Heterotic Compactifications with U(1) Bundles at 1-loop

- The SO(32) case —

Ralph Blumenhagen, G.H., Timo Weigand

hep-th/0504232 ($E_8 \times E_8$) and 0507041 ($SO(32)$).

Gabriele Honecker

gabriele@mppmu.mpg.de

MPI für Physik, München

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- Compactifications with intersecting D-branes

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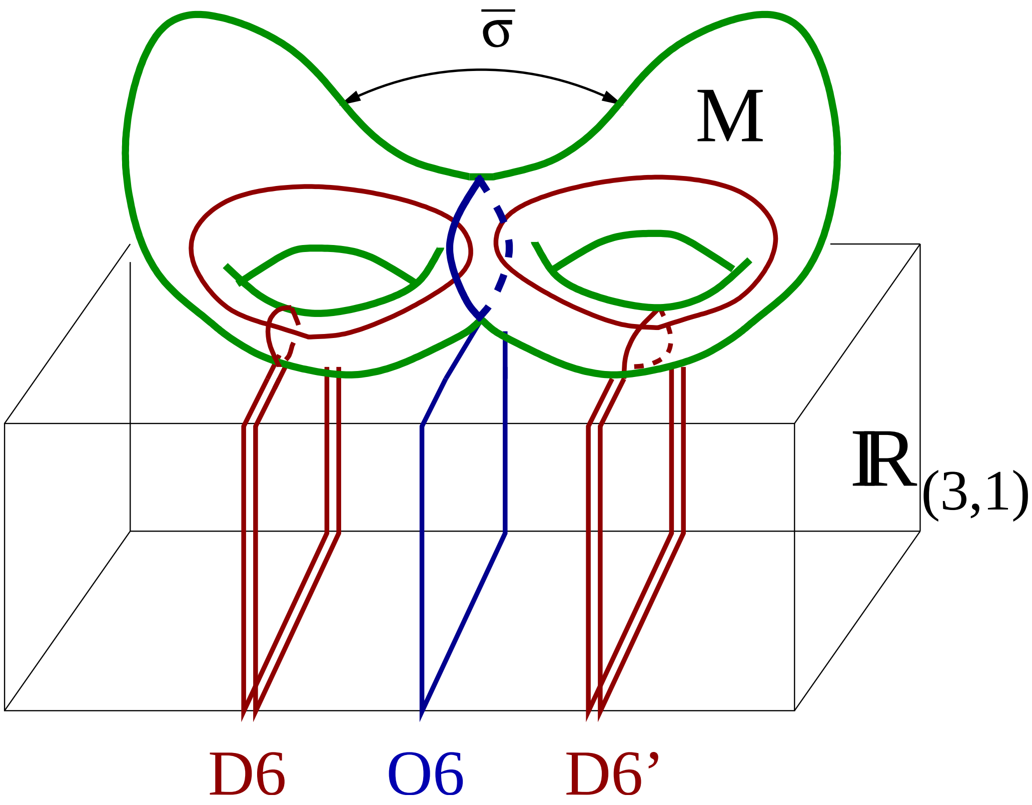

