

---

# SUSY QCD corrections to Higgs boson production via gluon gluon fusion

---

M. Margarete Mühlleitner  
(LAPTH Annecy-Le-Vieux)  
Coll. with M. Spira

## SUSY 2005

*The Millenium Window to Particle Physics*

**The 13th International Conference  
on Supersymmetry  
and Unification of Fundamental Interactions**

*July 18-23, 2005, IPPP Durham*

---

## Introduction

---

**Gluon gluon fusion:** dominant production process at existing and future hadron colliders.

**QCD corrections** have been calculated at

NLO (SM, MSSM):  $K \approx 1.5 - 1.7$  Spira, Djouadi, Graudenz, Zerwas  
Dawson  
Kauffman, Schaffer

NNLO (squarks, gluinos decoupled) Harlander, Kilgore  
Anastasiou, Melnikov  
Ravindran, Smith, vanNeerven

**NLO SUSY QCD corrections:** known so far in the heavy squark, gluino limit.

Dawson, Djouadi, Spira  
Harlander, Steinhauser  
Harlander, Hofmann

**$m_{\tilde{Q}} < 400$  GeV:** squarks play a significant role  $\rightsquigarrow$   
calculation of the full squark mass dependence at NLO.

---

## The MSSM Higgs sector

---

**MSSM Higgs sector** – supersymmetry & anomaly free theory  $\Rightarrow$  2 complex Higgs doublets

EW  
 $\xrightarrow{\text{SB}}$

neutral, CP-even  $h, H$

neutral, CP-odd  $A$

charged  $H^+, H^-$

---

---

## The MSSM Higgs sector

---

**MSSM Higgs sector** – supersymmetry & anomaly free theory  $\Rightarrow$  2 complex Higgs doublets

EW  
 $\xrightarrow{\text{SB}}$

neutral, CP-even  $h, H$

neutral, CP-odd  $A$

charged  $H^+, H^-$

### Higgs masses

$$M_h \lesssim 140 \text{ GeV}$$

$$M_{A,H,H^\pm} \sim \mathcal{O}(v) \dots 1 \text{ TeV}$$

Ellis et al; Okada et al; Haber, Hempfling;  
Hoang et al; Carena et al; Heinemeyer et al;  
Zhang et al; Brignole et al; ...

# The MSSM Higgs sector

**MSSM Higgs sector** – supersymmetry & anomaly free theory  $\Rightarrow$  2 complex Higgs doublets

EWSB  
 $\rightarrow$

neutral, CP-even  $h, H$       neutral, CP-odd  $A$       charged  $H^+, H^-$

## Higgs masses

$$M_h \lesssim 140 \text{ GeV}$$

$$M_{A,H,H^\pm} \sim \mathcal{O}(v) \dots 1 \text{ TeV}$$

Ellis et al; Okada et al; Haber, Hempfling;  
Hoang et al; Carena et al; Heinemeyer et al;  
Zhang et al; Brignole et al; ...

**Modified couplings with respect to the SM:** (decoupling limit)      Gunion, Haber

$\Phi$	$g_{\Phi u \bar{u}}$	$g_{\Phi d \bar{d}}$	$g_{\Phi V V}$
$h$	$c_\alpha / s_\beta \rightarrow 1$	$-s_\alpha / c_\beta \rightarrow 1$	$s_{\beta-\alpha} \rightarrow 1$
$H$	$s_\alpha / s_\beta \rightarrow 1 / \text{tg}\beta$	$c_\alpha / c_\beta \rightarrow \text{tg}\beta$	$c_{\beta-\alpha} \rightarrow 0$
$A$	$1 / \text{tg}\beta$	$\text{tg}\beta$	0

$$\tan \beta \uparrow \Rightarrow g_{\Phi u u} \downarrow$$

$$g_{\Phi d d} \uparrow$$

$$g_{\Phi V V}^{MSSM} \lesssim g_{\Phi V V}^{SM}$$

# The MSSM Higgs sector

**MSSM Higgs sector** – supersymmetry & anomaly free theory  $\Rightarrow$  2 complex Higgs doublets

EWSB  
 $\rightarrow$

neutral, CP-even  $h, H$       neutral, CP-odd  $A$       charged  $H^+, H^-$

## Higgs masses

$$M_h \lesssim 140 \text{ GeV}$$

$$M_{A,H,H^\pm} \sim \mathcal{O}(v) \dots 1 \text{ TeV}$$

Ellis et al; Okada et al; Haber, Hempfling;  
Hoang et al; Carena et al; Heinemeyer et al;  
Zhang et al; Brignole et al; ...

**Modified couplings with respect to the SM:** (decoupling limit)      Gunion, Haber

$\Phi$	$g_{\Phi u\bar{u}}$	$g_{\Phi d\bar{d}}$	$g_{\Phi VV}$
$h$	$c_\alpha/s_\beta \rightarrow 1$	$-s_\alpha/c_\beta \rightarrow 1$	$s_{\beta-\alpha} \rightarrow 1$
$H$	$s_\alpha/s_\beta \rightarrow 1/\text{tg}\beta$	$c_\alpha/c_\beta \rightarrow \text{tg}\beta$	$c_{\beta-\alpha} \rightarrow 0$
$A$	$1/\text{tg}\beta$	$\text{tg}\beta$	0

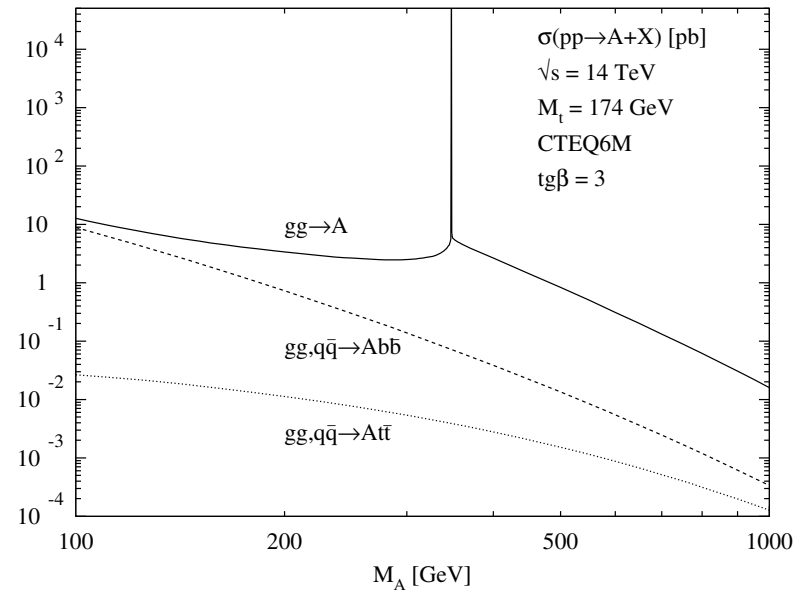
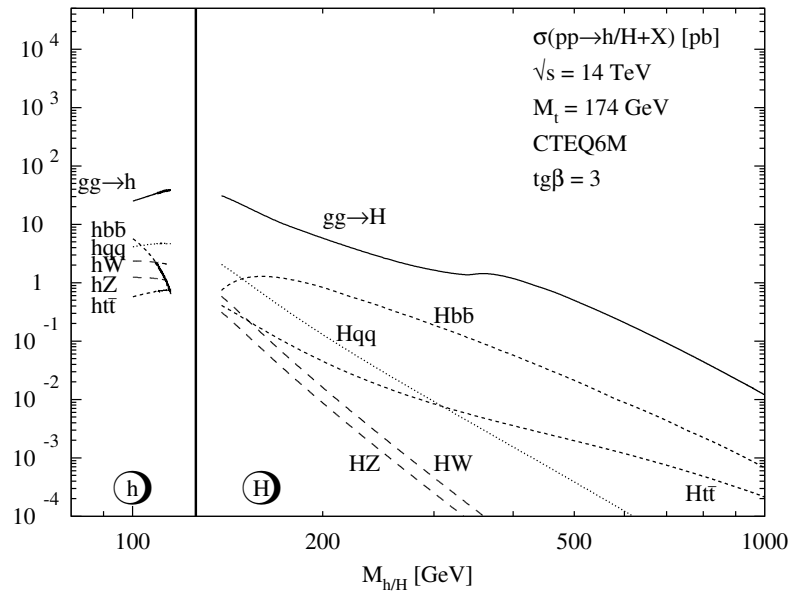
$$\tan \beta \uparrow \Rightarrow g_{\Phi uu} \downarrow$$

