SUSY-QCD CORRECTIONS TO $e^+e^- \to b\bar{b}/t\bar{t}$ + NEUTRAL HIGGS BOSON

I. Introduction

II. Motivation

III. $e^+e^- \to b\bar{b}/t\bar{t}\phi$

IV. Results

V. Conclusions

in collaboration with M. Spira / PSI Villigen

NPB 719: 35-52 (2005)

I. MSSM: Particles

minimal supersymmetric extension of SM = MSSM

supermultiplets with # fermionic degrees of freedom (dof) = # bosonic dof:

SM fermion (2 fermionic dof) $\Rightarrow$ MSSM complex scalar (2 bosonic dof)
SM gauge boson (2 bosonic dof) $\Rightarrow$ MSSM fermion (gaugino) (2 fermionic dof)

gauge multiplets		chiral multiplets	
$J = 1$	$J = 1/2$	$J = 1/2$	$J = 0$
Gluon g	Gluino $\tilde{g}$	Quark Q	Squark $\tilde{Q}_{1,2}$
$W^{\pm},\ W^0$	Wino $\widetilde{W}^{\pm},\ \widetilde{W}^0$	Lepton L	Slepton $\tilde{L}_{1,2}$
B^0	Bino $\widetilde{B}^0$	Higgsino $\widetilde{\phi}_1^{0,-}, \widetilde{\phi}_2^{+,0}$	Higgs ϕ_1, ϕ_2

gauginos (J = 1/2 gauge multiplets)

supermultiplets: ⊛ same quantum numbers
⊛ only spins differ by 1/2

after EWSB: mixing $\widetilde{W}^{\pm}, \widetilde{\phi}^{\pm}_{1,2}$ to charginos $\chi^{\pm}_{1,2}$
mixing $\widetilde{W}^0, \widetilde{B}^0$ and $\widetilde{\phi}^0_{1,2}$ to neutralinos $\chi^0_{1...4}$

I. MSSM: Higgs bosons

2 Higgs doublets
ϕ_1 and ϕ_2 (8 dof)

$\Longrightarrow$ EWSB $\Longrightarrow$

5 Higgs bosons: 2 neutral, scalar h, H
1 neutral, pseudoscalar A
2 charged $H^\pm$

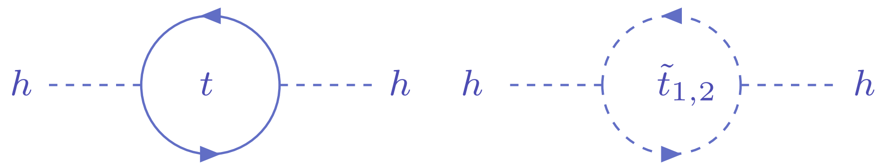
3 would-be goldstone bosons $\Rightarrow$ massive gauge bosons

EWSB $\Rightarrow$ 2 free parameters: $M_A, \tan\beta = \frac{v_2}{v_1}$

$$M_h^2 = \frac{1}{2}\left\{M_A^2 + M_Z^2 + \varepsilon - \left[(M_A^2 + M_Z^2 + \varepsilon)^2 - 4M_A^2 M_Z^2 \cos^2(2\beta) - 4\varepsilon\left(M_A^2 \sin^2\beta + M_Z^2 \cos^2\beta\right)\right]^{1/2}\right\}$$