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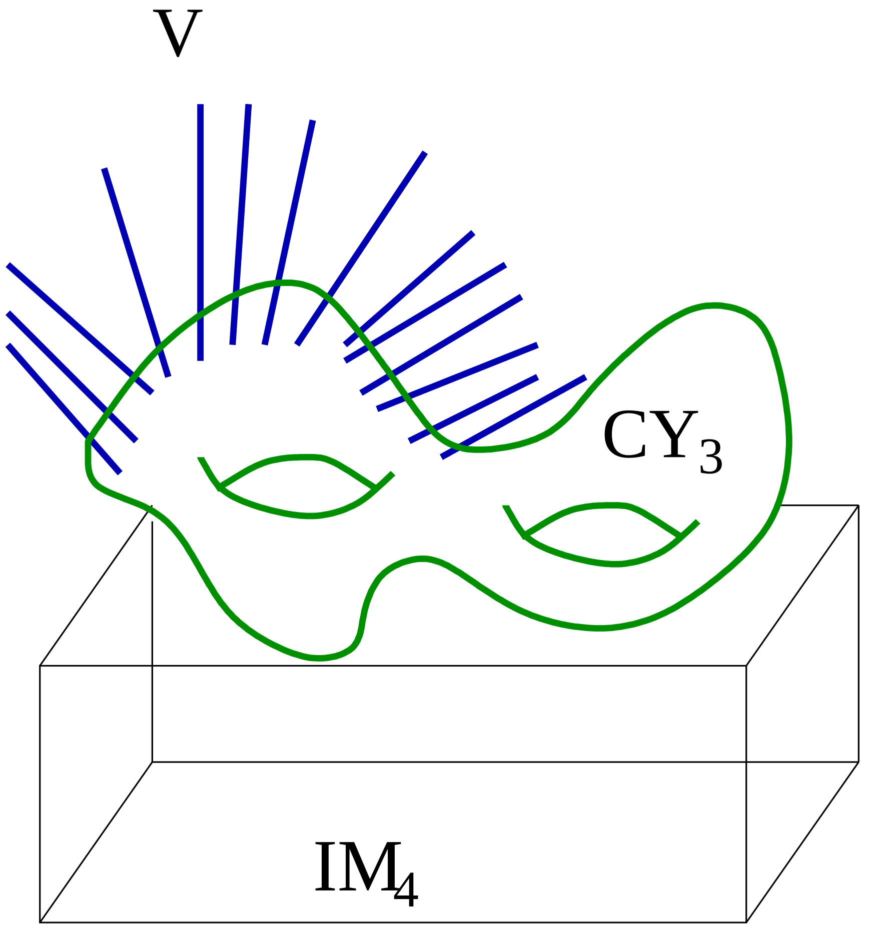

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  GS mechanism involves just the universal dilaton,
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- Conflict with S-duality $\rightarrow$ study heterotic string with abelian (line) bundles & Type I vacua with non-abelian bundles.

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Stability discussion:
Marino, Minasian, Moore, Strominger '99
Douglas, Fiol, Römelsberger '00
Enger, Lutken '03, Aspinwall '04

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Decompose $SO(32) \to SO(2M) \times \prod_{x=1}^{K+L} U(N_x)$ and
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Take vector bundles of the following form

$$W = \bigoplus_{m=K+1}^{K+L} V_m \oplus \bigoplus_{i=1}^{K} L_i,$$

where the V_m are $U(N_m)$ bundles and the L_i denote some complex line bundles with structure group $U(1)_i$.

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The observable gauge group is $SO(2M) \times \prod_{j=1}^{K} SU(N_j)$ and the anomaly-free part of $U(1)^{K+L}$, which is the commutant of the structure group G of the bundle W

Consistency conditions

Consistency conditions

- The vector bundle W has to admit spinors

$$(2\pi)\mathrm{tr}F = c_1(W) \in H^2(\mathcal{M}, 2\mathbb{Z}).$$

- At string tree level, the field strength of the vector bundle has to satisfy the hermitian Yang-Mills equations

$$F_{ab} = F_{\overline{a}\overline{b}} = 0, \quad g^{a\bar{b}} F_{a\bar{b}} = 0.$$

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- A necessary condition is the so-called Donaldson-Uhlenbeck-Yau (DUY) condition for all m, i,

$$\int_{\mathcal{M}} J \wedge J \wedge c_1(V_m) = 0, \qquad \int_{\mathcal{M}} J \wedge J \wedge c_1(L_i) = 0,$$

