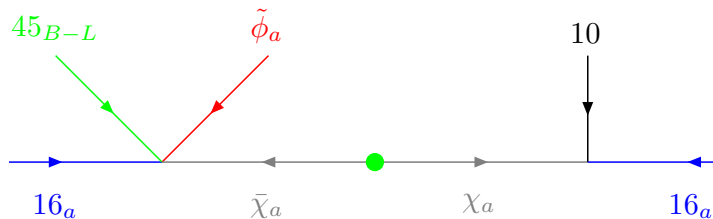
$$W_{ch.f.} = 16_3 \ 10 \ 16_3 + 16_a \ 10 \ \chi_a + \bar{\chi}_a \left(M_X \chi_a + 45 \frac{\phi_a}{\hat{M}} 16_3 + 45 \frac{\tilde{\phi}_a}{\hat{M}} 16_a + A 16_a \right)$$



$$\langle \phi \rangle = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \langle \tilde{\phi} \rangle = \begin{pmatrix} 0 \\ \tilde{\phi}_2 \end{pmatrix}$$

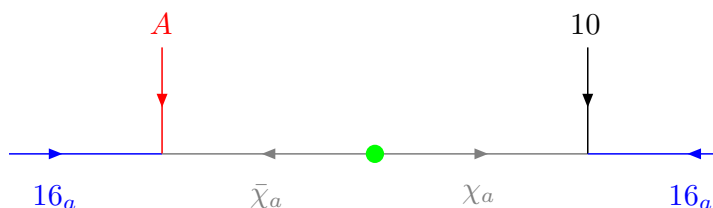


$$\bullet = M(\alpha X + \beta Y)$$



$$\epsilon \propto \phi_1/M_G, \quad \tilde{\epsilon} \propto \tilde{\phi}_2/M_G,$$

$$\xi = \phi_2/\phi_1, \quad \epsilon' \propto A/M_G, \quad \sigma = \frac{1+\alpha}{1-3\alpha}$$



$$Y_d = \begin{pmatrix} 0 & \epsilon' & -\epsilon\xi\sigma \\ -\epsilon' & \tilde{\epsilon} & -\epsilon\sigma \\ \epsilon\xi & \epsilon & 1 \end{pmatrix} \lambda$$

Neutrino sector

$$W_N = \overline{16} (\lambda_2 N_a 16_a + \lambda_3 N_3 16_3) + \frac{1}{2} (S_a N_a N_a + S_3 N_3 N_3)$$

$$\langle \overline{16} \rangle = v_{16}, \quad \langle S_a \rangle = M_a, \quad a = 1, 2, \quad \langle S_3 \rangle = M_3$$

$$W_{\nu, \bar{\nu}, N}^{eff} = \nu m_\nu \bar{\nu} + \bar{\nu} V N + \frac{1}{2} N M_N N$$

with

$$V = v_{16} \begin{pmatrix} 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}, \quad M_N = \text{diag}(M_1, M_2, M_3)$$

effectively:

$$M_R = V M_N^{-1} V^T \equiv \text{diag}(M_{R_1}, M_{R_2}, M_{R_3}),$$

$$M_{R_1} \ll M_{R_2} \ll M_{R_3}$$

Bi-large lepton mixing

$$W_{\nu, \bar{\nu}}^{eff} = \nu m_\nu \bar{\nu} + \frac{1}{2} \bar{\nu} M_R \nu$$

$$m_\nu \simeq \begin{pmatrix} 0 & b & b \\ b & a & a \\ b & a & 1 \end{pmatrix} \lambda v_u, \quad M_R = \begin{pmatrix} M_{R1} & 0 & 0 \\ 0 & M_{R2} & 0 \\ 0 & 0 & M_{R3} \end{pmatrix}$$

$$b \ll a \ll 1, \quad M_{R1} \ll M_{R2} \ll M_{R3}$$

effective left-handed neutrino Majorana mass matrix:

$$\mathcal{M} = m_\nu M_R^{-1} m_\nu^T \equiv \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3, \quad \mathcal{M}_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \frac{(b\lambda v_u)^2}{M_{R1}}, \dots$$

