Global fits of SUSY parameters from collider observables

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On behalf of the SFitter and Fittino authors:
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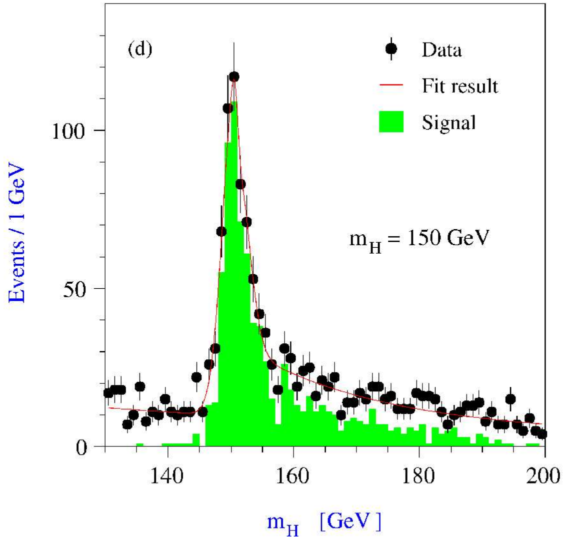
The task

Once SUSY has been established in experiments, Lagrangian parameters need to be extracted from measurements.

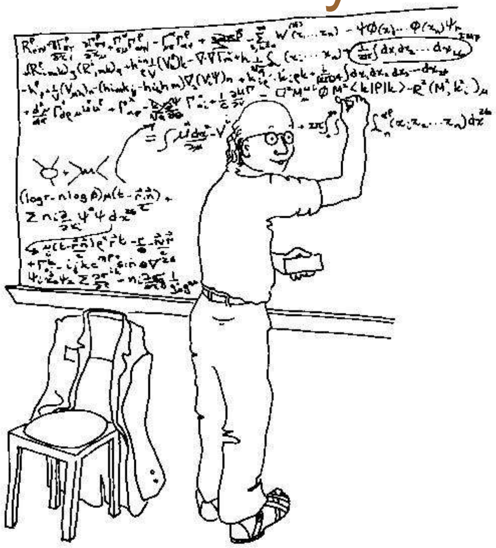
Stumbling block: Lagrangian parameters ≠ observables

Experiment:



Mapping

Theory:



Observables:

m(h)

BR(h→ gg)

$\sigma(e^+e^- \to \chi_1^+\chi_1^-)$ BR($\chi_1^+ \to \mathrm{Stau}_1\ \nu$) BR($\chi_1^- \to \mathrm{Stau}_1\ \nu$)

etc.

Lagrangian parameters:

tan β

μ

M_1

etc.

The challenge

Need a procedure to connect observables to Lagrangian parameters within a certain theoretical framework

At tree level, some sectors (e. g. chargino, chargino+neutralino) can be treated separately.

At loop level, in principle every observable depends on every parameter.

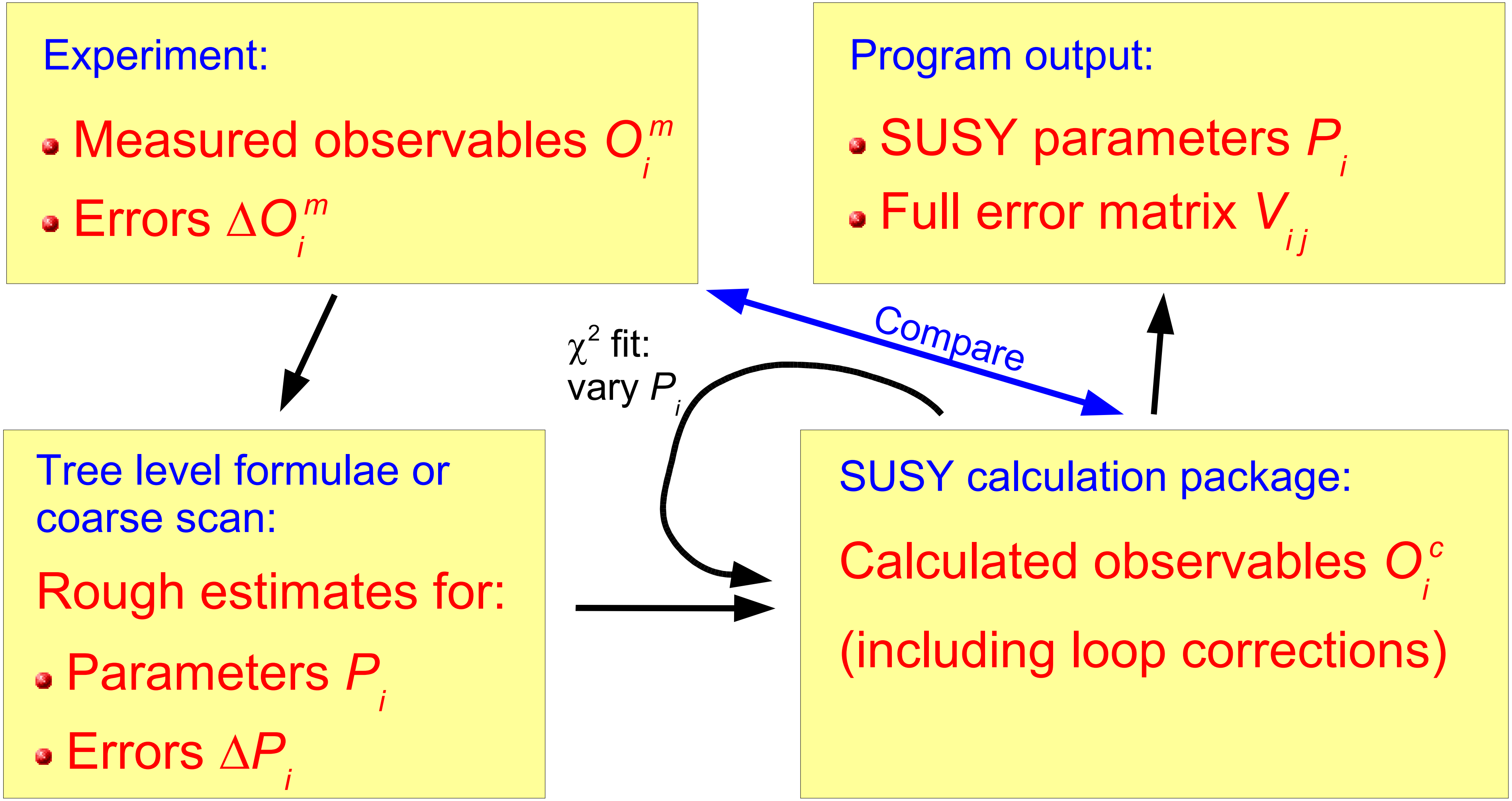
Complicated mutual dependence of the various parameters.

