Partial Breaking of $\mathcal{N} = 2$ Supersymmetry and of Gauge Symmetry in the $U(N)$ Gauge Model

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I). Introduction

superstrings: maximal SUSY

one parameter +VEV's

$\therefore$ desirable to accomplish partial spontaneous breaking of extended supersymmetries in field theory without gravity as well.

• appears not possible from the consideration of supercharges alone.

(early 1980's $\rightarrow$)

• basic mechanism:

$$\left\{\bar{Q}^{j}_{\dot{\alpha}}, \mathcal{S}^{m}_{\alpha i}(x)\right\} = 2(\sigma^{n})_{\alpha\dot{\alpha}}\delta_{i}{}^{j}T^{m}_{n}(x) + (\sigma^{m})_{\alpha\dot{\alpha}}C_{i}{}^{j}$$

· $C_{i}{}^{j}$: not a VEV but follows simply from the algebra.
· permitted by the Jacobi id. constraints.
· not modify the SUSY algebra on the elementary fields.

Our model:

- $\mathcal{N}=2$ proven in the text.
- $U(N)$ gauge group. (cf. $U(1)$ case Antoniadis-Partouche-Taylor in 1995)
- In $\mathcal{N}=1$ language $\Phi^a = (A^a, \psi^a, F^a),\ V^a = (v_m^a, \lambda^a, D^a)$
- noncanonical kinetic term from the Kähler ptl (∴ LEEA) and gauged .
- three parameter $e,\ m$ and ξ

The model predicts:

- $C_i{}^j = 4m\xi\tau_1 \xrightarrow{90^{\rm o}{\rm rot.}} 4m\xi\tau_3$
- The scalar ptl VEV
  $\langle\langle\mathcal{V}\rangle\rangle = \mp 2m\xi = 2|m\xi|$
- ∴ Half of the supercharges annihilates the vacuum while the remaining half takes $\infty \sim |m\xi| \int d^4x$ matrix elements.

∴ Partial Breaking of Extended SUSY is a Reality.

Contents

II). $\mathcal{N}=2$ Strategy and Our Model

- $\mathcal{N}=1$ superspace ensures $\mathcal{N}=1$ SUSY.

- Our action will be made invariant under

$$\mathfrak{R}:\begin{cases}\boldsymbol{\lambda}_i^a=\begin{pmatrix}\lambda^a\\ \psi^a\end{pmatrix}\to\boldsymbol{\lambda}^{ia}=\begin{pmatrix}\psi^a\\ -\lambda^a\end{pmatrix}=R\boldsymbol{\lambda}_i^a R^{-1}\\ \xi\to-\xi\end{cases}$$

- Our $\mathcal{N}=2$ transformations $\boldsymbol{\delta}$ are covariant w.r.t. $\mathfrak{R}$ $\quad\therefore\ \boldsymbol{\delta}S(\xi)=0\ \ \mathcal{N}=2$ SUSY

- To be more pedagogical, $R\delta^{(1,\xi)}_{\eta_1=\theta}R^{-1}\equiv\delta^{(2,-\xi)}_{\eta_2=\theta}$
  so that $0=\delta^{(2,\xi)}_{\eta_2=\theta}S(\xi)$ follows from $R\delta^{(1,\xi)}_{\eta_1=\theta}S(\xi)R^{-1}=0$

- Kähler: kinetic term for A

$$\mathcal{L}_K=\int d^2\theta d^2\bar{\theta}\ K(\Phi^a,\bar{\Phi}^a),\qquad K=\frac{i}{2}(\Phi^a\bar{\mathcal{F}}_a-\bar{\Phi}^a\mathcal{F}_a),$$

The Kähler metric $g_{ab^*}=\partial_a\partial_{b^*}K$ admits U(N) isometry generated by Killing vectors

$$k_a^b=-ig^{bc^*}\partial_{c^*}\mathfrak{D}_a,\quad \underline{\mathfrak{D}_a=-\frac{1}{2}(\mathcal{F}_b f^b_{ac}\bar{A}^c+\bar{\mathcal{F}}_b f^b_{ac}A^c)}\ .$$

• U(N) gauging $\mathcal{L}_\Gamma = \int d^2\theta d^2\bar\theta \Gamma, \quad \Gamma = \left[\int_0^1 d\alpha e^{\frac{i}{2}\alpha v^a(k_a - k_a^*)} v^c \mathfrak{D}_c\right]_{v^a \to V^a}$

• gauge kinetic action $\mathcal{L}_{\mathcal{W}^2} = -\frac{i}{4}\int d^2\theta \tau_{ab}\mathcal{W}^a\mathcal{W}^b + c.c. \quad , \quad \mathcal{W}_\alpha = -\frac{1}{4}\bar D \bar D e^{-V} D_\alpha e^V$