$$M_H^2 = M_A^2 + M_Z^2 - M_h^2 + \varepsilon$$

$$M_{H^\pm}^2 = M_A^2 + M_W^2$$

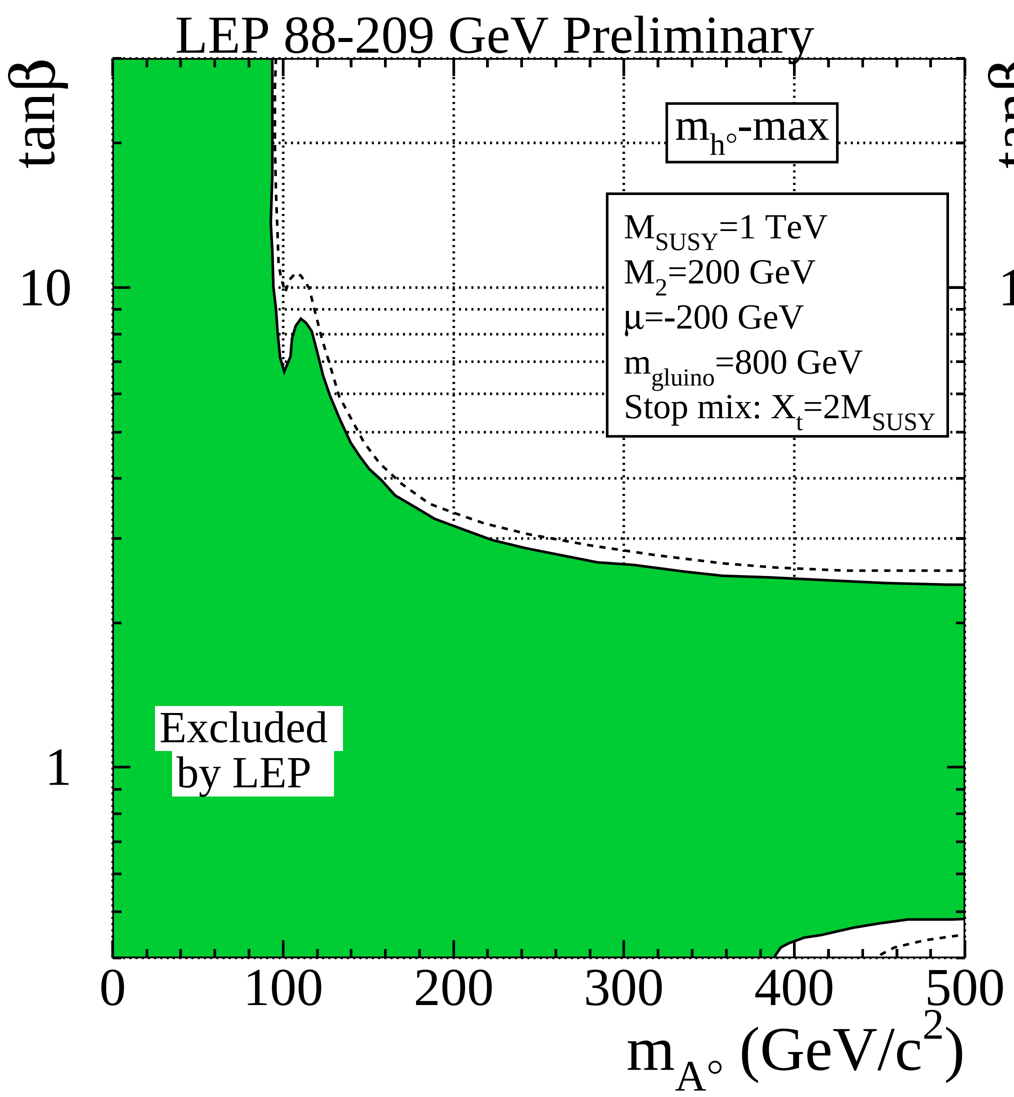


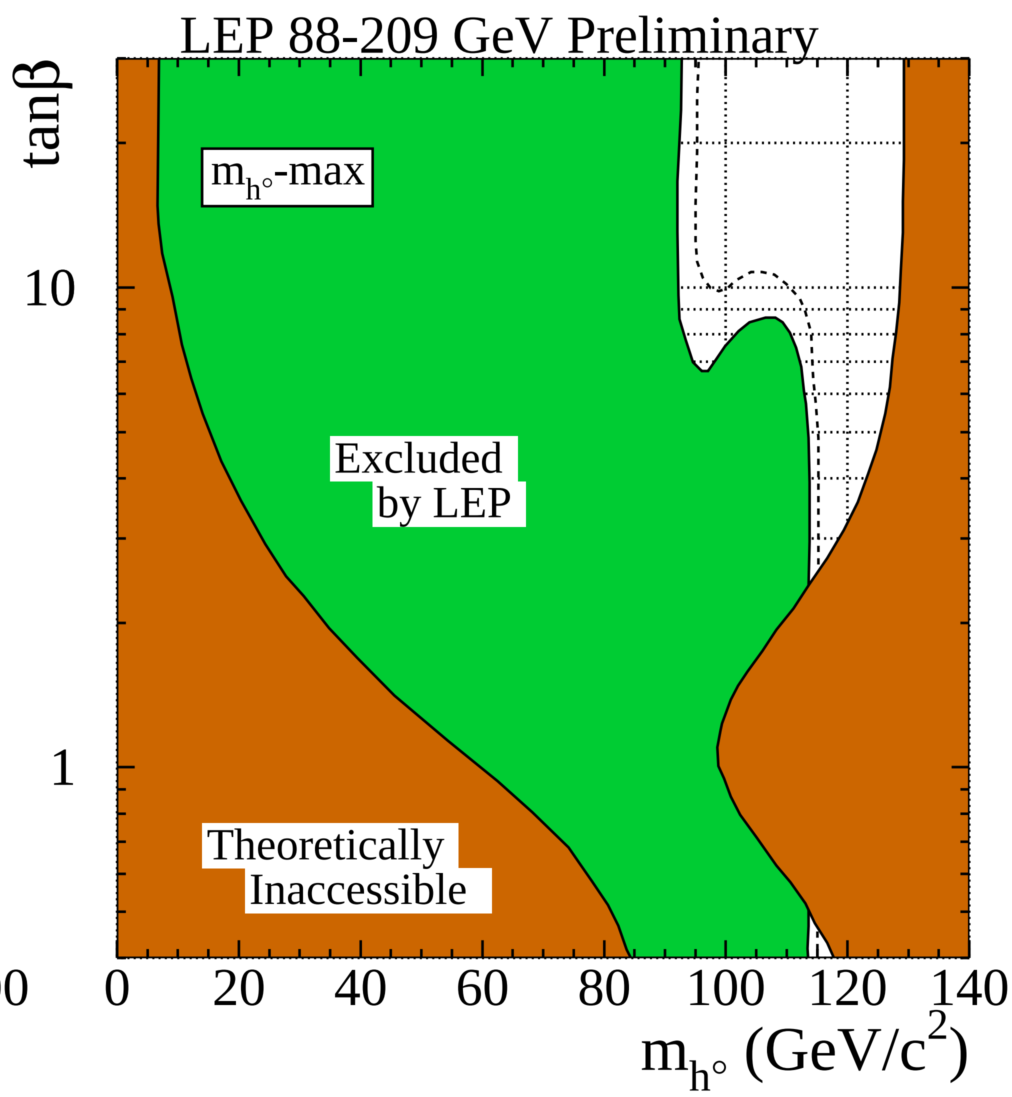
- ▷ leading order: $\varepsilon = 0$ $\Longrightarrow$ $M_h < M_Z$
- ▷ incl. radiative corrections $\Rightarrow$ $M_h \lesssim$ 140 GeV