Approximate picture (not quite correct since non-linear mapping):

$$\begin{bmatrix} P_1 \\ P_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} \square & & 0 \\ & \square & \\ 0 & \square & \ddots \end{bmatrix} \begin{bmatrix} O_1 \\ O_2 \\ \vdots \end{bmatrix} \qquad \begin{bmatrix} P_1 \\ P_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} \square & & \neq 0 \\ & \square & \\ \neq 0 & \square & \ddots \end{bmatrix} \begin{bmatrix} O_1 \\ O_2 \\ \vdots \end{bmatrix}$$

Tree level Loop level

The solution: Iterative approach



Experiment:

- Measured observables O_i^m
- Errors ΔO_i^m

Program output:

- SUSY parameters P_i
- Full error matrix V_{ij}

Compare

χ^2 fit:
vary P_i

Tree level formulae or coarse scan:

Rough estimates for:

- Parameters P_i
- Errors ΔP_i

SUSY calculation package:

Calculated observables O_i^c

(including loop corrections)

SUSY fit packages

At present two programs are publicly available which determine SUSY Lagrangian parameters from collider observables using the described iterative technique:

- SFitter (R. Lafaye, T. Plehn, D. Zerwas)
  `http://cern.ch/sfitter`
- Fittino (P. Bechtle, K. Desch, P. W.)
  `http://www-flc.desy.de/fittino`

The ingredients are:

SFitter:

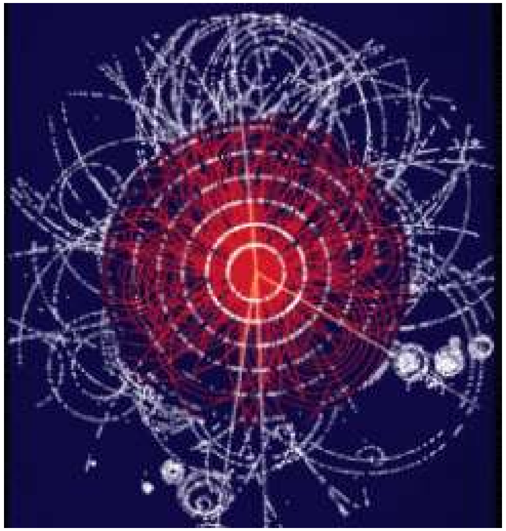
- SUSPECT or SOFTSUSY for masses
- MSMLIB for BR
- Prospino 2.0 for NLO σ_{pp}
- MINUIT for fit

Fittino:

- SPheno 2.2.2 for masses, BR, σ_{e+e-}
- Simulated Annealing + MINUIT for fit

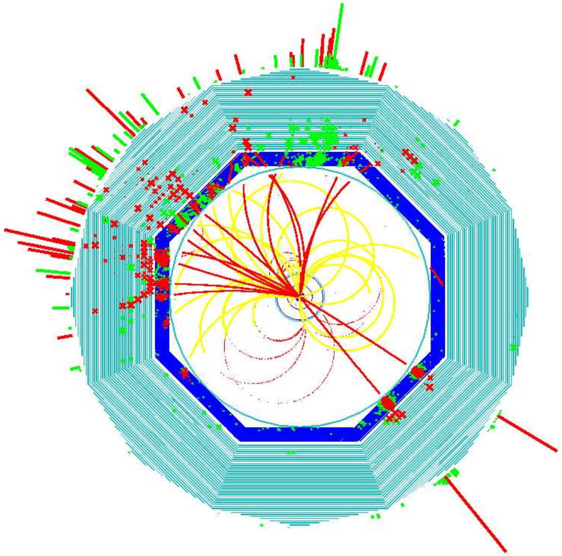
Both programs use SUSY Les Houches Accord for interfacing

Colliders to explore SUSY



Large Hadron Collider (LHC):

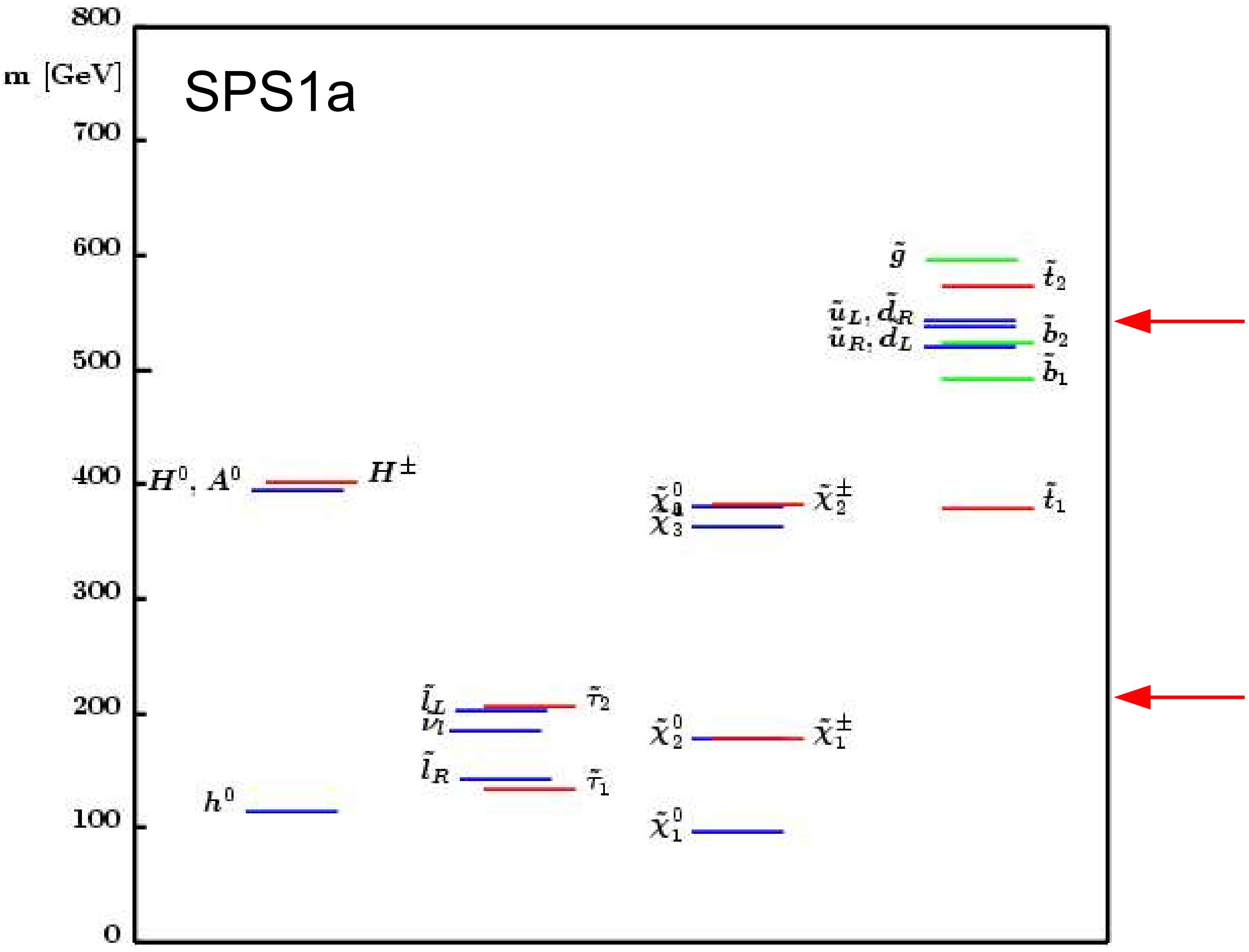
- high mass reach (several TeV) for squarks+gluinos
- colorless sparticles mainly through cascades
- modest accurary on masses 1-10 %
- rates subject to QCD/PDF uncertainties



International Linear Collider (ILC):

- precise spectroscopy: masses 0.1-1 % up to $\sum$ m = 1 TeV
- polarized cross-sections usable: ~ 1 %