• superpotential $\mathcal{L}_W = \int d^2\theta \; W(\Phi) + c.c.$

• Fayet-Iliopoulos D-term $\mathcal{L}_D = \xi \int d^2\theta d^2\bar\theta V^0 = \sqrt{2}\xi D^0,$

<u>The total action</u> $\mathcal{L} = \mathcal{L}_K + \mathcal{L}_\Gamma + \mathcal{L}_{\mathcal{W}^2} + \mathcal{L}_W + \mathcal{L}_D$

• Impose $\mathfrak{R}$ invariance. $-\frac{i}{4} g^{cd^*}\partial_c \tau_{ab}\partial_{d^*}\overline{W} = \frac{1}{2} g^{cd^*}\partial_c W g_{ad^*,b} - \frac{1}{2}\partial_a\partial_b W,$

which is solved by

$$W = eA^0 + m\mathcal{F}_0$$
$$\tau_{ab} = \mathcal{F}_{ab}$$

III). Transformation Laws

Doublet of fermions $\boldsymbol{\lambda}_I{}^a \equiv \begin{pmatrix} \lambda^a \\ \psi^a \end{pmatrix}$ & parameters $\boldsymbol{\eta}_I \equiv \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$.

covariantize δ_1 w.r.t. $\mathfrak{R}$

$$\boldsymbol{\delta\lambda}_J{}^a = (\sigma^{mn}\boldsymbol{\eta}_J)v^a_{mn} + \sqrt{2}i(\sigma^m\bar{\boldsymbol{\eta}}_J)\mathcal{D}_m A^a + i(\boldsymbol{\tau}\cdot\boldsymbol{D}^a)_J{}^K\boldsymbol{\eta}_K - \frac{1}{2}\boldsymbol{\eta}_J f^a_{\ bc}A^{*b}A^c \ .$$

$\boldsymbol{D}^a$; three-vectors

$$\boldsymbol{D}^a = \hat{\boldsymbol{D}}^a - \sqrt{2}g^{ab^*}\partial_{b^*}\left(\boldsymbol{\mathcal{E}}A^{*0} + \boldsymbol{\mathcal{M}}\mathcal{F}_0^*\right) .$$

fermion bilinears

$$\boldsymbol{\mathcal{E}} = (0,\ -e,\ \xi)\ , \quad \boldsymbol{\mathcal{M}} = (0,\ -m,\ 0),$$

- $\boldsymbol{D}^a$ is complex : $\mathrm{Im}\boldsymbol{D}^a = \delta^a{}_0(0, -\sqrt{2}m, 0)$

$\mathsf{C}_i{}^j = +4m\xi(\boldsymbol{\tau}_1)_i{}^j$

cf. $\boldsymbol{\tau}\cdot\boldsymbol{D}^{*b}\boldsymbol{\tau}\cdot\boldsymbol{D}^a = \boldsymbol{D}^{*b}\cdot\boldsymbol{D}^a\mathbf{1} + i\left(\boldsymbol{D}^{*b}\times\boldsymbol{D}^a\right)\cdot\boldsymbol{\tau}$

IV). Analysis of Vacua

The scalar potential of the model, $\mathcal{V} = -\mathcal{L}_{\rm pot}$ is

$$\mathcal{V} = g^{ab}\left(\frac{1}{8}\mathfrak{D}_a\mathfrak{D}_b + \xi^2\delta^0_a\delta^0_b + \partial_a W \partial_{b^*} W^*\right) = \frac{1}{8}g_{bc}\mathfrak{D}^b\mathfrak{D}^c + \frac{1}{2}g^{bc}\tilde{\boldsymbol{D}}^*_b \cdot \tilde{\boldsymbol{D}}_c$$

where

$$\mathfrak{D}^a = g^{ab}\mathfrak{D}_b = -if^a_{cd}A^{*c}A^d ,$$

$$\tilde{\boldsymbol{D}}_b \equiv g_{ba}\tilde{\boldsymbol{D}}^a = -\sqrt{2}\partial_{b^*}\left(\boldsymbol{\mathcal{E}}A^{*0} + \boldsymbol{\mathcal{M}}\mathcal{F}^*_0\right) = \sqrt{2}\begin{pmatrix} 0 \\ \partial_{b^*}W^* \\ -\xi\delta_b{}^0 \end{pmatrix} .$$

$$\frac{\partial\mathcal{V}}{\partial A^a} = \frac{1}{4}g_{bc}\partial_a\mathfrak{D}^b\mathfrak{D}^c + \frac{1}{8}\partial_a g_{bc}\mathfrak{D}^b\mathfrak{D}^c + \frac{i}{4}\mathcal{F}_{abc}\tilde{\boldsymbol{D}}^b \cdot \tilde{\boldsymbol{D}}^c.$$