$$g_{\Phi dd} \uparrow$$

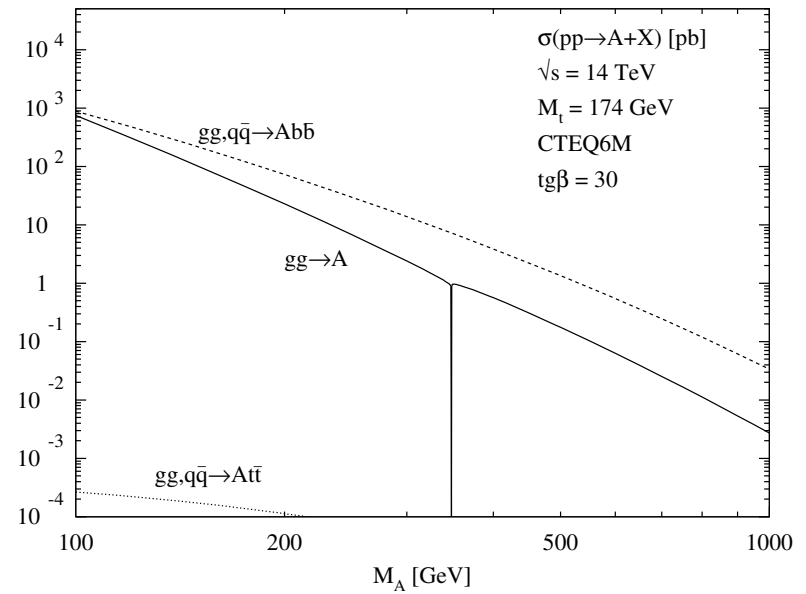
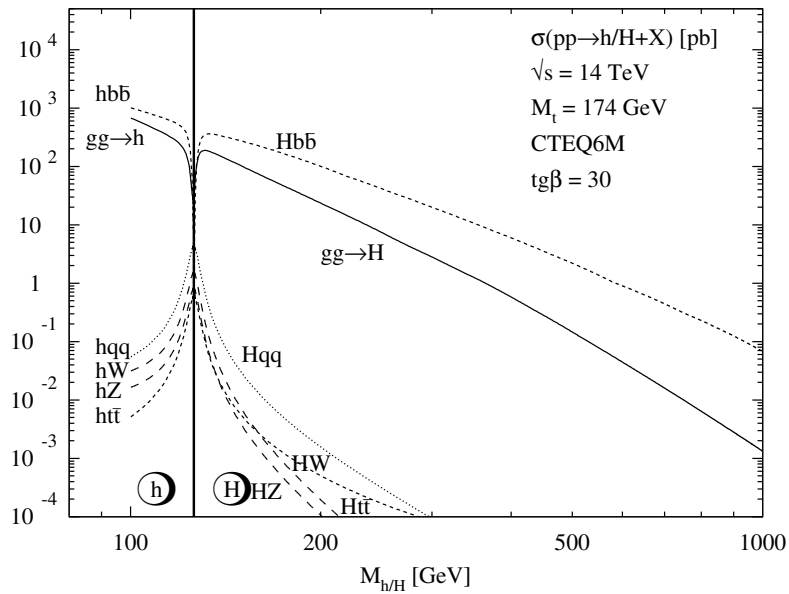
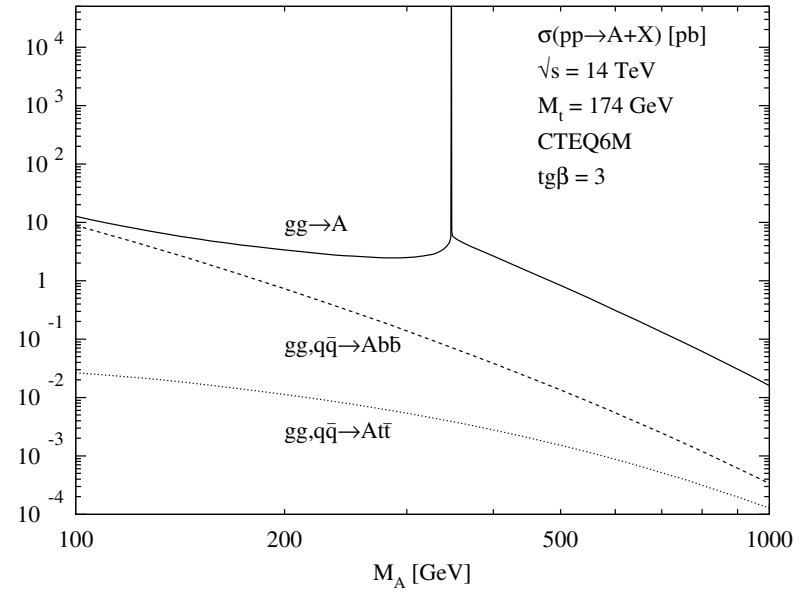
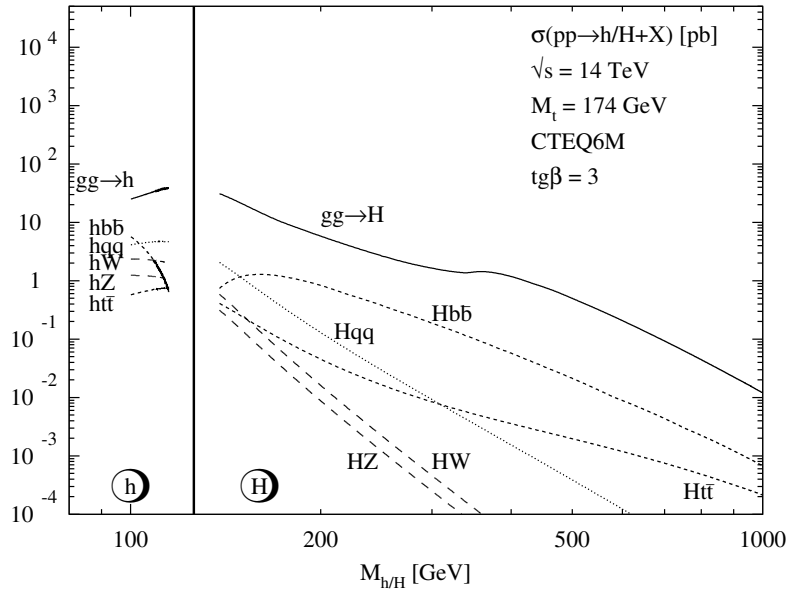
$$g_{\Phi VV}^{MSSM} \lesssim g_{\Phi VV}^{SM}$$