An example spectrum



well measurable at LHC

precise spectroscopy at ILC

Fit assumptions

Without assuming a certain SUSY breaking scenario, the MSSM contains 105 parameters (masses, phases, mixing angles)

→ infeasible to determine all of them
(technical difficulties, lack of sensitive observables)

Simplifying assumptions:

- no CP violation (all phases = 0)
- no mixing between generations
- no mixing within first two generations
- Universality of same type sfermion mass parameters in first two generations

⇒ 18 SUSY parameters remain

mSUGRA fit

At beginning of LHC running, even 18 parameters are too many. Therefore assume specific SUSY breaking scenario to further reduce number of parameters → mSUGRA

Only 4½ parameters remain: tan β, m_0, $m_{½}$, A_0, sign(μ)

Using masses only yields following precisions for SPS1a:

SFitter

	SPS1a	ΔLHC	ΔILC	ΔLHC+ILC
m_0	100	**3.9**	**0.09**	**0.08**
$m_{1/2}$	250	**1.7**	**0.13**	**0.11**
tanβ	10	**1.1**	**0.12**	**0.12**
A0	-100	**33**	**4.8**	**4.3**

sign(μ) fixed

- ΔILC ≈ 1/10 ΔLHC
- only slight improvement from combined analysis (unification reduces impact of missing strongly interacting sparticles at ILC)

Masses versus edges

LHC does not directly measure masses but positions of edges in spectra (= functions of various masses).

Fitting edge positions instead of masses yields:

SFitter

	SPS1a	ΔLHC masses	ΔLHC edges
m_0	100	**3.9**	**1.2**
$m_{1/2}$	250	**1.7**	**1.0**
tanβ	10	**1.1**	**0.9**
A0	-100	**33**	**20**

sign(μ) fixed

using edges yields sizable difference

Explanation:

Δm_0	Effect on $m\ell_R$	Effect on $m\ell\ell$
1GeV	**0.7/5=0.14**	**0.4/0.08=5**

similar effect for $m_{1/2}$

Inclusion of correlations is needed for precise determination from masses

Impact of theoretical uncertainties

Assumed uncorrelated theoretical uncertainties:

Higgs	sleptons	Squarks,gluinos	Neutralinos, charginos
3GeV	1%	**3%**	**1%**

SFitter

	SPS1a	ΔLHC+ ILCexp	ΔLH+ ILCth
m_0	100	**0.08**	**1.2**
$m_{1/2}$	250	**0.11**	**0.7**
tanβ	10	**0.12**	**0.7**
A0	-100	**4.3**	**17**

Sensitivity reduced by an order of magnitude due to theoretical uncertainties

SFitter

	SPS1a	SoftSUSYup	ΔLHC+LC
m_0	100	**95.2**	**1.1**
$m_{1/2}$	250	**249.8**	**0.5**
tanβ	10	**9.82**	**0.5**
A0	-100	**-97**	**10**

down/up effect:

spectrum calculated with SUSPECT,
fit with SOFTSUSY,
m_0 incompatible

MSSM fit

Even better: No assumption on SUSY breaking in fit

Fit LE parameters to data and learn about SUSY breaking from extrapolation to high scale ("bottom-up approach")

Disadvantage:
Requires many precision measurements. Only possible with combined LHC and ILC inputs.

18 SUSY parameters (→ slide 8) + m_{top} fit performed for SPS1a' scenario (Definition: `http://spa.desy.de/spa`)

Input observables:

- masses from LHC and ILC
- σ_{e+e-}
- σ_{e+e-} x BR
- BR

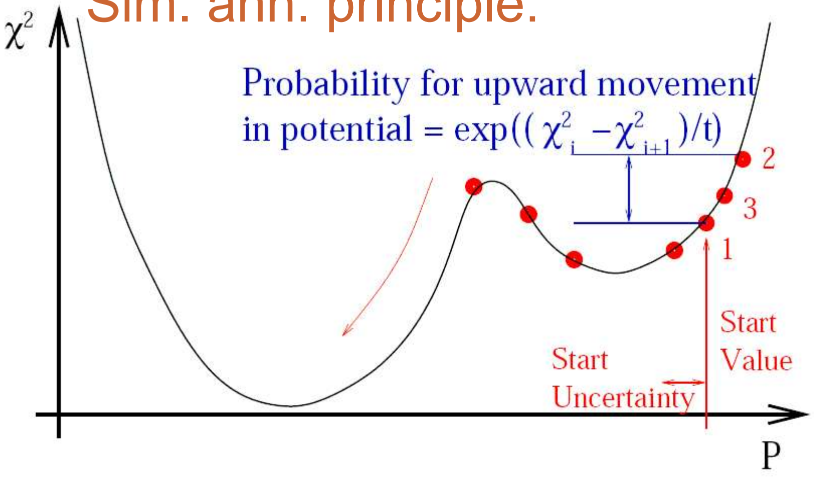
Fit strategy for MSSM fit

Fitting in high-dimensional space is a delicate business.