$$\varepsilon = \frac{3G_F}{\sqrt{2}\pi^2}\frac{m_t^4}{\sin^2\beta}\log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$$

Haber,... Carena,... Heinemeyer,... Zhang,...

m_t = 174.3 GeV



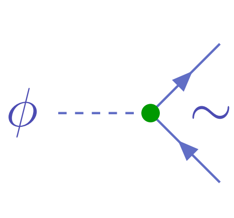


ADLO

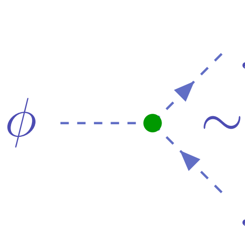
I. MSSM: Yukawa Couplings

MSSM coupling = SM coupling with 2 superpartners, same coupling strength

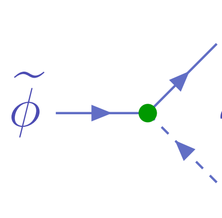
Yukawa coupling:



$\sim g_Y^f \cdot g_f^\phi$



$\sim \left(g_Y^f \cdot g_f^\phi\right)^2$



$\sim g_Y^f \cdot g_f^\phi$

$g_Y^f = m_f / v$

$v = \sqrt{v_1^2 + v_2^2}$

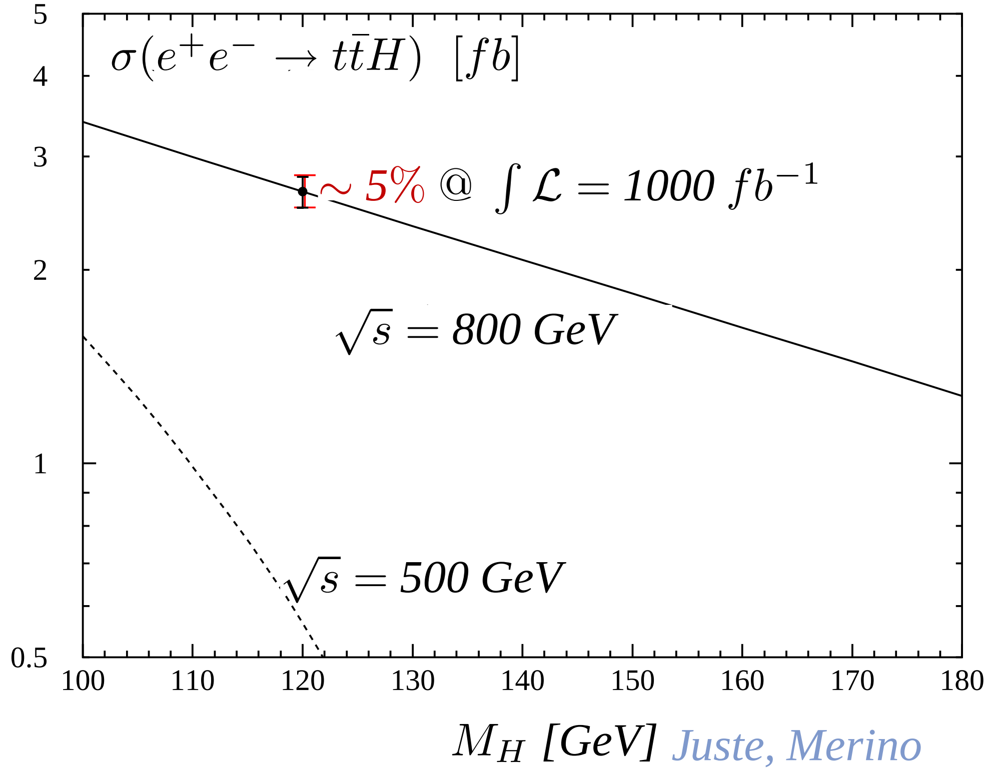
Limes	ϕ	g_u^ϕ	g_d^ϕ	g_V^ϕ
SM	H	1	1	1
MSSM	h	$\cos\alpha / \sin\beta$	$-\sin\alpha / \cos\beta$	$\sin(\beta - \alpha)$
	H	$\sin\alpha / \sin\beta$	$\cos\alpha / \cos\beta$	$\cos(\beta - \alpha)$
	A	$1/\tan\beta$	$\tan\beta$	0
$\lim m_A \to \infty$	h	1	1	1
	H	$-1/\tan\beta$	$\tan\beta$	0
	A	$1/\tan\beta$	$\tan\beta$	0