In order to examine general phases, let indices $a = i \cup r$ and $i\,(r)$ label the (non) Cartan generators of $U(N)$ and assume $\langle A^r\rangle = 0$.

$$\therefore \quad 0 = \frac{i}{4}\langle\mathcal{F}_{abc}\boldsymbol{D}^b \cdot \boldsymbol{D}^c\rangle \quad \text{(with tilde dropped)}$$

generic form of $\mathcal{F} \Rightarrow \langle\mathcal{F}_{rs}\rangle \propto \delta_{rs}$, $\langle\mathcal{F}_{ir}\rangle = 0$ $\therefore \langle g_{ir}\rangle = \langle g^{ir}\rangle = 0$ and $0 = \langle\mathcal{F}_{ijr}\rangle$

$$\therefore 0 = \frac{i}{4}\langle\mathcal{F}_{ijk}\boldsymbol{D}^j \cdot \boldsymbol{D}^k\rangle$$

• Eigenvalue bases

$$(\mathsf{t}_{\underline{i}})_j{}^k = \delta_{\underline{i}}{}^k \delta_j{}^{\underline{i}} \qquad \text{and} \qquad \langle\Phi\rangle = \langle\lambda^{\underline{i}}\rangle t_{\underline{i}}$$

$$\langle\mathcal{F}\rangle = \sum_{\underline{i}=1}^{N}\sum_{\ell}\frac{C_\ell}{\ell!}(\lambda^{\underline{i}})^\ell \ .$$

$$\begin{aligned}
\therefore \quad \langle\langle\mathcal{F}_{\underline{jj}}\rangle\rangle &= -2(\frac{e}{m} \mp i\frac{\xi}{m}) = -2\zeta, \\
\langle\langle g_{\underline{jj}}\rangle\rangle &= \mp 2\frac{\xi}{m}, \\
\langle\langle \boldsymbol{D}^{\underline{j}}\rangle\rangle &= \frac{m}{\sqrt{N}}\begin{pmatrix} 0 \\ -i \\ \pm 1 \end{pmatrix}
\end{aligned}$$

V). Partially Broken Supersymmetry and Nambu-Goldstone Fermion

$\langle\langle \boldsymbol{D}^r \rangle\rangle = 0$, $\quad \therefore \ \langle\langle \boldsymbol{\delta\lambda}_I^r \rangle\rangle = 0$.

For $\boldsymbol{\delta\lambda}_I^{\underline{i}}$, the 2×2 matrix $\boldsymbol{\tau} \cdot \boldsymbol{D}^{\underline{i}}$ diagonalized as

$$\langle\langle \delta \left(\frac{\lambda^{\underline{i}} \pm \psi^{\underline{i}}}{\sqrt{2}} \right) \rangle\rangle = \mp \frac{1}{\sqrt{2}} \langle\langle D_2^{\underline{i}} \mp i D_3^{\underline{i}} \rangle\rangle (\eta_1 \mp \eta_2) \ .$$

Suppose that $+$ sign is chosen for ${}^\forall \underline{i} = 1 \sim N$.

$$\begin{aligned} \langle\langle \delta \left(\frac{\lambda^{\underline{i}} + \psi^{\underline{i}}}{\sqrt{2}} \right) \rangle\rangle &= im\sqrt{\frac{2}{N}} (\eta_1 - \eta_2) \ , \\ \langle\langle \delta \left(\frac{\lambda^{\underline{i}} - \psi^{\underline{i}}}{\sqrt{2}} \right) \rangle\rangle &= 0 \ . \end{aligned}$$

$\mathcal{N} = 2$ supersymmetry is spontaneously broken to $\mathcal{N} = 1$ and we obtain the Nambu-Goldstone fermion $\frac{1}{\sqrt{2}}(\lambda^0 + \psi^0)$ associated with the overall $U(1)$ part.