- 2-3 mixing close to maximal, if \mathcal{M}_1 or \mathcal{M}_2 dominates
- bi-large mixing only if \mathcal{M}_1 and \mathcal{M}_2 contribute comparably with opposite overall phases
- $\sin \theta_{13} \simeq b/a \ll 1$

Fit to fermion masses and mixing

Observable (masses in GeV)	Data (σ)	Theory	Pull
$G_\mu \times 10^5$	1.16637 (0.1%)	1.16635	< 0.01
α_{EM}^{-1}	137.036 (0.1%)	137.031	< 0.01
$\alpha_s(M_Z)$	0.1187 (0.002)	0.1184	0.02
M_t	174.3 (5.1)	175.36	0.04
$m_b(M_b)$	4.25 (0.25)	4.252	< 0.01
$M_b - M_c$	3.4 (0.2)	3.513	0.32
$m_c(m_c)$	1.2 (0.2)	1.03	0.72
m_s	0.105 (0.025)	0.114	0.13
m_d/m_s	0.0521 (0.0067)	0.0627	2.53
$Q^{-2} \times 10^3$	1.934 (0.334)	1.763	0.26
M_τ	1.777 (0.1%)	1.777	< 0.01
M_μ	0.10566 (0.1%)	0.10566	< 0.01
$M_e \times 10^3$	0.511 (0.1%)	0.511	< 0.01
V_{us}	0.22 (0.0026)	0.2192	0.09
V_{cb}	0.0413 (0.0015)	0.0407	0.15
V_{ub}	0.00367 (0.00047)	0.00332	0.56
V_{td}	0.0082 (0.00082)	0.00819	< 0.01
ϵ_K	0.00228 (0.000228)	0.00238	0.19
$\sin(2\beta)$	0.736 (0.049)	0.6757	1.51
$\Delta m_{31}^2 \times 10^3$	2.3 (0.6)	2.407	0.03
$\Delta m_{21}^2 \times 10^5$	7.9 (0.6)	7.866	< 0.01
$\sin^2 \theta_{12}$	0.295 (0.045)	0.2851	0.05
$\sin^2 \theta_{23}$	0.51 (0.13)	0.546	0.08
TOTAL χ^2			6.70

Input parameters:

$$1/\alpha_G, M_G, \epsilon_3$$

$$\lambda, \epsilon, \sigma, \tilde{\epsilon}, \rho, \epsilon', \xi$$

$$\Phi_\sigma, \Phi_{\tilde{\epsilon}}, \Phi_\rho, \Phi_\xi$$

$$m_{16} = 3500 \text{ GeV}$$

$$M_{1/2} = 450 \text{ GeV}$$

$$A_0 = -6888.3 \text{ GeV}$$

$$\mu(M_Z) = 247.9 \text{ GeV}$$

$$(m_{H_d}/m_{16})^2 = 2.00$$

$$(m_{H_u}/m_{16})^2 = 1.71$$

$$\tan \beta = 49.98$$

$$M_{R_1} = 1.1 \times 10^{10} \text{ GeV}$$

$$M_{R_2} = -9.3 \times 10^{11} \text{ GeV}$$

$$M_{R_3} = 5.8 \times 10^{13} \text{ GeV}$$

Neutrino masses, $\sin^2 \theta_{13}$ and CP phases

m_{ν_3} (eV)	0.0492
m_{ν_2} (eV)	0.0096
m_{ν_1} (eV)	0.0037
$\sin^2 \theta_{13}$	0.0024
J	0.0107
$\sin \delta$	0.98
α_1 (rad)	-1.286
α_2 (rad)	1.821
$\langle m_{\beta\beta} \rangle$ (eV)	0.00021
$m_{\nu_e}^{eff}$ (eV)	0.0065
ϵ_1	-1.16×10^{-7}

Exp. limit:

$$\sin^2 \theta_{13} < 0.031$$

Chooz, solar+KamLAND

at limits of future reactor exp.

observable at long baseline acc. exp.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{ai} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$U_{ai} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} P$$

$$P = \text{diag} \left(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1 \right)$$