II. Why NLO for $e^+e^- \to Q\bar{Q}\phi$?

small cross section: $\sigma(e^+e^- \to Q\bar{Q}\phi) \sim \mathcal{O}(1\ fb) \iff \sigma(e^+e^- \to Z\phi) \sim \mathcal{O}(100\ fb)$

but

1. measuring top Yukawa coupling:
   - $M_\phi \leq 2m_t$: directly accessible in $e^+e^- \to t\bar{t}\phi$
   - $M_\phi > 2m_t$: from $\phi \to t\bar{t}$
2. measuring $tan\beta$: similar as top Yukawa coupling but with top $\leftrightarrow$ bottom

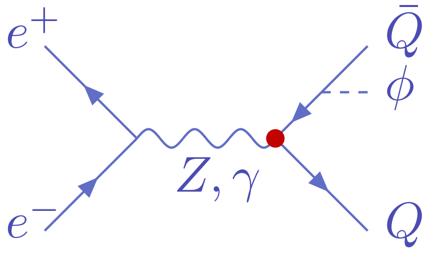


Juste, Merino

NLO needed:

- reduce theoretical uncertainties
- sizeable contributions

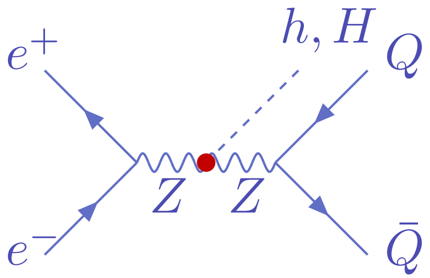
III. Leading Order $e^+e^- \to Q\bar{Q}\phi$



Higgs radiation off heavy (anti)quark

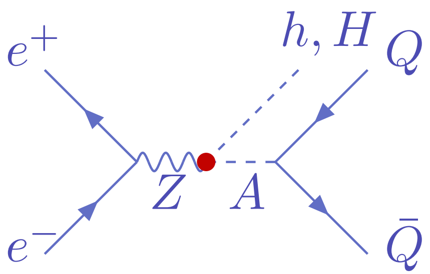
$\to g_Q^{h/H}\ \mathcal{C}_1^{LO}$

$\to g_Q^{A}\ \mathcal{D}_1^{LO}$



Higgs radiation off the Z boson
no AZZ coupling

$\to g_Z^{h/H}\ \mathcal{C}_2^{LO}$



Z boson splitting into h/H and A pair
with one dissociating into heavy quarks
$h, H \leftrightarrow A$

$\to g_Z^{A}\ \mathcal{C}_3^{LO}$

$\to g_Z^{H}\ \mathcal{D}_2^{LO},\ g_Z^{h}\ \mathcal{D}_3^{LO}$

$\triangleright$ *resonant contributions: Breit-Wigner propagators for $Z \to b\bar{b}$ and $\phi \to b\bar{b}/t\bar{t}$*

Leading Order cross section: $\sigma_{LO}^{\phi} = \int dPS_3 \sum_{spins,colours} \overline{|\mathcal{M}_{LO}^{\phi}|^2}$

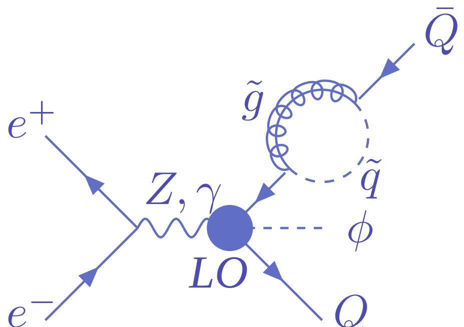
III. QCD Corrections

Dittmaier, Krämer, Liao, Spira, Zerwas
Denner, Dittmaier, Roth, Weber
Dawson, Reina

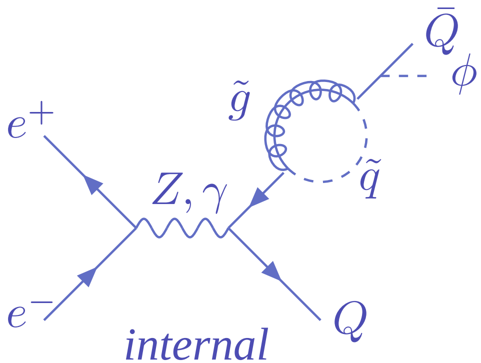
- α_s evaluated at NLO with 5 active flavours, normalised $\alpha_s(M_Z^2)$ = 0.119
- renormalisation scale $\mu_R = \sqrt{s}$
- renormalisation of $Q\bar{Q}\phi$ vertices $\Leftrightarrow$ renormalisation of quark mass
  - m_Q: on-shell
  - $g_{t\bar{t}\phi}$: on-shell
  - $g_{b\bar{b}\phi}$: $\overline{MS}$ running bottom mass $\overline{m}_b(Q_\phi^2)$
    $\longrightarrow$ absorbing large logarithmic contributions
- UV divergences consistently regularised in $D = 4-2\varepsilon$ dimensions
- γ_5 treated naively since no anomalies involved
- IR divergences from virtual corrections and cross section of real gluon emission:
  1. dimensional regularisation
  2. infinitesimal gluon mass
- $\alpha = 1/128$, $\sin^2\theta_W = 0.23$, $m_Z = 91.187$ GeV, $\Gamma_Z = 2.49$ GeV, $m_t = 175$ GeV and $m_b = 4.62$ GeV