**Dominant production mechanism at the LHC:**  $gg \rightarrow \Phi$   
 $\Phi b\bar{b}$       for  $\tan \beta$  large

# MSSM Higgs boson production at the LHC



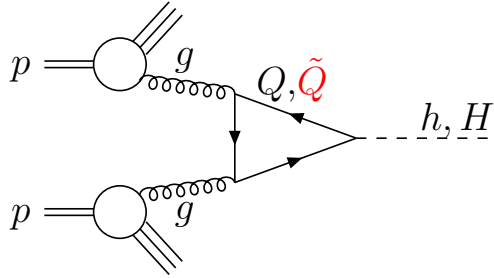
# MSSM Higgs boson production at the LHC





# gg → H, h at leading order

## Lowest order - 1 loop



$$\sigma(pp \rightarrow \Phi + X) = \sigma_0 \tau_\Phi \frac{d\mathcal{L}^{gg}}{d\tau_\Phi}$$

$$\tau_\Phi = \frac{M_\Phi^2}{s}$$

$$\sigma_0 = \frac{G_F \alpha_S^2}{288 \sqrt{2} \pi} \left| \sum_Q g_Q^\Phi F(\tau_Q) - \sum_{\tilde{Q}} g_{\tilde{Q}}^\Phi \tilde{F}(\tau_{\tilde{Q}}) \right|^2$$

$$\tau_{Q, \tilde{Q}} = \frac{4m_{Q, \tilde{Q}}^2}{M_\Phi^2}$$

$$F(\tau_Q) = \frac{3}{2} \tau_Q \left[ 1 + (1 - \tau_Q) f(\tau_Q) \right]$$

$$\tilde{F}(\tau_{\tilde{Q}}) = \frac{3}{4} \tau_{\tilde{Q}} \left[ 1 - \tau_{\tilde{Q}} f(\tau_{\tilde{Q}}) \right]$$

$$f(\tau) = \begin{cases} \arcsin^2 \frac{1}{\sqrt{\tau}} & \tau \geq 1 \\ -\frac{1}{4} \left[ \log \left( \frac{1 + \sqrt{1 - \tau}}{1 - \sqrt{1 - \tau}} \right) - i\pi \right]^2 & \tau < 1 \end{cases}$$

Remarks: - MSSM:  $\tan \beta \uparrow \Rightarrow b/\tilde{b} \uparrow + t/\tilde{t} \downarrow$

- heavy quarks dominant

$$\Phi Q Q \sim m_Q \rightsquigarrow t, b$$

$$- g_Q^\Phi \sim m_Q^2 / m_{\tilde{Q}}^2 \rightsquigarrow \tilde{t}, \tilde{b}$$

-  $gg \rightarrow A$  no  $\tilde{Q}$  contribution at LO

---

## The scenario

---

### The gluophic Higgs scenario

Carena, Heinemeyer, Wagner, Weiglein

$$m_t = 174 \text{ GeV}, M_{SUSY} = 350 \text{ GeV}, \mu = M_2 = 300 \text{ GeV}, A_b = A_t = -670 \text{ GeV}$$

$$\tan \beta = 3$$

$$m_{\tilde{t}_1} = 156 \text{ GeV} \quad m_{\tilde{t}_2} = 516 \text{ GeV}$$

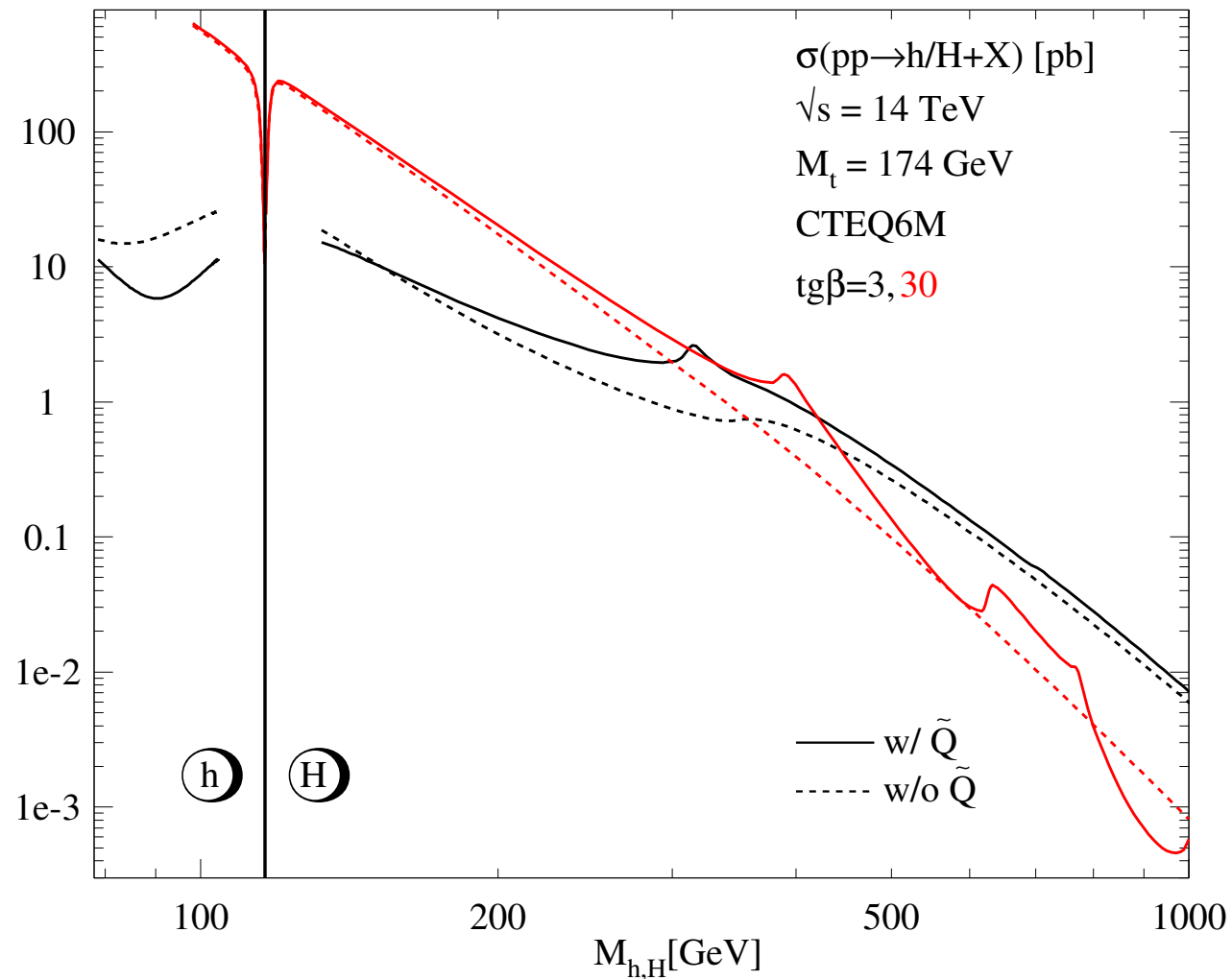
$$m_{\tilde{b}_1} = 346 \text{ GeV} \quad m_{\tilde{b}_2} = 358 \text{ GeV}$$