Neutrinoless double-beta decay, J , ...

m_{ν_3} (eV)	0.0492
m_{ν_2} (eV)	0.0096
m_{ν_1} (eV)	0.0037
$\sin^2 \theta_{13}$	0.0024
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Jarlskog parameter:

$$J = s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23} \sin \delta$$

may be observable at long baseline exp.

neutrinoless double-beta decay:

$$\langle m_{\beta\beta} \rangle = \left| \sum_i U_{ei}^2 m_{\nu_i} \right|$$

not observable in near-future exp.

eff. ν_e mass measured in low energy beta decay of tritium:

$$m_{\nu_e}^{eff} = \left(\sum_i |U_{ei}|^2 m_{\nu_i}^2 \right)^{1/2}$$

not observable in proposed exp.

Leptogenesis

m_{ν_3} (eV)	0.0492
m_{ν_2} (eV)	0.0096
m_{ν_1} (eV)	0.0037
$\sin^2 \theta_{13}$	0.0024
J	0.0107
$\sin \delta$	0.98
α_1 (rad)	-1.286
α_2 (rad)	1.821
$\langle m_{\beta\beta} \rangle$ (eV)	0.00021
$m_{\nu_e}^{eff}$ (eV)	0.0065
ϵ_1	-1.16×10^{-7}

thermal leptogenesis:

$$8 \times \epsilon_1 \approx -0.9 \times 10^{-6}$$

required $\sim -10^{-6}$

moreover:

$$m_{\nu_1} \simeq 0.004 \text{ eV and } M_{R_1} \simeq 10^{10} \text{ GeV}$$

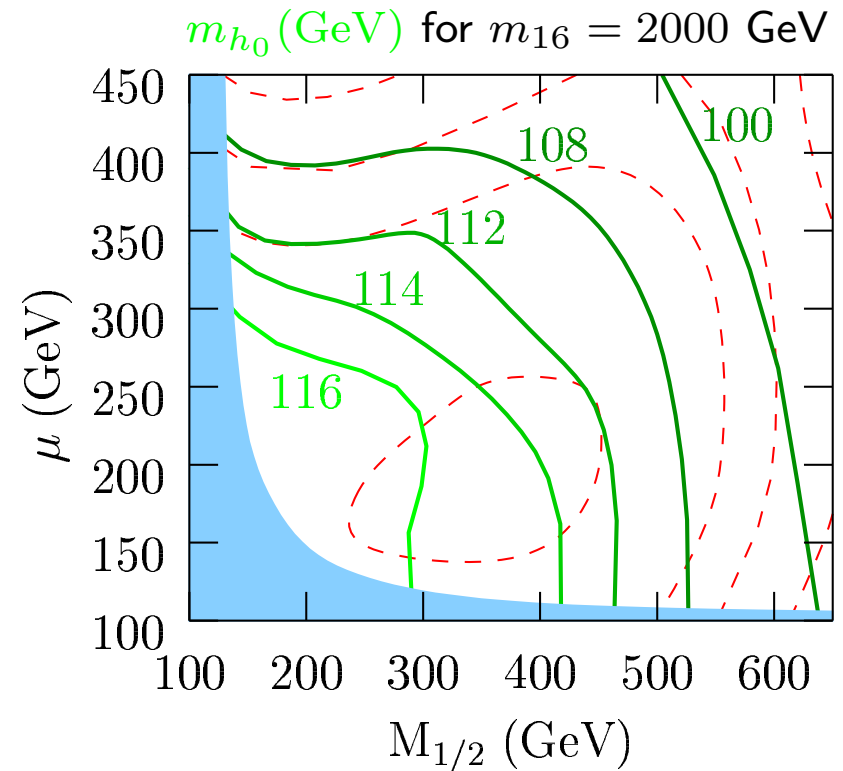
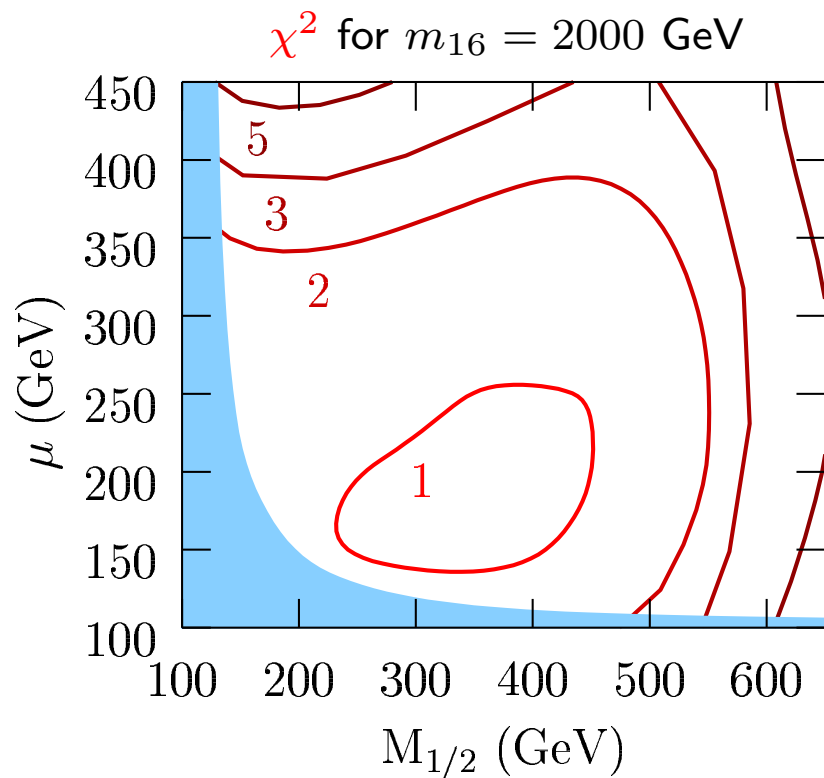
in the range where baryon asymmetry is entirely determined by neutrino properties!
(no dependence on initial neutrino abundance or initial baryon asymmetry)