MINUIT turned out to be insufficient for minimization (local minima) and error estimation (too complex correlations) for this MSSM fit.

Simulated annealing has proven to be a robust algorithm.

Sim. ann. principle:



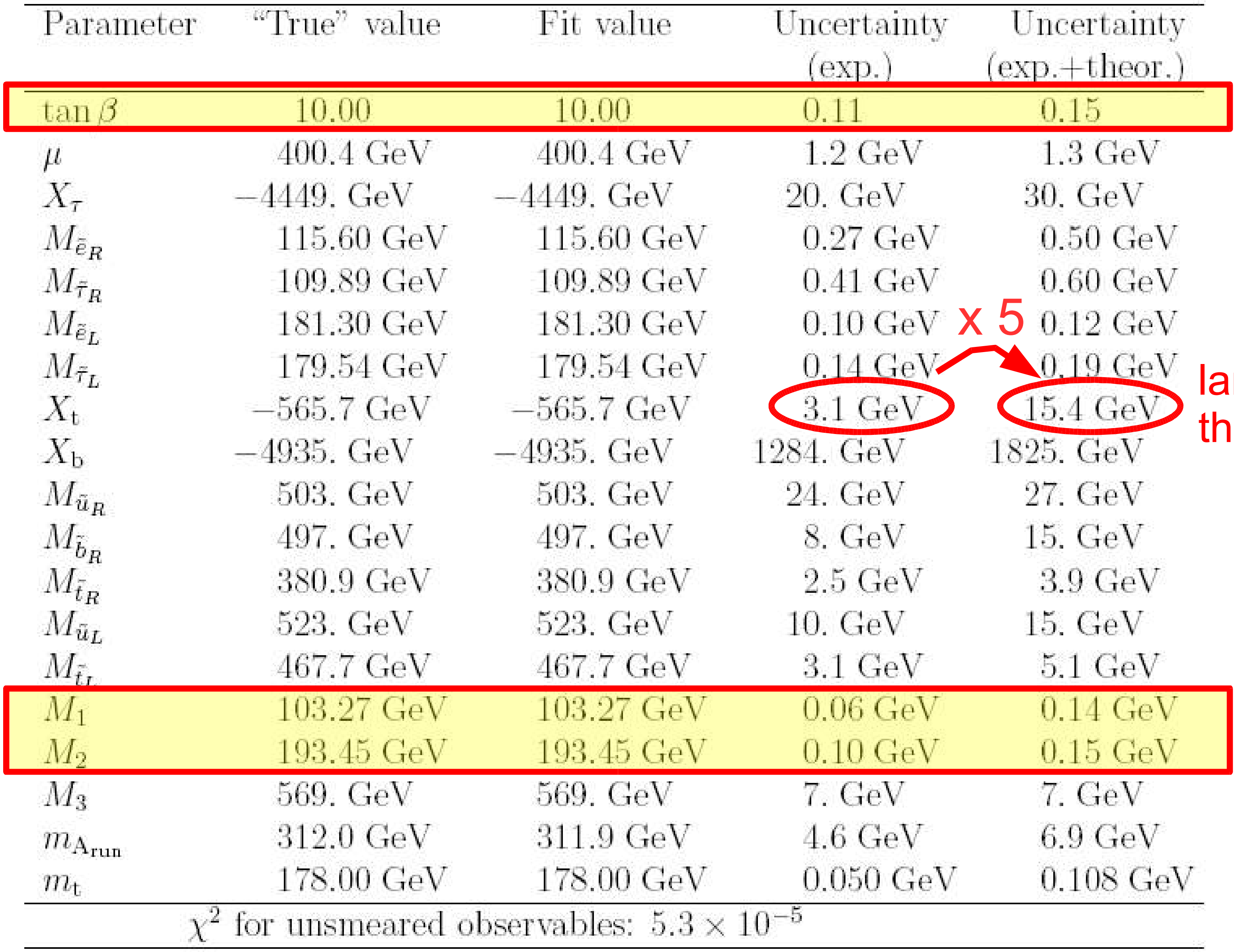
Fit strategy:

1. Sim. ann. minimization
2. MINUIT fit with start values from sim. ann.
3. Covariance matrix from many fits with smeared inputs

Disadvantage: CPU intensive

(but these days we have the grid!)