$$\tan \beta = 30$$

$$m_{\tilde{t}_1} = 200 \text{ GeV} \quad m_{\tilde{t}_2} = 502 \text{ GeV}$$

$$m_{\tilde{b}_1} = 315 \text{ GeV} \quad m_{\tilde{b}_2} = 387 \text{ GeV}$$

## The LO cross section w/ and w/o Squarks



## QCD corrections

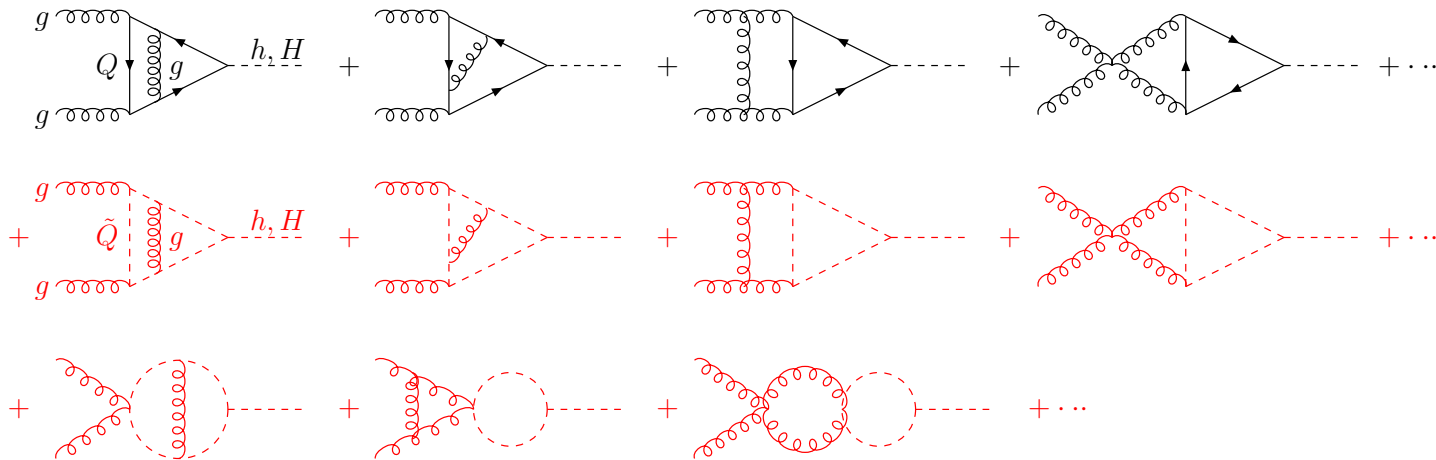
$$\Delta\hat{\sigma}_{ij} = \sigma_0 \left\{ C_{ij}\delta(1 - \hat{\tau}) + D_{ij}\Theta(1 - \hat{\tau}) \right\} \frac{\alpha_s}{\pi}$$

$$\hat{\tau} = \frac{M_{\Phi}^2}{\hat{s}}$$

$\nearrow$   
 virtual+soft  
 corrections

$\uparrow$   
 real corrections

### Virtual corrections [2 loops, no gluino contributions]



UV-,IR-,Coll-singularities in  $n = 4 - 2\epsilon$  dimensions.

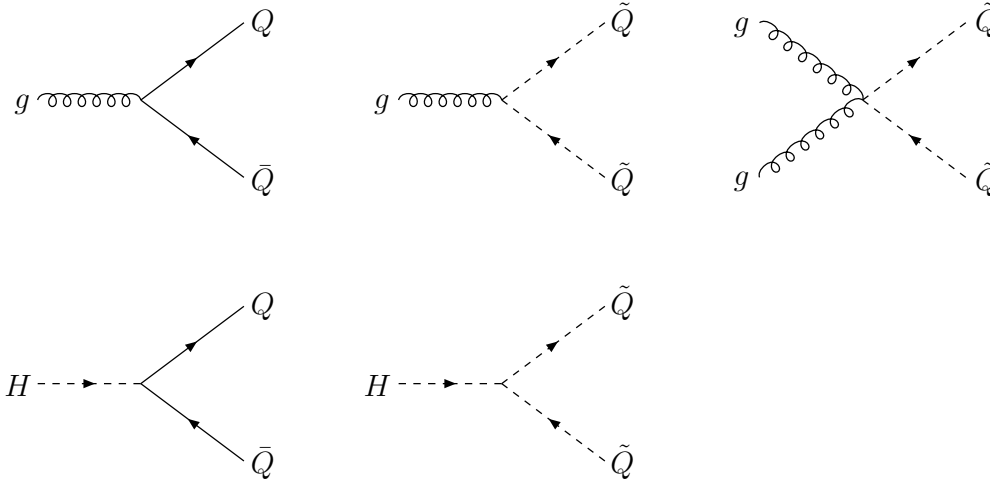
# Renormalization

## Lagrangian

$$\mathcal{L} = -\frac{1}{4}G^{a\mu\nu}G_{\mu\nu}^a + \bar{Q}(i\not{D} - m_Q)Q + |D_\mu\tilde{Q}|^2 - m_{\tilde{Q}}^2|\tilde{Q}|^2 - g_Q^H \frac{m_Q}{v}\bar{Q}QH - g_{\tilde{Q}}^H \frac{m_{\tilde{Q}}^2}{v}|\tilde{Q}|^2H + \frac{1}{2}(\partial_\mu H)^2 - \frac{M_H^2}{2}H^2$$

$$iD_\mu = i\partial_\mu - g_S G_\mu^a T^a$$

## Interaction vertices:



---

## Renormalization - Suite

---

- Quark/Squark mass  $m_{Q,\tilde{Q}}$ : on-shell

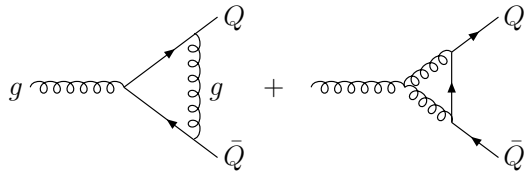
---

## Renormalization - Suite

---

- Quark/Squark mass  $m_{Q, \tilde{Q}}$ : on-shell

-  $gQ\bar{Q}$  vertex:



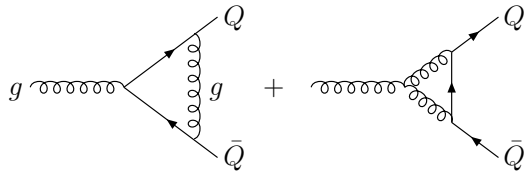
$$Z_1 = Z_{\alpha_S}^{1/2} Z_3^{1/2} Z_2$$

[Slavnov-Taylor identity]

## Renormalization - Suite

- Quark/Squark mass  $m_{Q,\tilde{Q}}$ : on-shell

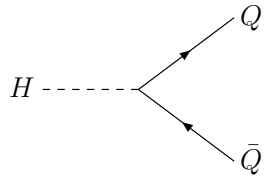
-  $gQ\bar{Q}$  vertex:



$$Z_1 = Z_{\alpha_S}^{1/2} Z_3^{1/2} Z_2$$

[Slavnov-Taylor identity]