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- The DUY is modified by 1-loop effects (details later)
- $Re(f_x) = g_x^{-2} > 0$ has to be well defined (details later)
- The Bianchi identity for $H = dB - \frac{\alpha'}{4}(\omega_Y - \omega_L)$,

$$dH = \frac{\alpha'}{4}\left(\mathrm{tr}(R^2) - \mathrm{tr}(F^2)\right),$$

imposes the so-called tadpole condition in cohomology

$$\sum_{m=K+1}^{K+L} \mathrm{ch}_2(V_m) + \sum_{i=1}^{K} N_i\, \mathrm{ch}_2(L_i) = -c_2(T).$$

Massless spectrum

Massless spectrum

The adjoint of $SO(32)$ decomposes as $(M + \sum_x N_x = 16)$

$$
\mathbf{496} \rightarrow \left(\begin{array}{c}
(\mathbf{Adj}_{SO(2M)})_0 \\
\sum_{x=1}^{K+L} (\mathbf{Adj}_{SU(N_x)})_0 + (K+L) \times (\mathbf{1})_0 \\
\sum_{x=1}^{K+L} (\mathbf{Anti}_{SU(N_x)})_{2(x)} + h.c. \\
\sum_{x<y} \left[(\mathbf{N}_x, \mathbf{N}_y)_{1(x),1(y)} + (\mathbf{N}_x, \overline{\mathbf{N}}_y)_{1(x),-1(y)} \right] + h.c. \\
\sum_{x=1}^{K+L} (2M, \mathbf{N}_x)_{1(x)} + h.c.
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$$

The massless spectrum is determined by various cohomology classes

$$
H^*(\mathcal{M}, \mathcal{W}) \qquad \mathcal{W} = \bigotimes_m \bigwedge^p V_m \otimes \bigotimes_i L_i^q ,
$$

where the charges p and q can be derived from decomposition, e.g. $(\mathbf{N}_i, \mathbf{N}_j)_{1(i),1(j)}$ is counted by $\mathcal{W} = L_i \otimes L_j$.

Massless spectrum

Massless spectrum

The net-number of chiral matter multiplets is given by the Euler characteristic of the respective bundle $\mathcal{W}$

$$\begin{aligned}\chi(\mathcal{M},\mathcal{W}) &= \sum_{i=0}^{3}(-1)^i \dim(H^i(\mathcal{M},\mathcal{W})) \\ &= \int_{\mathcal{M}} \left[\mathrm{ch}_3(\mathcal{W}) + \frac{1}{12}\, c_2(T)\, c_1(\mathcal{W})\right].\end{aligned}$$

Gauge anomalies

Gauge anomalies

- All non-abelian cubic gauge anomalies cancel upon tadpole cancellation, whereas the mixed abelian-non-abelian, the mixed abelian-gravitational and the cubic abelian ones do not:

$$
\begin{aligned}
A_{U(1)_x - SU(N_i)^2} &\sim f_x \wedge \mathrm{tr}_{SU(N_i)} F^2 \\
&\int_{\mathcal{M}} \left(\mathrm{tr} \overline{F}_x^3 + 3\, \mathrm{tr} \overline{F}_x \wedge \overline{f}_i^2 - \frac{1}{8} \mathrm{tr} \overline{F}_x \wedge \mathrm{tr} \overline{R}^2 \right) \\
\mathcal{A}_{U(1)_x - G^2} &\sim f_x \wedge \mathrm{tr} R^2 \\
&\int_{\mathcal{M}} \left(\mathrm{tr} \overline{F}_x^3 - \frac{1}{16} \mathrm{tr} \overline{F}_x \wedge \mathrm{tr} \overline{R}^2 \right)
\end{aligned}
$$

Need to be cancelled by a generalized Green-Schwarz mechanism.