Conclusion: $\langle\langle g_{\underline{jj}} \rangle\rangle = \mp 2 \frac{\xi}{m} = 2 \left| \frac{\xi}{m} \right|$. $\qquad \mathcal{N} = 1$ vacua

VII). Simultaneous Occurrence of Gauge Symmetry Breaking

- The vacuum condition:

$$\langle\langle \mathcal{F}_{\underline{ii}} \rangle\rangle + 2\zeta = 0 \quad \text{with} \quad \zeta \equiv \frac{e}{m} \pm i\frac{\xi}{m} \quad \text{and} \quad \langle\langle \mathcal{F}_{\underline{ii}} \rangle\rangle = \sum_{\ell}^{k=\deg \mathcal{F}} \frac{C_l}{(\ell-2)!} (\lambda^{\underline{i}})^{\ell-2}$$

Thus each $\lambda^{\underline{i}}$ is one of these $k-2$ complex roots denoted by $\lambda^{(\ell,\pm)}$. This defines a grouping of N eigenvalues into $k-2$ sets and hence determines a breaking pattern of $U(N)$ gauge symmetry into a product gauge group $\prod_{i=1}^{k-2} U(N_i)$ with $\sum_{i=1}^{k-2} N_i = N$

VIII). Mass Spectrum

index labelling $a, b \cdots = \begin{cases} 1)\ \underline{i}, \underline{j} \cdots \text{ for the diagonal generators} \\ 2)\ r, s \cdots \text{ for the non-diagonal generators} \\ \quad \begin{cases} 2)\text{-}1)\ r', s' \cdots \text{ unbroken ones} \\ 2)\text{-}2)\ \mu, \nu \cdots \text{ broken ones} \end{cases} \end{cases}$ $\alpha, \beta, \cdots = 1) \cup 2)\text{-}1)$

fermion mass spectrum

i) $\frac{1}{2}\boldsymbol{\lambda}^{\underline{i}I}(M_{\underline{ii}})_I{}^J\boldsymbol{\lambda}^{\underline{i}}_J$ term with $(M_{\underline{ii}})_I{}^J = \frac{1}{2\sqrt{2}}\langle\langle\mathcal{F}_{\underline{iii}}\rangle\rangle(\boldsymbol{\tau}\cdot\langle\langle\boldsymbol{D}^{\underline{i}}\rangle\rangle)$

$$\Rightarrow \quad \begin{array}{ll} \frac{1}{\sqrt{2}}\left(\lambda^{\underline{i}} \pm \psi^{\underline{i}}\right) & \text{massless,} \\ \frac{1}{\sqrt{2}}\left(\lambda^{\underline{i}} \mp \psi^{\underline{i}}\right) & \text{mass } \ |m\langle\langle g^{\underline{ii}}\rangle\rangle\langle\langle\mathcal{F}_{0\underline{ii}}\rangle\rangle| \end{array}$$

ii) $\frac{1}{2}\boldsymbol{\lambda}^{rI}(M_{rs})_I{}^J\boldsymbol{\lambda}^s_J$ term with $(M_{rs})_I{}^J = \begin{pmatrix} \mp m_{rs} + m'_{rs} & m_{rs} \\ -m_{rs} & \pm m_{rs} + m'_{rs} \end{pmatrix}$

	$m_{rs} = -\frac{1}{2}m\langle\langle\mathcal{F}_{0rs}\rangle\rangle$	$m'_{rs} = \frac{1}{2\sqrt{2}}\left(\langle\langle g_{tr}\rangle\rangle f^t_{s\underline{i}}\lambda^{*\underline{i}} - \langle\langle g_{ts}\rangle\rangle f^t_{r\underline{i}}\lambda^{*\underline{i}}\right)$
$r's'$	the same as i)	0
$\mu\nu$	0	mass $\left\lvert \frac{1}{\sqrt{2}} f^{\nu}_{\mu\underline{i}}\lambda^{\underline{i}} \right\rvert$

$$\because \ \langle\langle\mathcal{F}_{0\mu\mu}\rangle\rangle = \frac{\langle\langle\mathcal{F}_{\underline{ii}}\rangle\rangle - \langle\langle\mathcal{F}_{\underline{jj}}\rangle\rangle}{2\sqrt{2N}(\lambda^{\underline{i}} - \lambda^{\underline{j}})} = \frac{-2\zeta - (-2\zeta)}{2\sqrt{2N}(\lambda^{\underline{i}} - \lambda^{\underline{j}})} = 0\ .$$

boson mass spectrum $\qquad \mathcal{V} = \mathcal{V}_1 + \mathcal{V}_2 \qquad \overrightarrow{\delta A^{\mu}} = \begin{pmatrix} \delta A^{\mu} \\ \delta A^{*\mu} \end{pmatrix}$

- $\langle\langle \delta\delta \mathcal{V}_1 \rangle\rangle = \frac{1}{8} \sum_{\tilde{\mu}} \langle\langle \delta \mathfrak{D}^{\tilde{\mu}} \rangle\rangle \langle\langle g_{\tilde{\mu}\tilde{\mu}} \rangle\rangle \langle\langle \delta \mathfrak{D}^{\tilde{\mu}} \rangle\rangle$