-  $HQ\bar{Q}$  vertex:



$$\mathcal{L}_{\text{int}} = -g_Q^H \frac{m_{Q_0}}{v} \bar{\Psi}_0 \Psi_0 H = -g_Q^H \frac{m_Q}{v} \bar{\Psi} \Psi H \underbrace{\left[ Z_2 - \frac{\delta m_Q}{m_Q} \right]}_{Z_{HQQ}} + \mathcal{O}(\alpha_S^2)$$

$$\Gamma_{H\bar{Q}Q}(q^2 = 0) \neq Z_{HQQ}$$

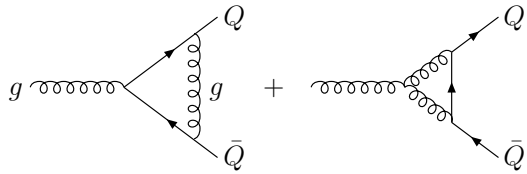
Braaten, Leveille



## Renormalization - Suite

- Quark/Squark mass  $m_{Q,\tilde{Q}}$ : on-shell

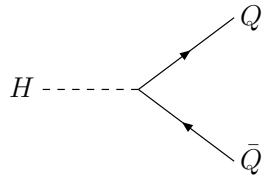
-  $gQ\bar{Q}$  vertex:



$$Z_1 = Z_{\alpha_S}^{1/2} Z_3^{1/2} Z_2$$

[Slavnov-Taylor identity]

-  $HQ\bar{Q}$  vertex:

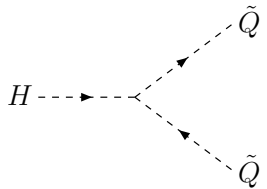


$$\mathcal{L}_{\text{int}} = -g_Q^H \frac{m_{Q_0}}{v} \bar{\Psi}_0 \Psi_0 H = -g_Q^H \frac{m_Q}{v} \bar{\Psi} \Psi H \underbrace{\left[ Z_2 - \frac{\delta m_Q}{m_Q} \right]}_{Z_{HQQ}} + \mathcal{O}(\alpha_S^2)$$

$$\Gamma_{H\bar{Q}Q}(q^2 = 0) \neq Z_{HQQ}$$

Braaten, Leveille

-  $H\tilde{Q}\tilde{Q}$  vertex:



$$\mathcal{L}_{\text{int}} = -g_{\tilde{Q}}^H \frac{m_{\tilde{Q}_0}^2}{v} \tilde{Q}_0^* \tilde{Q}_0 H = -g_{\tilde{Q}}^H \frac{m_{\tilde{Q}}^2}{v} \tilde{Q}^* \tilde{Q} H \underbrace{\left[ Z_2^{\tilde{Q}} - \frac{\delta m_{\tilde{Q}}^2}{m_{\tilde{Q}}^2} \right]}_{Z_{H\tilde{Q}\tilde{Q}}} + \mathcal{O}(\alpha_S^2)$$

$$\Gamma_{H\tilde{Q}\tilde{Q}}(q^2 = 0) \neq Z_{H\tilde{Q}\tilde{Q}}$$

disregard renorm. of  $g_{\tilde{Q}}^H$ !

---

## Virtual corrections - heavy squark limit

---

Total virtual correction [heavy squark limit, without b, t loops]:

$$C_{\text{virt}} = \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu^2}{M_H^2} \right)^\epsilon \left\{ -\frac{3}{\epsilon^2} - \frac{33-2N_F}{6\epsilon} \left( \frac{\mu^2}{M_H^2} \right)^{-\epsilon} + \pi^2 + \frac{23}{3} \right\}$$

↑      ↑  
IR     Coll

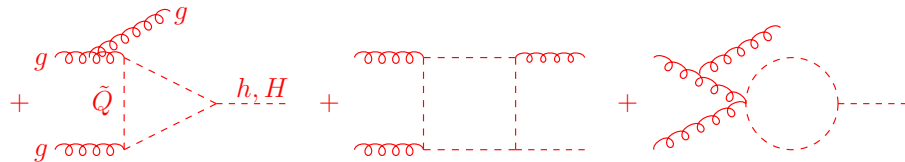
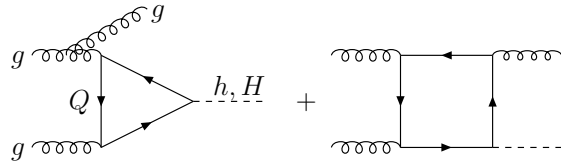
$[\frac{23}{3} \rightarrow \frac{11}{2}]$  in the heavy quark limit (without squark loops)

To get a finite cross section the real corrections have to be added.

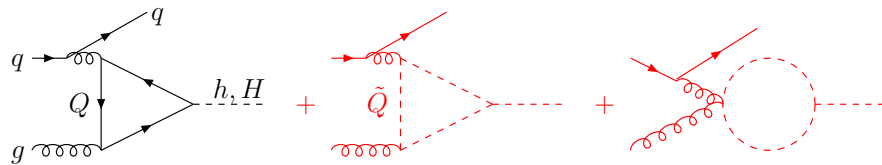
# Real Corrections

## 3 incoherent processes:

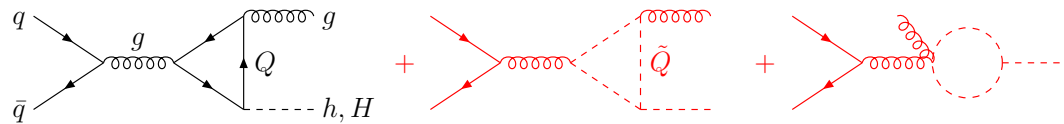
$gg \rightarrow Hg$ :



$gq \rightarrow Hq$ :



$q\bar{q} \rightarrow Hg$ :



Phase space integration in  $n = 4 - 2\epsilon$  dimensions  $\rightsquigarrow$  IR, Coll. singularities: poles in  $\epsilon$

---

## Real corrections - heavy squark limit

---

Total real corrections [heavy squark limit, without b, t loops]:

$$C_{\text{real}} = \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu^2}{m_{\Phi}^2} \right)^{\epsilon} \left\{ \frac{3}{\epsilon^2} + \frac{33-2N_F}{6\epsilon} \right\}$$

$$D_{gg} = -\frac{\hat{\tau}}{\epsilon} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu^2}{\hat{s}} \right)^{\epsilon} P_{gg}(\hat{\tau}) - \frac{11}{2}(1-\hat{\tau})^3 \\ + 12 \left\{ \left( \frac{\log(1-\hat{\tau})}{1-\hat{\tau}} \right)_+ - \hat{\tau}[2-\hat{\tau}(1-\hat{\tau})] \log(1-\hat{\tau}) \right\}$$

$$D_{gq} = -\left\{ \frac{1}{2\epsilon} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu^2}{\hat{s}} \right)^{\epsilon} - \log(1-\hat{\tau}) \right\} \hat{\tau} P_{gq}(\hat{\tau}) - 1 + 2\hat{\tau} - \frac{\hat{\tau}^2}{3}$$

$$D_{q\bar{q}} = \frac{32}{27}(1-\hat{\tau})^3$$

- IR, Coll. poles in  $C_{\text{real}}$  subtract the corresponding ones of the virtual corrections.
- Coll. poles in the real corrections (Altarelli-Parisi kernels as coefficients)  
     $\rightsquigarrow$  absorbed in NLO structure functions.

