GEOMETRY OF RANK REDUCTION

based on SF, Nilles, Wingerter,
Phys. Rev D, hep-th/0504117

1. ROTATIONAL EMBEDDING
2. RANK REDUCTION
3. CONCLUSION

① ROTATIONAL EMBEDDING

Ibáñez, Nilles, Quevedo 1987

- 5d GUT on $S^1/\mathbb{Z}_2$ $(y \to -y)$
- GUT group in Cartan-Weyl

$$H_1, \ldots, H_r \ , \ E_\alpha, E_\beta, \ldots$$

- $$[H_I, E_\alpha] = \alpha_I E_\alpha$$

embed $\mathbb{Z}_2$ action

$$\vec{H} \longrightarrow \Theta \vec{H}, \quad \vec{\alpha} \longrightarrow \Theta \vec{\alpha}$$

$\Theta = r \times r$ matrix

$\Theta^2 = 1 \quad (\mathbb{Z}_2)$

$\Theta \vec{\alpha}$ is again root
(lattice automorphism)

$$E_{\vec{\alpha}} \longrightarrow c_\alpha E_{\Theta \vec{\alpha}}$$

- projects out non-invariant Cartan directions
- but does not reduce the rank (yet)
- new Cartan generators

$$E_\alpha + c_\alpha E_{\theta\alpha}$$

Example: E_6

$E_6 = SO(10) \times U(1) + \text{spinors}$

$78 = 45 + 1 + 16 + \overline{16}$

6 Cartans: $(H_1, H_1, H_1, H_2, H_3, H_4, H_5, H_6)$

SO(10) roots: $(0,0,0,\underline{\pm 1, \pm 1, 0, 0, 0})$ 40

16 $\left(\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}\right)$

$\overline{16}$ $\left(-\tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}\right)$

16 & $\overline{16}$ even # minus

$\rightarrow$ 72 roots of E_6

take:

$$\Theta = \begin{pmatrix} \mathbb{1}_{3\times 3} & & & & \\ & 0 & 1 & & \\ & 1 & 0 & & \\ & & & 1 & & \\ & & & & 0 & -1 \\ & & & & -1 & 0 \end{pmatrix}$$

flips $4 \leftrightarrow 5$

& $7 \leftrightarrow -8$ entry

unbroken gauge group

(recall: $(H_1, H_1, H_1, H_2, H_3, H_4, H_5, H_6)$)

- H_1, $H_2 + H_3$, H_4, $H_5 - H_6$ 4
- E_α with $\Theta\alpha = \alpha$ 12

$\pm(0,0,0,1,1,0,0,0)$ & $\pm(0,0,0,0,0,0,+1,-1)$ (4)

$\left(\pm\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right), \pm\left(\frac{1}{2},\frac{1}{2}\right), \pm\frac{1}{2}, \pm\left(\frac{1}{2},-\frac{1}{2}\right)\right)$ $\left(\frac{1}{2}\cdot 2^4 = 8\right)$

- for $\alpha \neq \Theta\alpha$: $E_\alpha + c_\alpha E_{\Theta\alpha}$

$(72-12)/2 =$ 30

dimension of unbroken gauge group is 46.

new Cartan algebra

$H_1,\ H_2+H_3,\ H_4,\ H_5-H_6$

& $E_\delta + c_\delta E_{-\delta},\ E_\gamma + c_\gamma E_{-\gamma}$

with

$\delta = (0,0,0,1,-1,0,0,0)$

$\gamma = (0,0,0,0,0,0,1,1)$

$\rightarrow$ rank 6

fits with $SO(10) \times U(1)$

But: proper treatment requires construction of algebra autom.

(e.g. Schellekens & Warner 1988)

Θ = product of Weyl reflections

Weyl reflection $\longleftrightarrow$ conjugation with

$$\tau_\beta = \exp\left\{\frac{i\pi}{2}(E_\beta + E_{-\beta})\right\}$$

$\Rightarrow$ order 4 automorphism ↯

$\Rightarrow$ supplement with shift

$\tau \to \tau \exp\{v \cdot H\} \Rightarrow$ order 2 ✓

(SF, Nilles, Wingerter '05)

WILSON LINES

discrete WL: $A_5 \sim \vec{a}\,\vec{H}$ with $\Theta \vec{H} = \vec{H}$

$\mathbb{Z}_2$ project.: $A_5 \overset{!}{=} -A_5 \rightarrow \underline{2\vec{a} \in Q^{\vee}}$

contin. WL: $A_5 \sim \vec{\lambda}\,\vec{H}$ with $\Theta \vec{H} = \underline{-}\vec{H}$

$\mathbb{Z}_2$ project.: $A_5 \overset{!}{=} A_5$

$\rightarrow$ $\underline{\vec{\lambda} \text{ continuous}}$

alternative pict.: $y \rightarrow y + 2\pi R_5$

accompanied by conjugation with

$\exp\{2\pi i\,\vec{a}\cdot\vec{H}\}$ or $\exp\{2\pi i\,\vec{\lambda}\cdot\vec{H}\}$

cont. WL: $\lambda_1(H_2-H_3)+\lambda_2(H_5+H_6)$

· take $\lambda_1 \neq 0$ & $\lambda_2 = 0$

· CSA of $SO(10)\times U(1)$:

$H_1 \cdot H_2+H_3,\ H_4,\ H_5-H_6,\ \underline{E_\delta + c_\delta E_{-\delta}},\ E_\gamma + c_\gamma E_\gamma$

with: $\delta=(0,0,0,1,-1,0,0,0)$ & $\gamma=(0,0,0,0,0,0,1,1)$

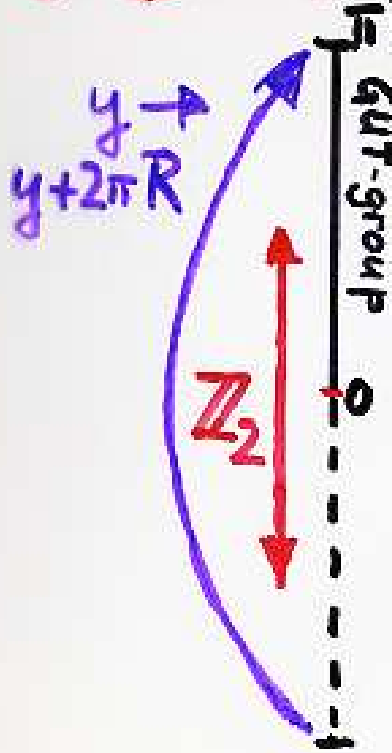
→ $E_\delta + c_\delta E_{-\delta}$ projected out.

→ <u>rank reduced</u>

include the rest ...

$SO(10)\times U(1) \longrightarrow SU(5)\times U(1)$

GEOMETRIC PICTURE



"sees" WL

$$E_\alpha + c_\alpha e^{2\pi i \vec{\lambda}\vec{\alpha}} E_{\Theta\alpha}$$

"sees" $\mathbb{Z}_2$

$$E_\alpha + c_\alpha E_{\Theta\alpha}$$

$\Rightarrow$ same algebra, embedded differently into bulk group.

symmetry breaking

$SO(10)\times U(1)$

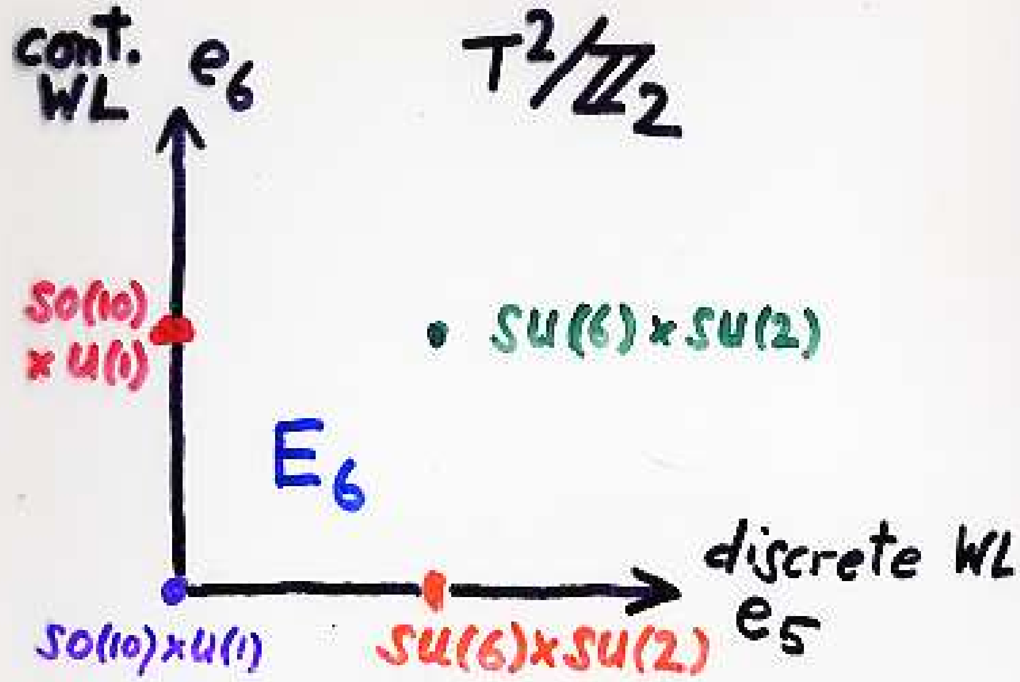
E_6

$SO(10)\times U(1)$

$\left.\begin{matrix} \\ \\ \end{matrix}\right\}$ intersect in $SU(5)\times U(1)$

- continuous breaking.

+ one direction with discrete WL



$G' \cap G' = SU(5) \times U(1)$

$G'' \cap G' = SU(4) \times SU(2) \times SU(2) \times U(1)$

$\bigcap_{\text{ALL}} = SU(3) \times SU(2) \times U(1) \times U(1)$ in 4d

CONCLUSIONS

- geometric picture of continuous Wilson line breaking drawn.
- model:

$$E_6 \xrightarrow{\mathbb{Z}_2} SO(10) \xrightarrow{\text{d. WL}} PS \xrightarrow{\text{c. WL}} SM$$

- outlook: realise electro-weak symmetry breaking with continuous Wilson line.

work in progress.