10D Supergravity

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- Bosonic part of the tree level action for heterotic strings is

$$S_{kin} = \frac{1}{2\,\kappa_{10}^2} \int e^{-2\phi} \Big(R \wedge *1 + 4 d\phi \wedge *d\phi - \frac{1}{2} H \wedge \star_{10} H - \frac{\alpha'}{4} \mathrm{tr}(F \wedge \star_{10} F) \Big)$$

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- Green-Schwarz counter term at 1-loop contains

$$S_{GS} = \frac{1}{48\,(2\pi)^5\,\alpha'}\int B\wedge X_8$$

$$X_8 = \frac{1}{24}\mathrm{Tr}F^4 - \frac{1}{7200}\left(\mathrm{Tr}F^2\right)^2 - \frac{1}{240}\left(\mathrm{Tr}F^2\right)\left(\mathrm{tr}R^2\right) + \frac{1}{8}\mathrm{tr}R^4 + \frac{1}{32}\left(\mathrm{tr}R^2\right)^2$$

The Green-Schwarz mechanism

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Dimensionally reducing the 10D 1-loop counter term

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leads to vertex couplings for the internal axions

$$S_{GS} \;=\; \sum_{k=1}^{h_{11}} \sum_{i=1}^{K} \frac{1}{2\pi} \int_{\mathbb{R}_{1,3}} \left(b_k^{(0)} \, \mathrm{tr}_{SU(N_i)} F^2 \right) \; \left(\frac{1}{4} \overline{f}_i^2 - \frac{1}{192} \mathrm{tr} \overline{R}^2 \right)_k$$

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and a mass term for the external (universal) axion:

$$S^0_{mass} \;=\; \sum_{x=1}^{K+L}\frac{\mathcal{Q}_0^x}{2\pi\alpha'}\int_{\mathbb{R}_{1,3}}\left(b_0^{(2)}\wedge f_x\right),$$

$$\mathcal{Q}_0^x \;=\; \frac{1}{6\,(2\pi)^4}\int_{\mathcal{M}}\left(\mathrm{tr}\overline{F}_x^3 - \frac{1}{16}\mathrm{tr}\overline{F}_x\wedge\mathrm{tr}\overline{R}^2\right)$$

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and to mass terms for the internal axions

$$\begin{aligned} S_{\text{mass}} &= \sum_{k=1}^{h_{11}} \sum_{x=1}^{K+L} \frac{\mathcal{Q}_k^x}{2\pi\alpha'} \int_{\mathbb{R}_{1,3}} \left(b_k^{(2)} \wedge f_x \right), \\ \mathcal{Q}_k^x &= \frac{1}{2\pi} (\mathrm{tr}\overline{F}_x)_k \end{aligned}$$

Massive gauge fields + axions

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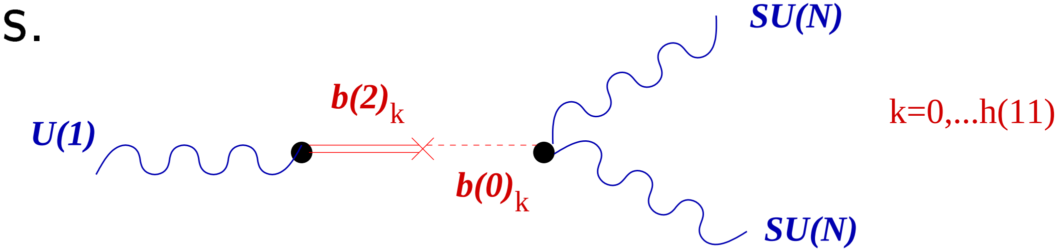
- GS diagrams precisely cancel the mixed abelian anomalies.



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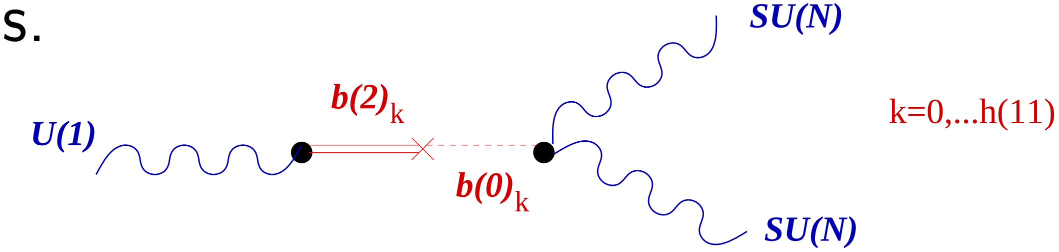