MSSM fit



Parameter	"True" value	Fit value	Uncertainty (exp.)	Uncertainty (exp.+theor.)
$\tan\beta$	10.00	10.00	0.11	0.15
μ	400.4 GeV	400.4 GeV	1.2 GeV	1.3 GeV
X_τ	−4449. GeV	−4449. GeV	20. GeV	30. GeV
$M_{\tilde{e}_R}$	115.60 GeV	115.60 GeV	0.27 GeV	0.50 GeV
$M_{\tilde{\tau}_R}$	109.89 GeV	109.89 GeV	0.41 GeV	0.60 GeV
$M_{\tilde{e}_L}$	181.30 GeV	181.30 GeV	0.10 GeV	0.12 GeV
$M_{\tilde{\tau}_L}$	179.54 GeV	179.54 GeV	0.14 GeV	0.19 GeV
X_{t}	−565.7 GeV	−565.7 GeV	3.1 GeV	15.4 GeV
X_{b}	−4935. GeV	−4935. GeV	1284. GeV	1825. GeV
$M_{\tilde{u}_R}$	503. GeV	503. GeV	24. GeV	27. GeV
$M_{\tilde{b}_R}$	497. GeV	497. GeV	8. GeV	15. GeV
$M_{\tilde{t}_R}$	380.9 GeV	380.9 GeV	2.5 GeV	3.9 GeV
$M_{\tilde{u}_L}$	523. GeV	523. GeV	10. GeV	15. GeV
$M_{\tilde{t}_L}$	467.7 GeV	467.7 GeV	3.1 GeV	5.1 GeV
M_1	103.27 GeV	103.27 GeV	0.06 GeV	0.14 GeV
M_2	193.45 GeV	193.45 GeV	0.10 GeV	0.15 GeV
M_3	569. GeV	569. GeV	7. GeV	7. GeV
$m_{\mathrm{A_{run}}}$	312.0 GeV	311.9 GeV	4.6 GeV	6.9 GeV
m_{t}	178.00 GeV	178.00 GeV	0.050 GeV	0.108 GeV

χ^2 for unsmeared observables: 5.3×10^{-5}

< 2 %

x 5

large impact of theory uncertainty

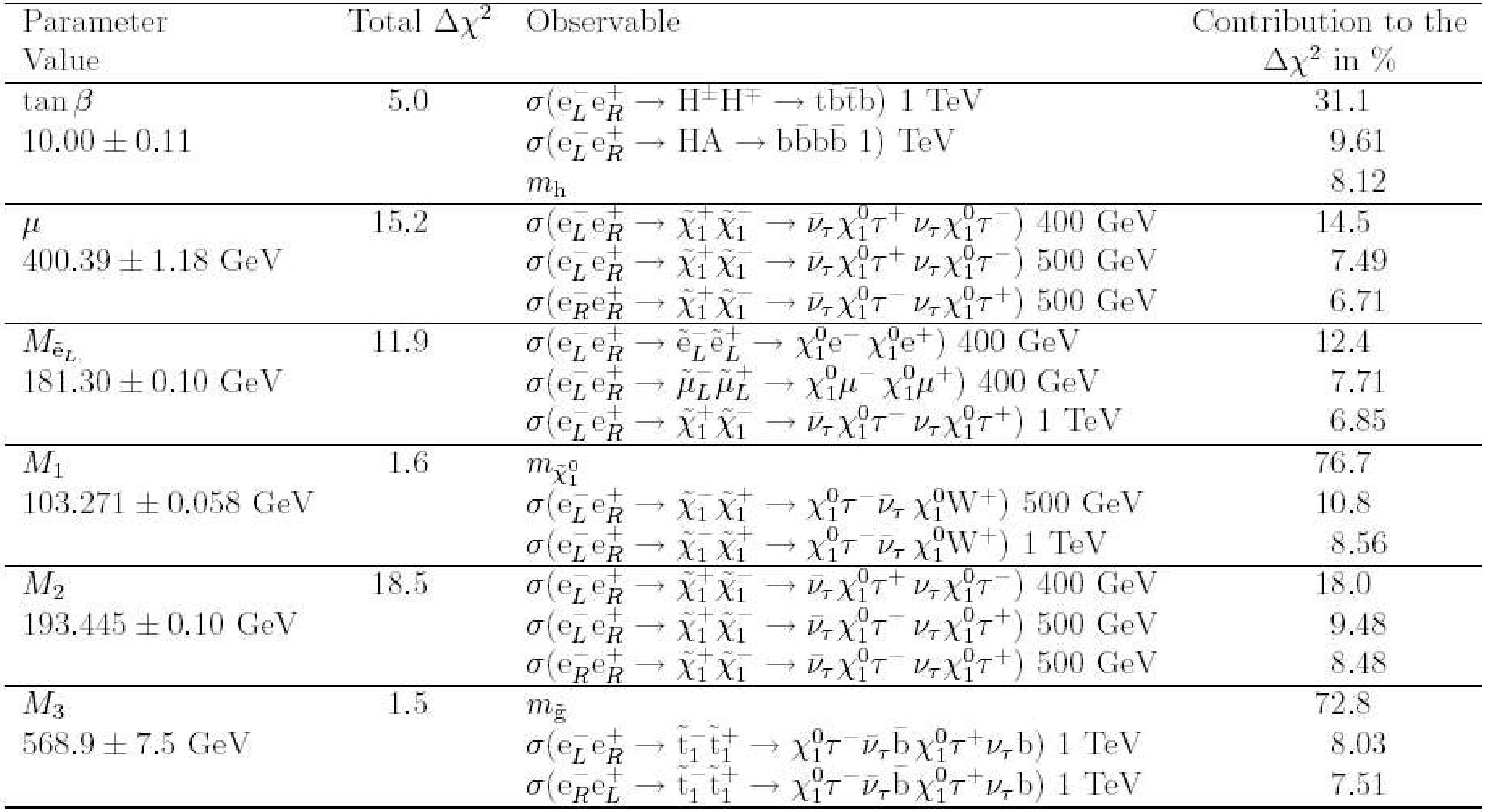
< 0.2 %

Important observables

What observables determine the precision of a parameter?