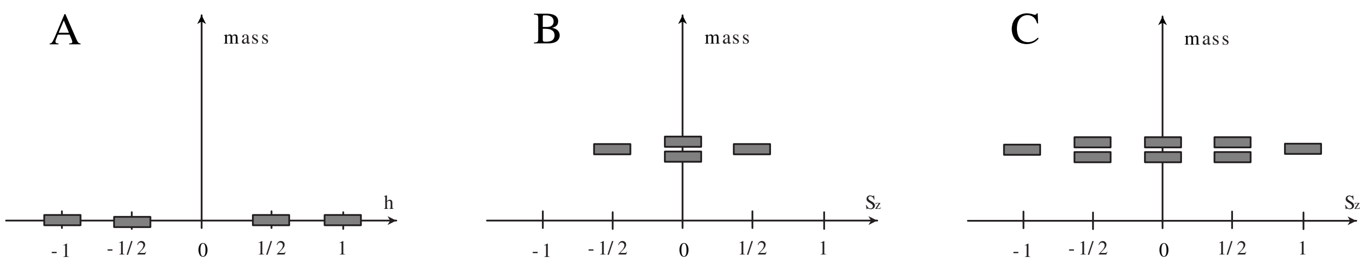
after the separation of the NG zero modes $= \frac{1}{4} \sum_{\tilde{\mu}} \langle\langle g_{\tilde{\mu}\tilde{\mu}} \rangle\rangle | f^{\tilde{\mu}}_{\underline{j}\mu} \lambda^{\underline{j}} |^2 \left(\mathcal{P}^{\tilde{\mu}}{}_{\mu} \overrightarrow{\delta A^{\mu}} \right)^{*t} \cdot \left(\mathcal{P}^{\tilde{\mu}}{}_{\mu} \overrightarrow{\delta A^{\mu}} \right)$

- $\langle\langle \delta\delta \mathcal{V}_2 \rangle\rangle = 2m^2 \delta A^{*\alpha} \langle\langle \mathcal{F}^{*}_{0\alpha\alpha} \rangle\rangle \langle\langle g^{\alpha\alpha} \rangle\rangle \langle\langle \mathcal{F}_{0\alpha\alpha} \rangle\rangle \delta A^{\alpha}$ (zero for the μ directions)

- the table

field	mass	label	# of polarization states
v^{α}_{m}	0	A	$2d_u (d_u \equiv \dim \prod_i U(N_i))$
v^{μ}_{m}	$\lvert \frac{1}{\sqrt{2}} f^{\nu}_{\mu \underline{i}} \lambda^{\underline{i}} \rvert$	C	$3(N^2 - d_u)$
$\frac{1}{\sqrt{2}}(\lambda^{\alpha} \pm \psi^{\alpha})$	0	A	$2d_u$
$\frac{1}{\sqrt{2}}(\lambda^{\alpha} \mp \psi^{\alpha})$	$\lvert m \langle\langle g^{\alpha\alpha} \rangle\rangle \langle\langle \mathcal{F}_{0\alpha\alpha} \rangle\rangle \rvert$	B	$2d_u$
$\boldsymbol{\lambda}^{\mu}_{I}$	$\lvert \frac{1}{\sqrt{2}} f^{\nu}_{\mu \underline{i}} \lambda^{\underline{i}} \rvert$	C	$4(N^2 - d_u)$
A^{α}	$\lvert m \langle\langle g^{\alpha\alpha} \rangle\rangle \langle\langle \mathcal{F}_{0\alpha\alpha} \rangle\rangle \rvert$	B	$2d_u$
$\mathcal{P}^{\tilde{\mu}}_{\mu} A^{\mu}$	$\lvert \frac{1}{\sqrt{2}} f^{\nu}_{\mu \underline{i}} \lambda^{\underline{i}} \rvert$	C	$N^2 - d_u$

- $\mathcal{N} = 1$ supermultiplet



IX). Perspectives

• how to add $U(N)$ adjoint matters (Harmonic superspace formalism),

$$\mathcal{N}=4 \rightarrow \mathcal{N}=2 \rightarrow \mathcal{N}=1.$$

• how to add fundamental matters, making the mirror particles heavy.

• supply $\langle \text{the prepotential derivatives} \rangle$ via string theory and/or Rieman surface calculation.

• the role played by $e,\ m$ and ξ in comparison with Dijkgraaf-Vafa (Λ)

• setting up strings/branes context.

cf.Kaste,Partouche hep-th/0409303

and many more.

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