## Result - heavy squark limit (without b, t loops)

$$\sigma(pp \rightarrow \Phi + X) = \sigma_0 \left[ 1 + C \frac{\alpha_S}{\pi} \right] \tau_\Phi \frac{d\mathcal{L}_{gg}}{d\tau_\Phi} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}}$$

$$C = \pi^2 + \frac{23}{3} + \frac{33-2N_F}{6} \log \frac{\mu^2}{M_\Phi^2}$$

$$\Delta\sigma_{gg} = \int_{\tau_\Phi}^1 d\tau \frac{d\mathcal{L}_{gg}}{d\tau} \frac{\alpha_S}{\pi} \sigma_0 \left\{ -\hat{\tau} P_{gg}(\hat{\tau}) \log \frac{Q^2}{\hat{s}} - \frac{11}{2} (1 - \hat{\tau})^3 \right. \\ \left. + 12 \left[ \left( \frac{\log(1-\hat{\tau})}{1-\hat{\tau}} \right)_+ - \hat{\tau} [2 - \hat{\tau}(1-\hat{\tau})] \log(1-\hat{\tau}) \right] \right\}$$

$$\Delta\sigma_{gq} = \int_{\tau_\Phi}^1 d\tau \sum_{q, \bar{q}} \frac{d\mathcal{L}_{gq}}{d\tau} \frac{\alpha_S}{\pi} \sigma_0 \left\{ -\frac{\hat{\tau}}{2} P_{gq}(\hat{\tau}) \left[ \log \frac{Q^2}{\hat{s}} - 2 \log(1-\hat{\tau}) \right] \right. \\ \left. - 1 + 2\hat{\tau} - \frac{\hat{\tau}^2}{3} \right\}$$

$$\Delta\sigma_{q\bar{q}} = \int_{\tau_\Phi}^1 d\tau \sum_q \frac{d\mathcal{L}_{q\bar{q}}}{d\tau} \frac{\alpha_S}{\pi} \sigma_0 \frac{32}{27} (1 - \hat{\tau})^3$$

$[\mu = \text{Ren. scale}, Q = \text{Fact. scale}]$

natural scales:  $\mu^2 = Q^2 = M_\Phi^2$

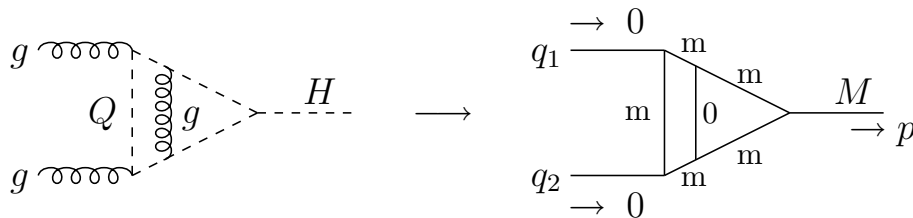
---

## General case (arbitrary $M_\Phi, m_Q, m_{\tilde{Q}}$ )

---

- Interference  $b, t, \tilde{b}, \tilde{t}$
- 5-dim. Feynman integrals  $\rightarrow$  1-dimensional [Trilogarithms]

### Example:



$$S = \int \frac{d^n k d^n q}{(2\pi)^{2n}} \frac{1}{(k^2 - m^2)[(k - q_1)^2 - m^2][(k + q_2)^2 - m^2][(k + q - q_1)^2 - m^2][(k + q + q_2)^2 - m^2]q^2}$$

$$= -\frac{\Gamma(2 + 2\epsilon)}{(4\pi)^4 m^4} \left(\frac{4\pi\mu^2}{m^2}\right)^{2\epsilon} \times I$$

$$I = \int_0^1 dx dy dz dr ds \frac{xz}{N^2} \quad \rho = \frac{M_\Phi^2}{m_{\tilde{Q}}^2} (1 + i0)$$

$$N = 1 + \rho \{ rx(1-x)(1-y-z)(1-y-zs) - [y + (1-y-z)x][1-y-x(1-y-zs)] \}$$

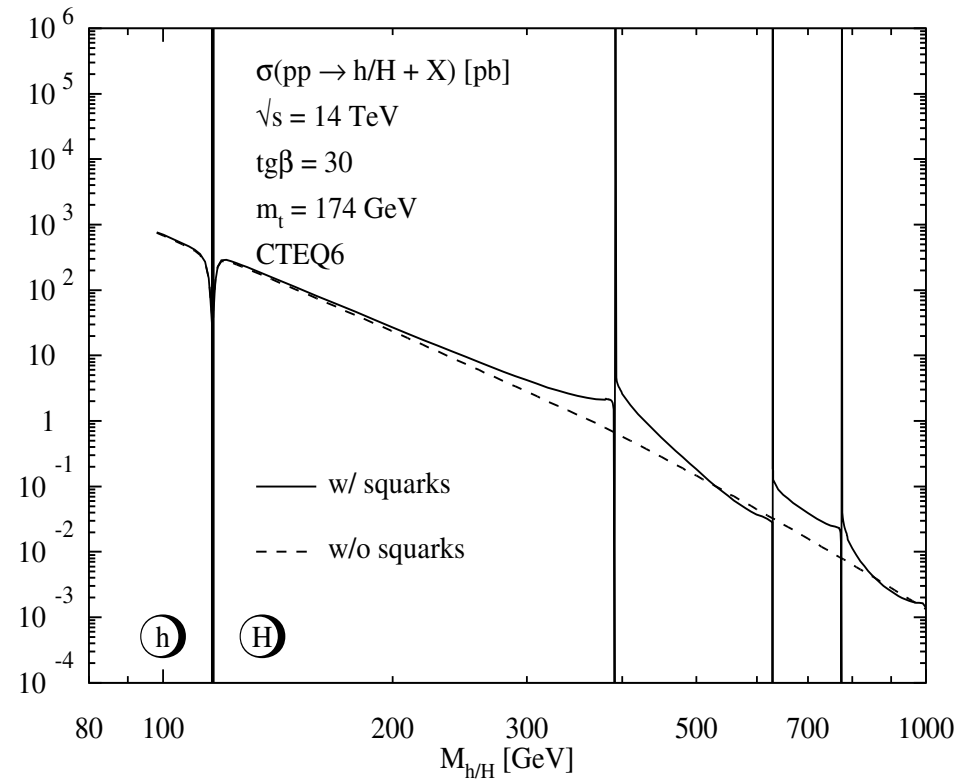
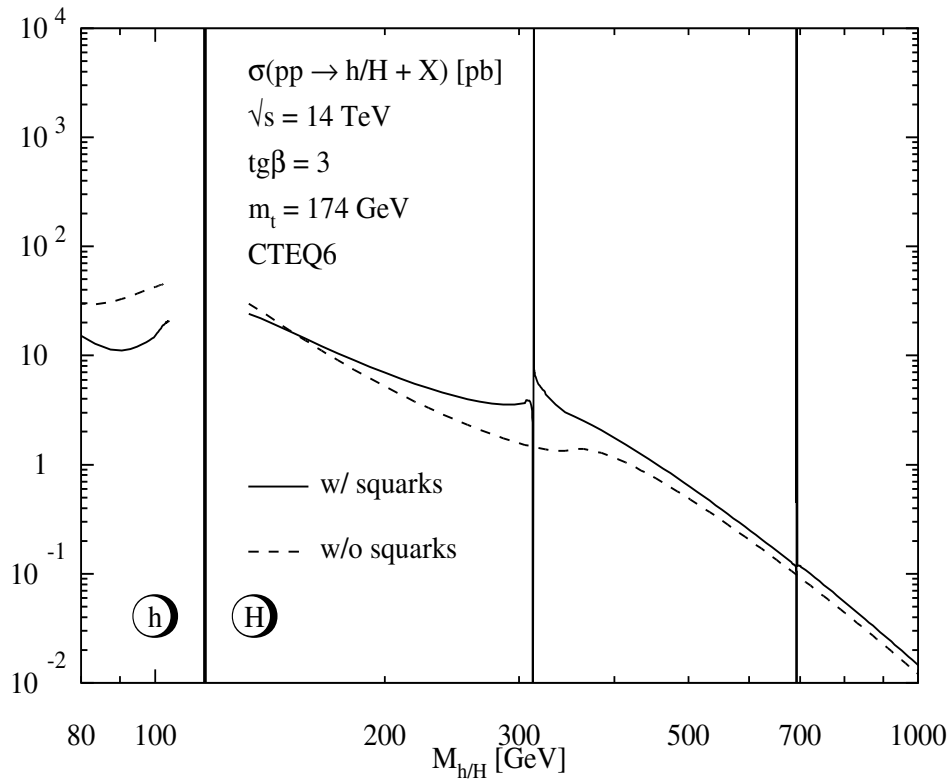
## Result