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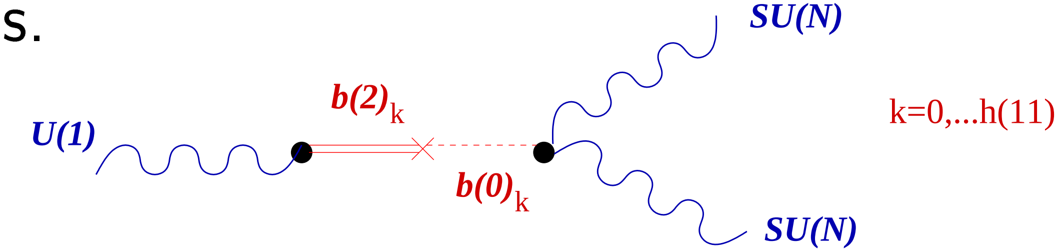

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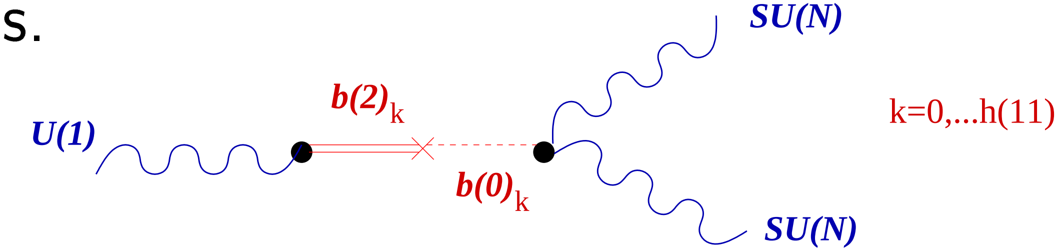

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- All mass terms are of the same order in both string and sigma model perturbation theory.
- Massive axions $b_k^{(0)}$ are longitudinal modes of U(1)s. Since axions complexify the Kähler moduli and the dilaton, supersymmetry dictates that the same $\#$ is massive.
  Only source for mass terms is the DUY equation.

Gauge kinetic function

Gauge kinetic function

- Defining the 4D complexified dilaton

$$S \;=\; \frac{1}{2\pi}\left[e^{-2\phi_{10}}\frac{\mathrm{Vol}(\mathcal{M})}{\ell_s^6} + i\, b_0^{(0)}\right].$$

and the 4D complexified Kähler moduli

$$T_k \;=\; \frac{1}{2\pi}\left[-\alpha_k + i b_k^{(0)}\right],$$

the GS terms induce the axionic part of the gauge kinetic function, e.g.

$$S_{kin} = \frac{1}{4\pi}\int_{\mathbb{R}_{1,3}} b_0^{(0)} \wedge \mathrm{tr}_{SU(N_i)} F^2$$

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- The 1-loop corrected gauge kinetic function for the non-abelian gauge fields are

$$f_{SU(N_i)} = 2S + \sum_k T_k \left((\overline{f}_i^2) - \frac{1}{48} \mathrm{tr} \overline{R}^2 \right)_k .$$

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- For the abelian ones we find

$$f_x = 2N_x S + \sum_k T_k \left(\mathrm{tr} \overline{F}_x^2 - \frac{N_x}{48} \mathrm{tr} \overline{R}^2 \right)_k .$$

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- For the SO(2M) group

$$f_{SO(2M)} = S - \frac{1}{96} \sum_{k=1}^{h_{11}} T_k \, (\mathrm{tr} \overline{R}^2)_k$$

Fayet-Iliopoulos terms

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- Since we are dealing with anomalous $U(1)$ gauge factors, there are potential Fayet-Iliopoulos (FI) terms generated. Employing the standard supersymmetric field theory formula

$$\frac{\xi_m}{g_m^2} = \frac{\partial \mathcal{K}}{\partial V_m}\bigg|_{V=0},$$

  the FI parameters ξ_m can be computed from the Kähler potential $\mathcal{K}$, which in our case takes the following gauge invariant form.
  V_m denotes the vector superfields.