Look at $\Delta\chi^2 = \chi^2_{\pm1\sigma} - \chi^2_{min}$

Some examples:

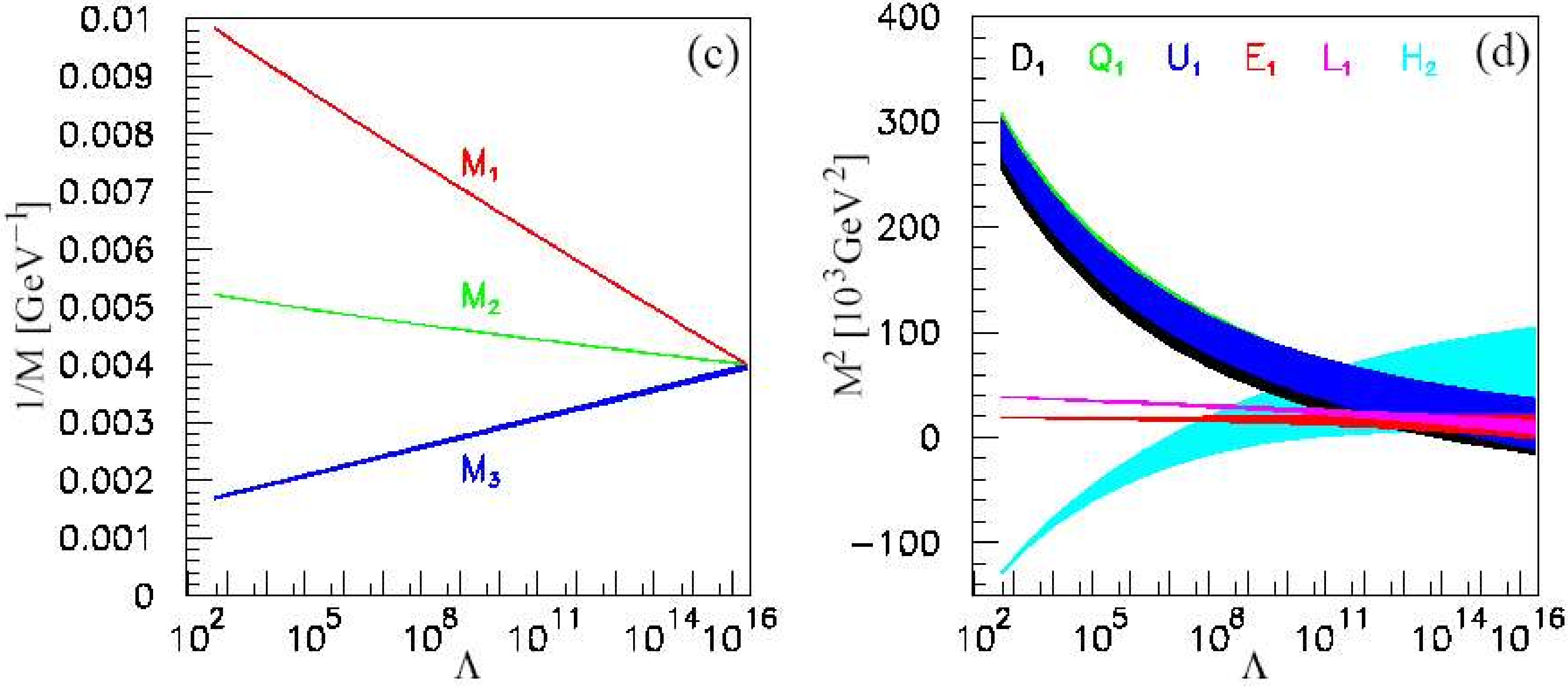


Parameter Value	Total $\Delta\chi^2$	Observable	Contribution to the $\Delta\chi^2$ in %
$\tan\beta$	5.0	$\sigma(\mathrm{e}_L^-\mathrm{e}_R^+ \to \mathrm{H}^\pm\mathrm{H}^\mp \to \mathrm{t}\bar{\mathrm{b}}\bar{\mathrm{t}}\mathrm{b})$ 1 TeV	31.1
10.00 ± 0.11		$\sigma(\mathrm{e}_L^-\mathrm{e}_R^+ \to \mathrm{HA} \to \mathrm{b}\bar{\mathrm{b}}\mathrm{b}\bar{\mathrm{b}}$ 1) TeV	9.61
		m_h	8.12
μ	15.2	$\sigma(\mathrm{e}_L^-\mathrm{e}_R^+ \to \tilde{\chi}_1^+\tilde{\chi}_1^- \to \bar{\nu}_\tau\chi_1^0\tau^+\,\nu_\tau\chi_1^0\tau^-)$ 400 GeV	14.5
400.39 ± 1.18 GeV		$\sigma(\mathrm{e}_L^-\mathrm{e}_R^+ \to \tilde{\chi}_1^+\tilde{\chi}_1^- \to \bar{\nu}_\tau\chi_1^0\tau^+\,\nu_\tau\chi_1^0\tau^-)$ 500 GeV	7.49
		$\sigma(\mathrm{e}_R^-\mathrm{e}_R^+ \to \tilde{\chi}_1^+\tilde{\chi}_1^- \to \bar{\nu}_\tau\chi_1^0\tau^-\,\nu_\tau\chi_1^0\tau^+)$ 500 GeV	6.71
$M_{\tilde{\mathrm{e}}_L}$	11.9	$\sigma(\mathrm{e}_L^-\mathrm{e}_R^+ \to \tilde{\mathrm{e}}_L^-\tilde{\mathrm{e}}_L^+ \to \chi_1^0\mathrm{e}^-\,\chi_1^0\mathrm{e}^+)$ 400 GeV	12.4
181.30 ± 0.10 GeV		$\sigma(\mathrm{e}_L^-\mathrm{e}_R^+ \to \tilde{\mu}_L\tilde{\mu}_L^+ \to \chi_1^0\mu^-\,\chi_1^0\mu^+)$ 400 GeV	7.71
		$\sigma(\mathrm{e}_L^-\mathrm{e}_R^+ \to \tilde{\chi}_1^+\tilde{\chi}_1^- \to \bar{\nu}_\tau\chi_1^0\tau^-\,\nu_\tau\chi_1^0\tau^+)$ 1 TeV	6.85
M_1	1.6	$m_{\tilde{\chi}_1^0}$	76.7
103.271 ± 0.058 GeV		$\sigma(\mathrm{e}_L^-\mathrm{e}_R^+ \to \tilde{\chi}_1^-\tilde{\chi}_1^+ \to \chi_1^0\tau^-\bar{\nu}_\tau\,\chi_1^0\mathrm{W}^+)$ 500 GeV	10.8