BUT gravitino problem!

CP asymmetry:

$$\epsilon_1 = -\frac{3}{16\pi} \frac{1}{(Y_\nu^T Y_\nu^*)_{11}} \sum_{i=2,3} \text{Im}\{([Y_\nu^\dagger Y_\nu]_{1i})^2 \frac{M_{R_1}}{M_{R_i}}\}$$

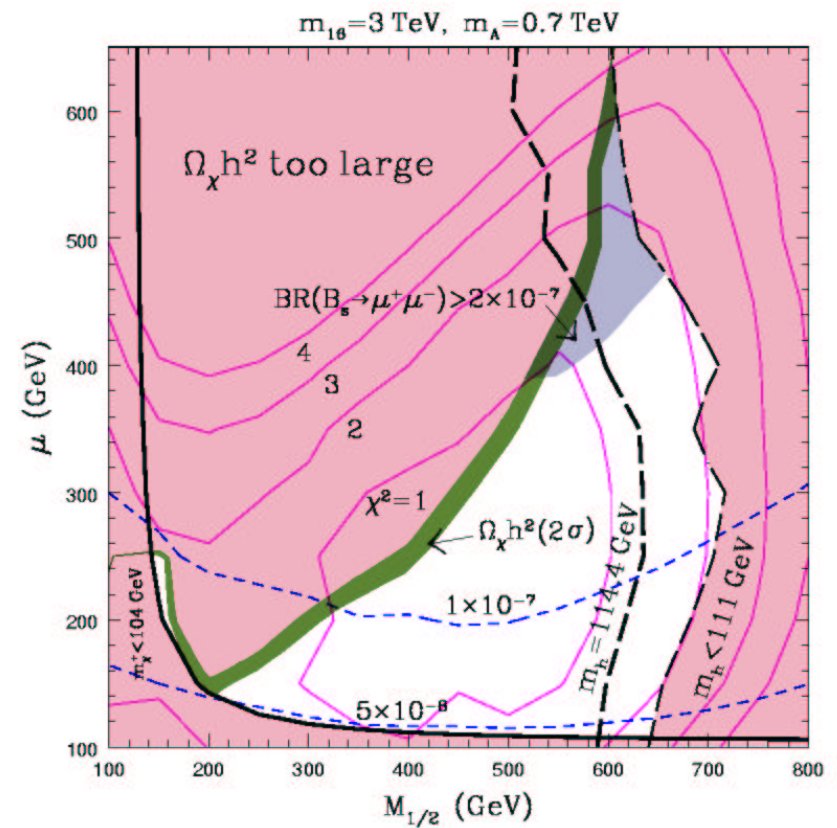
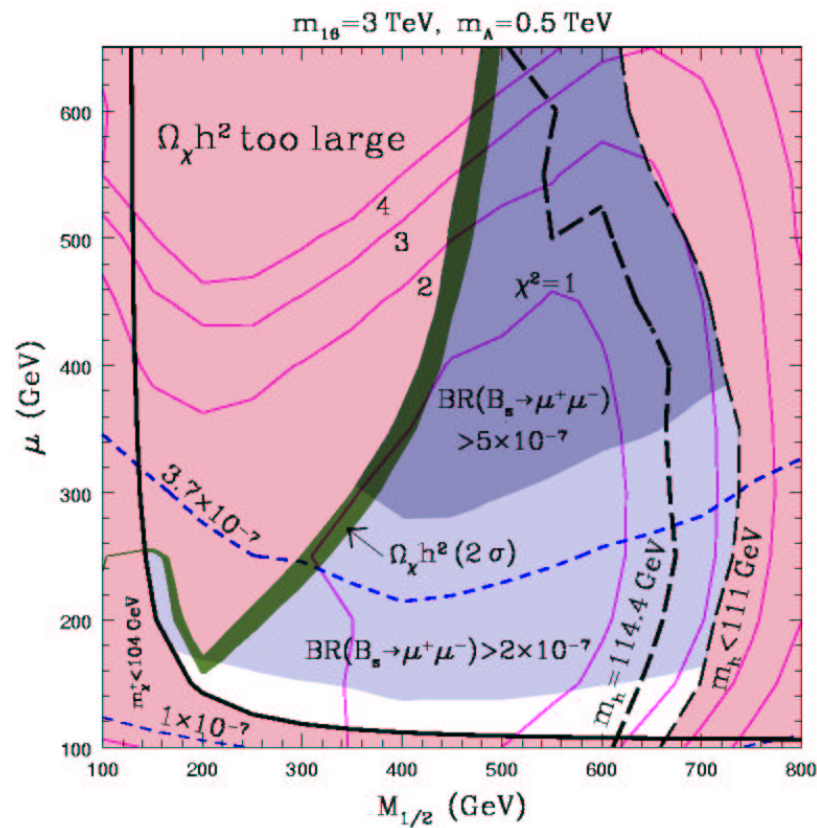
SUSY & Higgs, $\Omega_\chi h^2$, $B_s \rightarrow \mu^+ \mu^-$, ...



$$\mu, M_{1/2} \ll m_{16}, \quad -A_0 \simeq \sqrt{2}m_{10} \simeq 2m_{16}, \quad \tan\beta \simeq 50$$

T. Blazek, R.D. and S. Raby, (2002)

SUSY & Higgs, $\Omega_\chi h^2$, $B_s \rightarrow \mu^+ \mu^-$, ...



R.D., S. Raby, L. Roszkowski and R. Ruiz De Austri, (2003), (2005)

Conclusions

- SO(10) model with D_3 family symmetry
 - ▷ bi-large lepton mixing
 - ▷ excellent fit for fermion masses and mixing
- Predictions:
 - ▷ small 1-3 mixing
 - ▷ optimal for leptogenesis
 - ▷ tiny neutrinoless double beta decay, J , ...
 - ▷ accommodates $\Omega_\chi h^2$, $B_s \rightarrow \mu^+ \mu^-$, ...
 - ▷ suppresses dim. 5 proton decay and flavor violation

R.D., A. Mafi, S. Raby, (2001)