- $\alpha_S$ :  $\overline{\text{MS}}$  scheme, 5 active flavours
- $\lim_{m_{\tilde{Q}} \rightarrow \infty}$  recovered
- calculation analogous to  $m_{\tilde{Q}} \rightarrow \infty$

$$\begin{aligned}
 \sigma(pp \rightarrow \Phi + X) &= \sigma_0 \left[ 1 + C \frac{\alpha_S}{\pi} \right] \tau_\Phi \frac{d\mathcal{L}_{gg}}{d\tau_\Phi} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}} \\
 C &= \pi^2 + C_1(\tau_Q, \tau_{\tilde{Q}}) + \frac{33-2N_F}{6} \log \frac{\mu^2}{M_\Phi^2} \\
 \Delta\sigma_{gg} &= \int_{\tau_\Phi}^1 d\tau \frac{d\mathcal{L}_{gg}}{d\tau} \frac{\alpha_S}{\pi} \sigma_0 \left\{ -\hat{\tau} P_{gg}(\hat{\tau}) \log \frac{Q^2}{\hat{s}} + d_{gg}(\hat{\tau}, \tau_Q, \tau_{\tilde{Q}}) \right. \\
 &\quad \left. + 12 \left[ \left( \frac{\log(1-\hat{\tau})}{1-\hat{\tau}} \right)_+ - \hat{\tau} [2 - \hat{\tau}(1-\hat{\tau})] \log(1-\hat{\tau}) \right] \right\} \\
 \Delta\sigma_{gq} &= \int_{\tau_\Phi}^1 d\tau \sum_{q, \bar{q}} \frac{d\mathcal{L}_{gq}}{d\tau} \frac{\alpha_S}{\pi} \sigma_0 \left\{ -\frac{\hat{\tau}}{2} P_{gq}(\hat{\tau}) \left[ \log \frac{Q^2}{\hat{s}} - 2 \log(1-\hat{\tau}) \right] \right. \\
 &\quad \left. + d_{gq}(\hat{\tau}, \tau_Q, \tau_{\tilde{Q}}) \right\} \\
 \Delta\sigma_{q\bar{q}} &= \int_{\tau_\Phi}^1 d\tau \sum_q \frac{d\mathcal{L}_{q\bar{q}}}{d\tau} \frac{\alpha_S}{\pi} \sigma_0 d_{q\bar{q}}(\hat{\tau}, \tau_Q, \tau_{\tilde{Q}})
 \end{aligned}$$

# The NLO cross section w/ and w/o Squarks

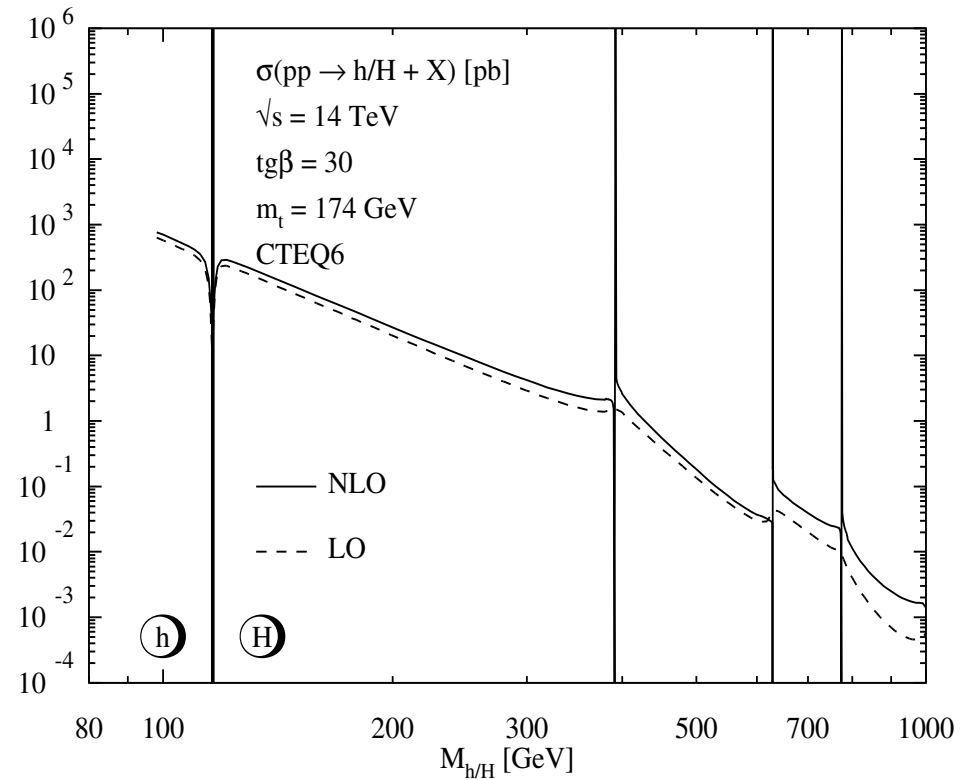
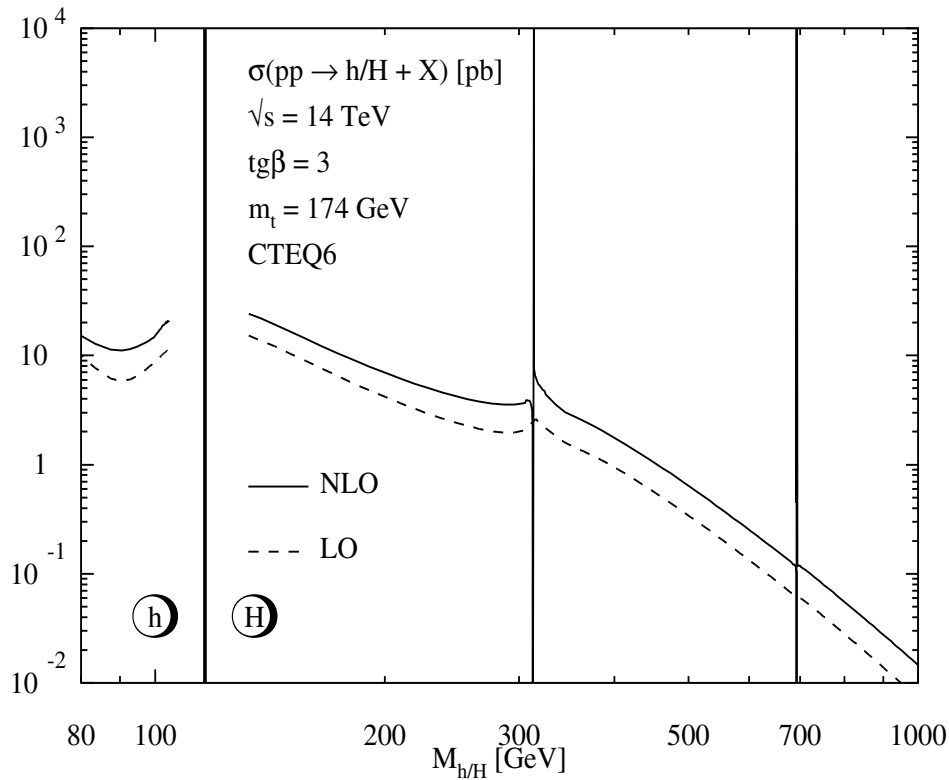
PRELIMINARY