		$\sigma(\mathrm{e}_L^-\mathrm{e}_R^+ \to \tilde{\chi}_1^-\tilde{\chi}_1^+ \to \chi_1^0\tau^-\bar{\nu}_\tau\,\chi_1^0\mathrm{W}^+)$ 1 TeV	8.56
M_2	18.5	$\sigma(\mathrm{e}_L^-\mathrm{e}_R^+ \to \tilde{\chi}_1^+\tilde{\chi}_1^- \to \bar{\nu}_\tau\chi_1^0\tau^+\,\nu_\tau\chi_1^0\tau^-)$ 400 GeV	18.0
193.445 ± 0.10 GeV		$\sigma(\mathrm{e}_L^-\mathrm{e}_R^+ \to \tilde{\chi}_1^+\tilde{\chi}_1^- \to \bar{\nu}_\tau\chi_1^0\tau^-\,\nu_\tau\chi_1^0\tau^+)$ 500 GeV	9.48
		$\sigma(\mathrm{e}_R^-\mathrm{e}_R^+ \to \tilde{\chi}_1^+\tilde{\chi}_1^- \to \bar{\nu}_\tau\chi_1^0\tau^-\,\nu_\tau\chi_1^0\tau^+)$ 500 GeV	8.48
M_3	1.5	$m_{\tilde{\mathrm{g}}}$	72.8
568.9 ± 7.5 GeV		$\sigma(\mathrm{e}_L^-\mathrm{e}_R^+ \to \tilde{\mathrm{t}}_1^-\tilde{\mathrm{t}}_1^+ \to \chi_1^0\tau^-\bar{\nu}_\tau\bar{\mathrm{b}}\,\chi_1^0\tau^+\nu_\tau\mathrm{b})$ 1 TeV	8.03
		$\sigma(\mathrm{e}_R^-\mathrm{e}_L^+ \to \tilde{\mathrm{t}}_1^-\tilde{\mathrm{t}}_1^+ \to \chi_1^0\tau^-\bar{\nu}_\tau\bar{\mathrm{b}}\,\chi_1^0\tau^+\nu_\tau\mathrm{b})$ 1 TeV	7.51

Extrapolation to high scale

Use fitted LE parameters and extrapolate to the high scale using RGE:

W. Porod *et al.*



Compare behavior with expectations from SUSY breaking models

Summary

- With SFitter and Fittino powerful tools are available to extract SUSY parameters from collider observables.
- LHC and ILC nicely complement one another to pin down the SUSY model. Stringent checks rely on inputs from both machines.
- Precision determination of parameters requires - apart from loop corrections - also correlations between input observables to be included.
- In order to fully benefit from ILC precision, theoretical uncertainties need to be reduced.
- We are eagerly awaiting data from LHC and ILC.