III. SUSY-QCD Corrections

Self Energies

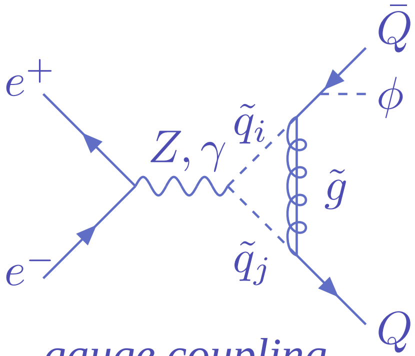


external

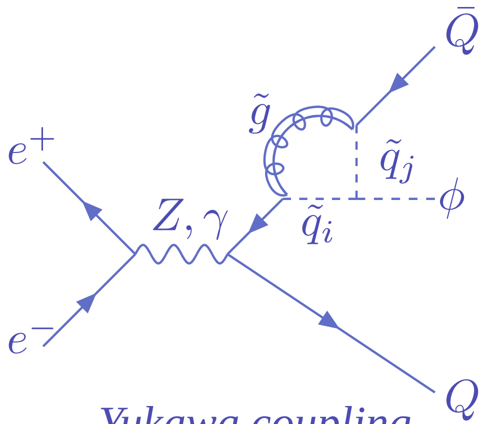


internal

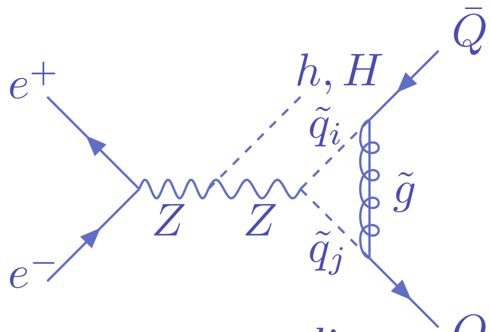
Vertex Corrections



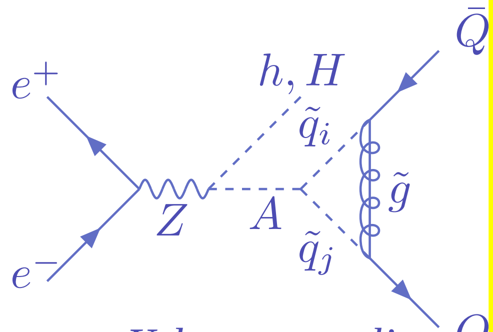
gauge coupling



Yukawa coupling

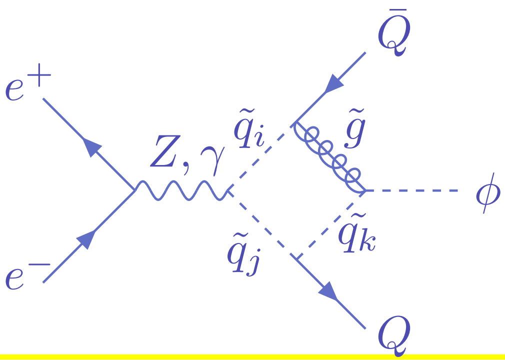


gauge coupling
no AZZ coupling



Yukawa coupling
$h, H \leftrightarrow A$

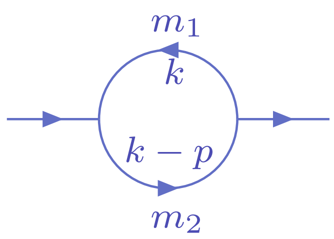
Boxes



III. SUSY-QCD Corrections

- massive particles: only virtual corrections and no infrared nor collinear singularities

- D = 4–2ε dimensions $\Longrightarrow$ UV-div $\propto 1/\varepsilon$

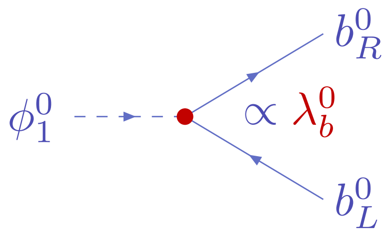


$B_0(p; m_1, m_2) \propto \frac{1}{\varepsilon}$

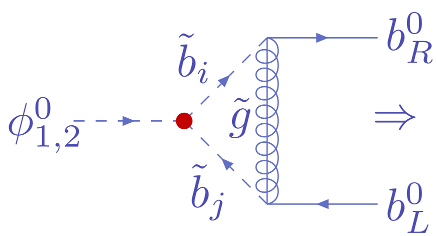
- Renormalisation:
  - m_Q: on-shell
  - $g_{Q\bar{Q}\phi}$: on-shell