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$$\mathcal{K} \sim -\ln\Big(S + S^* - \sum_m \mathcal{Q}_0^m\, V_m\Big) -$$

$$\ln\Big(\sum_{i,j,k=1}^{h_{11}} \frac{d_{ijk}}{6}\big(T_i + T_i^* - \sum_m \mathcal{Q}_i^m\, V_m\big)\big(T_j + \cdots\big)\big(T_k + \cdots\big)\Big).$$

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For the FI term one gets

$$\frac{\xi_m}{g_m^2} \sim e^{-2\phi_{10}}\frac{1}{2}\int_{\mathcal{M}} J^2 \wedge \mathrm{tr}\overline{F}_x - \frac{(2\pi\alpha')^2}{3!}\int_{\mathcal{M}}\Big(\mathrm{tr}\overline{F}_x^3 - \frac{1}{16}\mathrm{tr}\overline{F}_x \wedge \mathrm{tr}\overline{R}^2\Big)$$

Apparently, the first term arises at string tree-level, whereas the second term is a 1-loop term.

1-loop DUY equation

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- Interpretation: $\xi_m = 0$ contains a 1-loop correction to the DUY condition. In contrast to earlier claims that abelian gauge fluxes freeze some combinations of the Kähler moduli, we now realize that actually combinations of the dilaton and the Kähler moduli are frozen.

1-loop DUY equation

- Interpretation: $\xi_m = 0$ contains a 1-loop correction to the DUY condition. In contrast to earlier claims that abelian gauge fluxes freeze some combinations of the Kähler moduli, we now realize that actually combinations of the dilaton and the Kähler moduli are frozen.
- Support from Type I - HS duality:

$$
\begin{aligned}
e^{\phi^I_{10}} &= e^{-\phi^H_{10}}, \\
J^I &= J^H \, e^{-\phi^H_{10}},
\end{aligned}
$$

S-duality

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The dual gauge couplings match those of magnetised branes

$$\mathrm{Re}(f_x^I) = \frac{1}{\pi \ell_s^6 g_s}\left[\frac{N_x}{3!}\int_{\mathcal{M}} J^3 - \frac{(2\pi\alpha')^2}{2}\int_{\mathcal{M}} J\wedge\left(\mathrm{tr}\overline{F}_x^2 - \frac{N_x}{48}\mathrm{tr}\overline{R}^2\right)\right]$$

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DUY transforms into non-abelian generalization of MMMS for D9-branes in curved background ($\mathcal{F} = 2\pi\alpha' F$)

$$\frac{1}{2}\int_{\mathcal{M}} J^2\ \mathrm{tr}\overline{\mathcal{F}}_x - \frac{1}{3!}\int_{\mathcal{M}}\left(\mathrm{tr}\overline{\mathcal{F}}_x^3 - \frac{1}{16}\mathrm{tr}\overline{\mathcal{F}}_x\ \mathrm{tr}\overline{R}^2\right) = 0$$

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$$\frac{1}{2}\int_{\mathcal{M}} J^2\, \mathrm{tr}\overline{\mathcal{F}}_x - \frac{1}{3!}\int_{\mathcal{M}}\left(\mathrm{tr}\overline{\mathcal{F}}_x^3 - \frac{1}{16}\mathrm{tr}\overline{\mathcal{F}}_x\, \mathrm{tr}\overline{R}^2\right) = 0$$

It can be compactly written as ($\varphi = 0$)

$$\mathrm{Im}\left[\int_{\mathcal{M}} \mathrm{tr}_{U(N)}\ \left(e^{i\varphi}\, e^{J\mathrm{id}+i\mathcal{F}}\sqrt{\hat{A}(\mathcal{M})}\right)\right] = 0$$