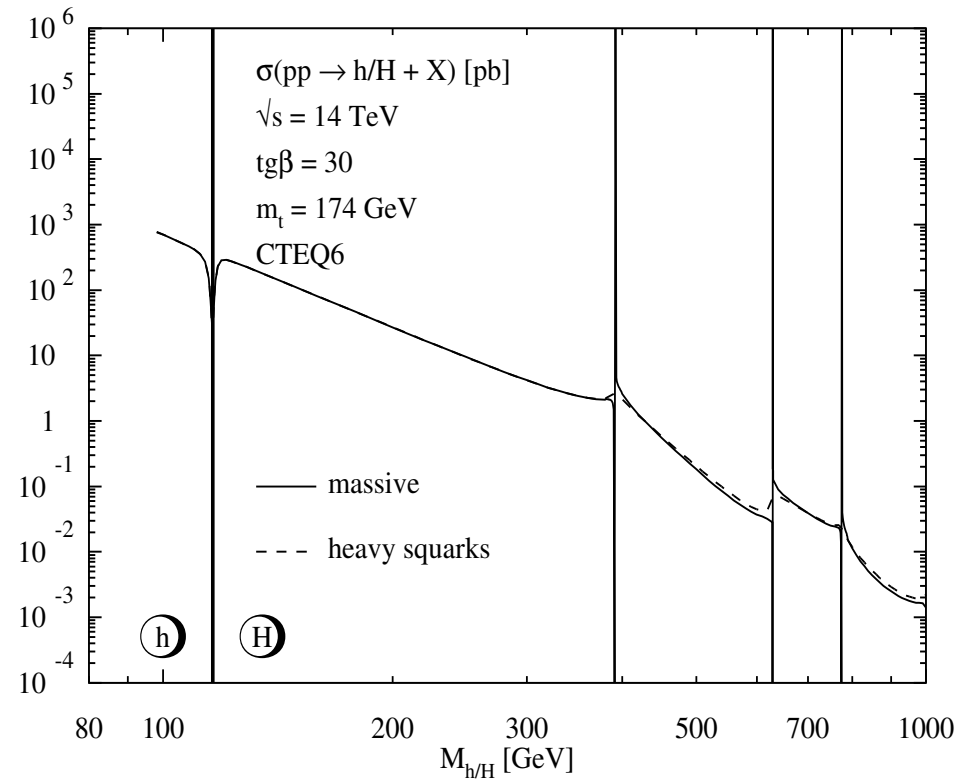
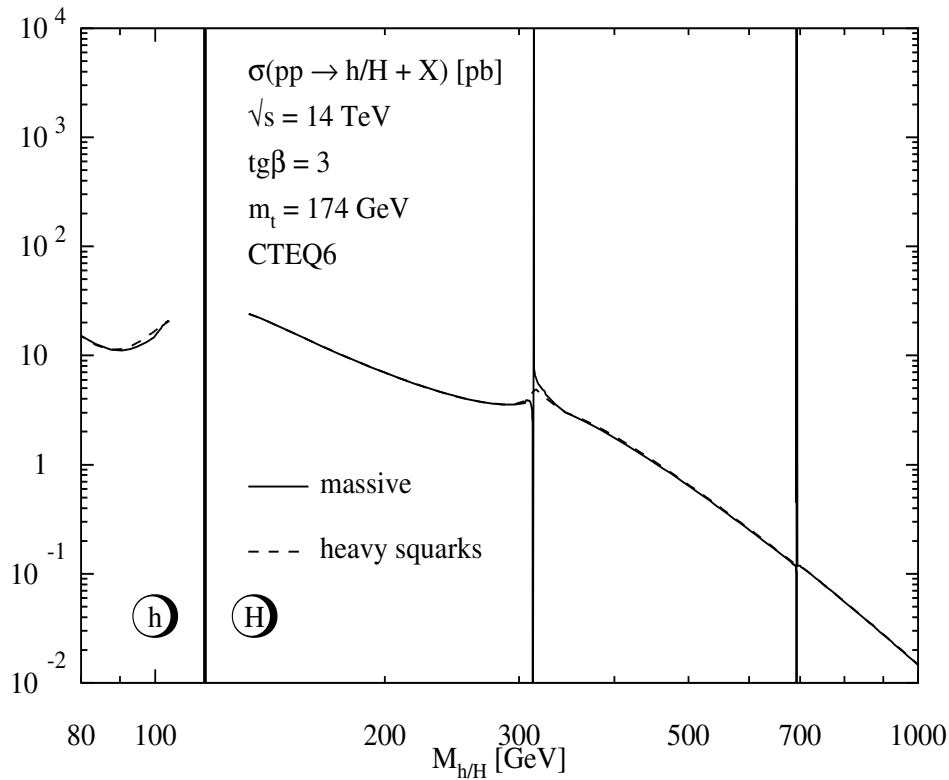
# The LO and NLO cross section w/ Squarks

PRELIMINARY



# The NLO cross section for massive squarks and in the heavy squark limit

PRELIMINARY



---

## Conclusions

---

- Calculated NLO corrections to  $gg \rightarrow h, H$  including the full squark mass dependence.
- K-factor due to squark inclusion is important:  $K \lesssim 2$ .
- K-factor very similar to the case of quark loops alone  $\rightsquigarrow$  large corrections to squark loops, too.
- Inclusion of full squark mass dependence has significant effects on the K-factor compared to the heavy squark mass limit. The deviation can be as large as 10–20 %.