- Resummation:
  - dominant contributions $\mathcal{O}(A_b, \mu \tan\beta)$ at NLO can be of $\mathcal{O}(1)$ for large $\tan\beta$
  - large $\tan\beta$ motivated by approximate unification of the tau and bottom Yukawa couplings at high energies
  - corrections of $\mathcal{O}(\alpha_s \mu \tan\beta / M_{SUSY})^n$ and $\mathcal{O}(\alpha_s A_b / M_{SUSY})^m$ are included to all orders $n, m = 1,2,\ldots$ $\Longrightarrow$ perturbativity improved

LO:



NLO:



$\Rightarrow \lambda_b^0 \, A_b \, \phi_1^0, \, \lambda_b^0 \, \mu \, \phi_2^{0*}$

$$\mathcal{L}^{LO} = -\lambda_b^0 \overline{b_R^0} \phi_1^0 b_L^0 + h.c. \qquad \mathcal{L}_{eff} = -\lambda_b^0 \overline{b_R^0} \left[(1+\Delta_1)\phi_1^0 + \Delta_2 \phi_2^{0*} \right] b_L^0 + h.c.$$

NLO = loop-suppressed, but once $\phi_{1,2}^0$ acquire VEV $v_{1,2}$: μ enhanced by tanβ and trilinear scalar coupling A_b can be of $\mathcal{O}(\mu \tan\beta)$ (e.g. no-mixing scenarios)

Leading parts in A_b and μ are finite in NLO:

$$\Delta_1 = -\frac{C_F}{2}\frac{\alpha_s}{\pi} m_{\tilde{g}} A_b I(m^2_{\tilde{b}_i}, m^2_{\tilde{b}_j}, m^2_{\tilde{g}})$$

$$\Delta_2 = \frac{C_F}{2}\frac{\alpha_s}{\pi} m_{\tilde{g}} \mu I(m^2_{\tilde{b}_i}, m^2_{\tilde{b}_j}, m^2_{\tilde{g}})$$

$$I(a,b,c) = \frac{ab\log\left(\frac{a}{b}\right) + bc\log\left(\frac{b}{c}\right) + ca\log\left(\frac{c}{a}\right)}{(a-b)(b-c)(a-c)}$$

all mass parameters of equal size:

$$\Delta_1 \sim \pm\Delta_2 \tan\beta = \frac{\alpha_s(M_{SUSY})}{3\pi}\tan\beta \sim \mathcal{O}(1) \Rightarrow \textit{resummation}$$

▷ *transform to the physical Higgs bosons:*

$$\Longrightarrow \quad \mathcal{L}_{eff} = -\frac{m_b}{v}\bar{b}\left\{\tilde{g}_b^h h + \tilde{g}_b^H H - i\gamma_5 \tilde{g}_b^A A\right\} b$$

resummed SUSY factors: $\tilde{g}_b^h = \frac{g_b^h}{1+\Delta_b \tan\beta}\left(1 - \frac{\Delta_b}{\tan\alpha}\right)$,

$$\tilde{g}_b^H = \frac{g_b^H}{1+\Delta_b \tan\beta}(1+\Delta_b \tan\alpha) \quad \textit{and} \quad \tilde{g}_b^A = \frac{g_b^A}{1+\Delta_b \tan\beta}\left(1 - \frac{\Delta_b}{\tan\beta}\right)$$

contain effective change $1/(1+\Delta_b \tan\beta)$

$\Longrightarrow$ *in* $\mathcal{L}_{eff}$ *all terms* $\mathcal{O}[(\alpha_s/M_{SUSY})^n(\mu\tan\beta)^m A_b^{n-m}]$ *resummed*