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There is a *unique* solution if the gauge bundle is π-stable w.r.t. the slope

$$\pi(V) = \mathrm{Arg}\left(\int_{\mathcal{M}} \mathrm{tr}_{U(N)}\left[e^{J\,\mathrm{id}+i\mathcal{F}}\,\sqrt{\hat{A}(\mathcal{M})}\right]\right),$$

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- A line bundle L on $\mathcal{M}$ is specified completely by its first Chern class which takes values in $H^2(\mathcal{M}, \mathbb{Z})$ and can be expanded in terms of a basis ω_i with $n_i \in \mathbb{Z}$

$$c_1(L) = \sum_{i=1}^{h_{11}} n_i \, \omega_i .$$

One also denotes such a line bundle as $\mathcal{O}(n_1, \ldots, n_k)$.

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- Then a vector bundle V of rank r is defined by the cohomology of the monad $(i = 1, \ldots, h_{11})$

$$0 \to \mathcal{O}|_{\mathcal{M}} \xrightarrow{g} \bigoplus_{a=1}^{r+2} \mathcal{O}(n_{a,i})|_{\mathcal{M}} \xrightarrow{f} \mathcal{O}(m_i)|_{\mathcal{M}} \to 0,$$

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- For $SU(N)$ bundles one has $c_1(V) = 0$, whereas for $U(N)$ bundles at first sight no condition.

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Tadpole is cancelled, gauge group $SO(26) \times U(1)_{massive}$ with chiral spectrum

reps.	Cohomology	χ
$(\mathbf{1})_2$	$H^*(\mathcal{M}, \bigwedge^2 V)$	155
$(\mathbf{26})_1$	$H^*(\mathcal{M}, V)$	-80

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1-loop corrected gauge couplings are

$$\left.\frac{1}{g^2_{SO(26)}}\right|_{1-loop} = \frac{95}{18\pi} r \sim 4.5$$

$$\left.\frac{1}{g^2_{U(1)_{massive}}}\right|_{1-loop} = \frac{245}{3\pi} r \sim 70$$

Ex. 2: $SO(16) \times U(1)^2$ model on a CICY

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Chiral Spectrum

reps.	Cohomology	χ	reps.	Cohomology	χ
$(\mathbf{16})_{1,0}$	$H^*(\mathcal{M}, V_1)$	-24	$(\mathbf{1})_{1,1}$	$H^*(\mathcal{M}, V_1 \otimes V_2)$	-120
$(\mathbf{16})_{0,1}$	$H^*(\mathcal{M}, V_2)$	-24	$(\mathbf{1})_{1,-1}$	$H^*(\mathcal{M}, V_1 \otimes V_2^*)$	0

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$$\left.\frac{1}{g^2_{SO(16)}}\right|_{1-loop} = \frac{1}{\pi}\frac{\frac{50}{18}r_1^2 + 8r_1r_2 - 2r_2^2}{4r_2 - r_1},$$

$$\left.\frac{1}{g^2_{U(1)_i}}\right|_{1-loop} = \frac{1}{\pi}\frac{\frac{2}{9}r_1^2 + 140r_2^2 + 32r_2^2}{4r_2 - r_1}, \quad i = 1, 2$$

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\end{aligned}$$

$U(1)_1 - U(1)_2$ is massless, the orthogonal combination is anomalous

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- 3-form fluxes can stabilize also complex structure moduli $W_{flux} \sim \int \Omega \wedge (H + i\, dJ)$
- Yukawa couplings: normalization depends on Kähler potential, difficult to compute — superpotential gives selection rules for non-vanishing couplings $H^p(\mathcal{M}, \mathcal{W}_1) \times H^q(\mathcal{M}, \mathcal{W}_2) \times H^r(\mathcal{M}, \mathcal{W}_3) \to \mathbb{C}$

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