Carena, Garcia, Nierste, Wagner
Guasch, Häfliger, Spira

▷ *extension of the previous resummation to* A_b *terms*

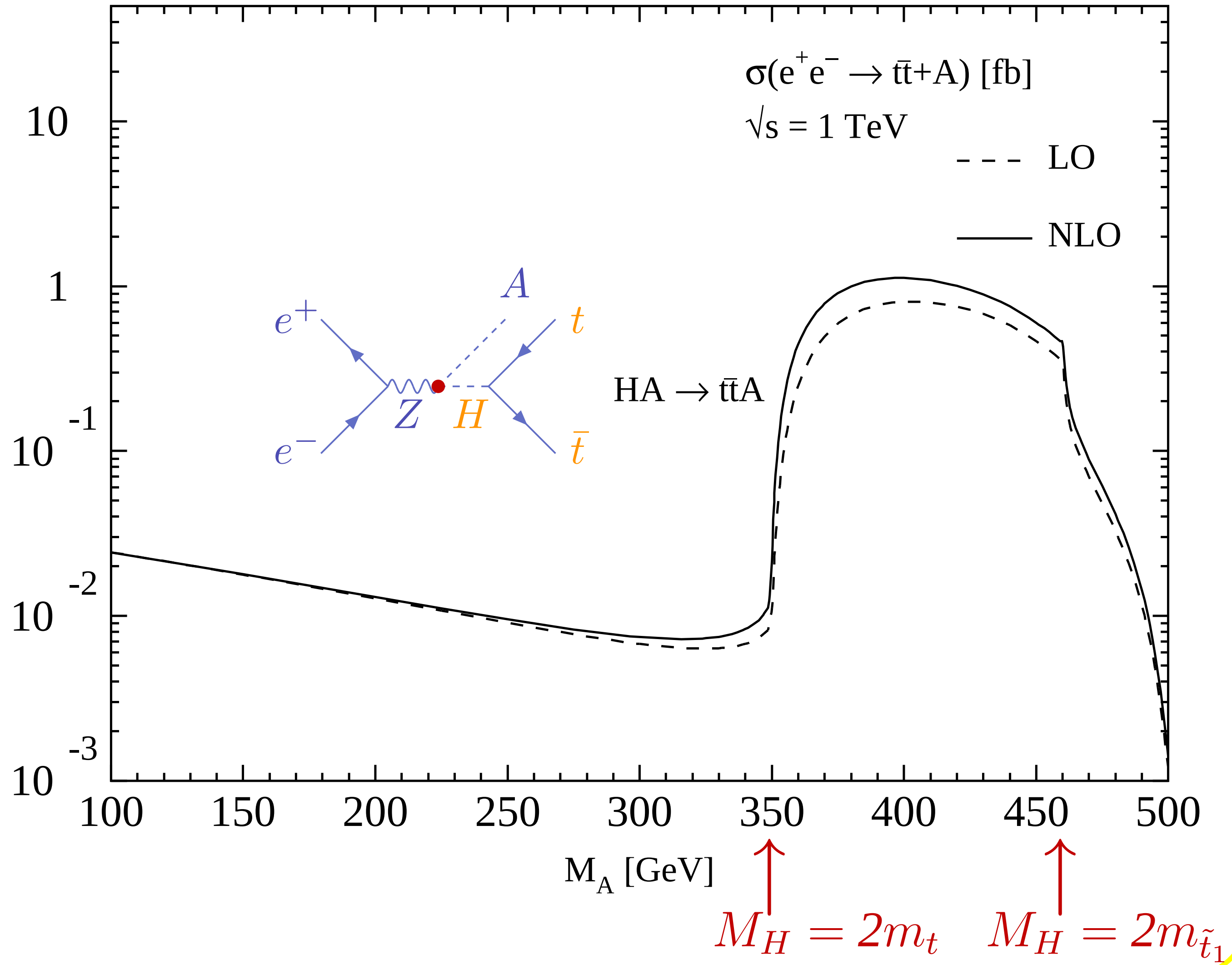
▷ *explains absence of* A_b *in NLO SUSY corrections:*
leading A_b *absorbed in eff. Yukawa coupling in the low-energy eff. Lagrangian*

IV. $e^+e^- \longrightarrow t\bar{t}A$: Total Cross Section

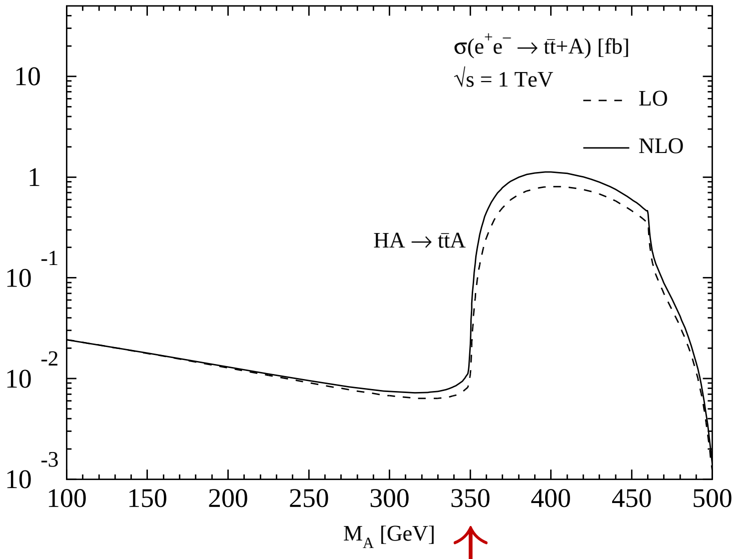
SPS 5 (m_0 = 150 GeV, $m_{1/2}$ = 300 GeV, A_0 = −1000 GeV, $\mu > 0$, $\tan\beta$ = 5) Allanach et al

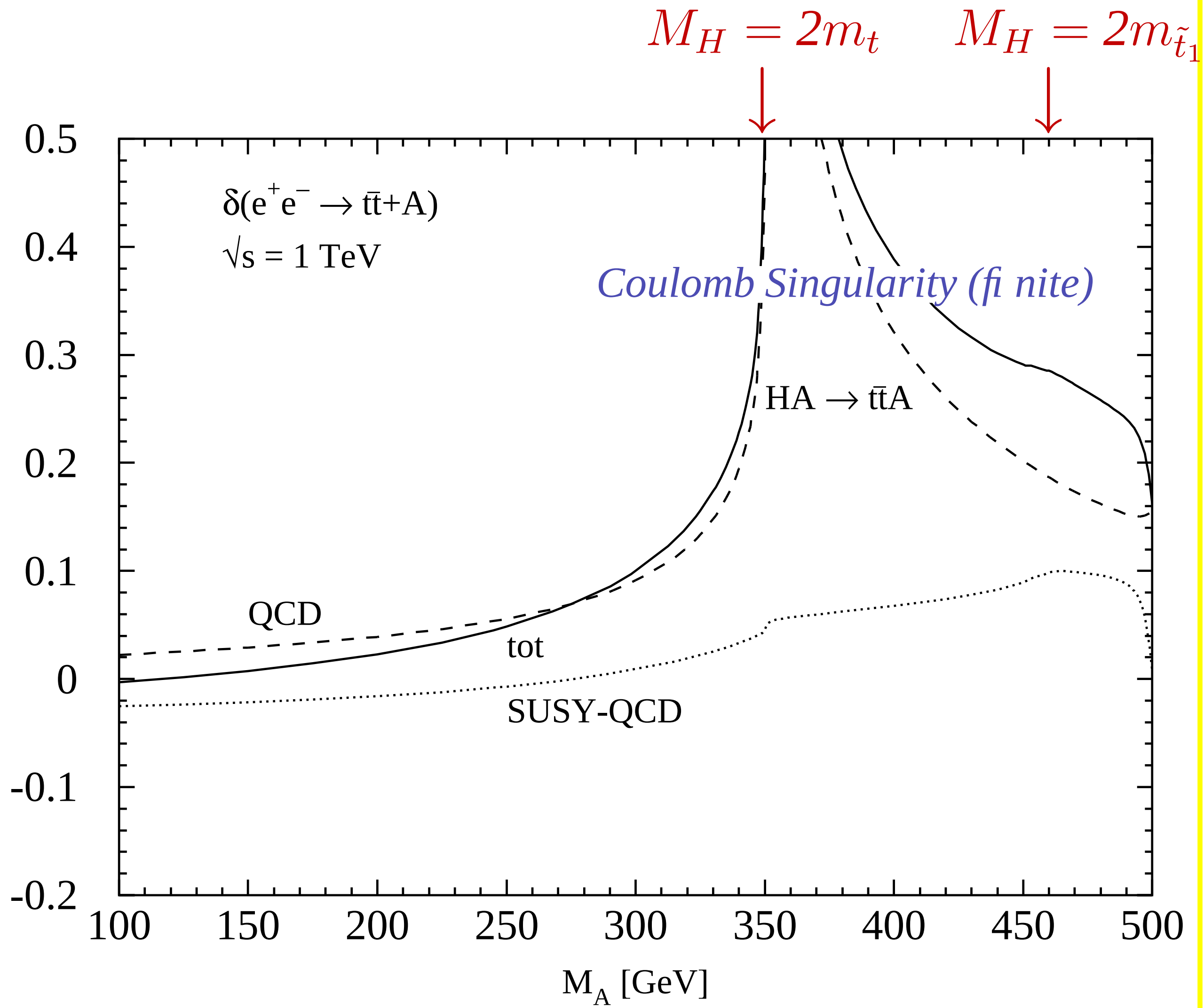
$\tan\beta = 5$
$\mu = 639.8$ GeV
$A_t = -905.6$ GeV
$A_b = -671.4$ GeV
$m_{\tilde{g}} = 710.3$ GeV
$m_{\tilde{q}_L} = 535.2$ GeV
$m_{\tilde{b}_R} = 620.5$ GeV
$m_{\tilde{t}_R} = 360.5$ GeV

$\longrightarrow m_{\tilde{t}_1} = 230.4$ GeV
$m_{\tilde{t}_2} = 637.8$ GeV



IV. $e^+e^- \to t\bar{t}A$: Relative Corrections QCD and SUSY-QCD





$\lesssim$ 10% SUSY-QCD corrections $\Rightarrow$ measureable at ILC

IV. $e^+e^- \to b\bar{b}A$: Total Cross Sections w/ and w/o Resummation

$$\sigma_{w/o} = g_b^2 \sigma_0 (1 + \delta_{NLO})$$
$$\sigma_{w/} = \tilde{g}_b \sigma_0 \left[\tilde{g}_b + g_b(\delta_{NLO} - \delta_{NLO}^{LE})\right]$$

SPS 1b

(m_0 = 200 GeV, $m_{1/2}$ = 400 GeV, A_0 = 0 GeV, $\mu > 0$, $\tan\beta$ = 30)

$\tan\beta = 30$

$\mu =$ 495.6 GeV

$A_t = -729.3$ GeV

$A_b = -987.4$ GeV

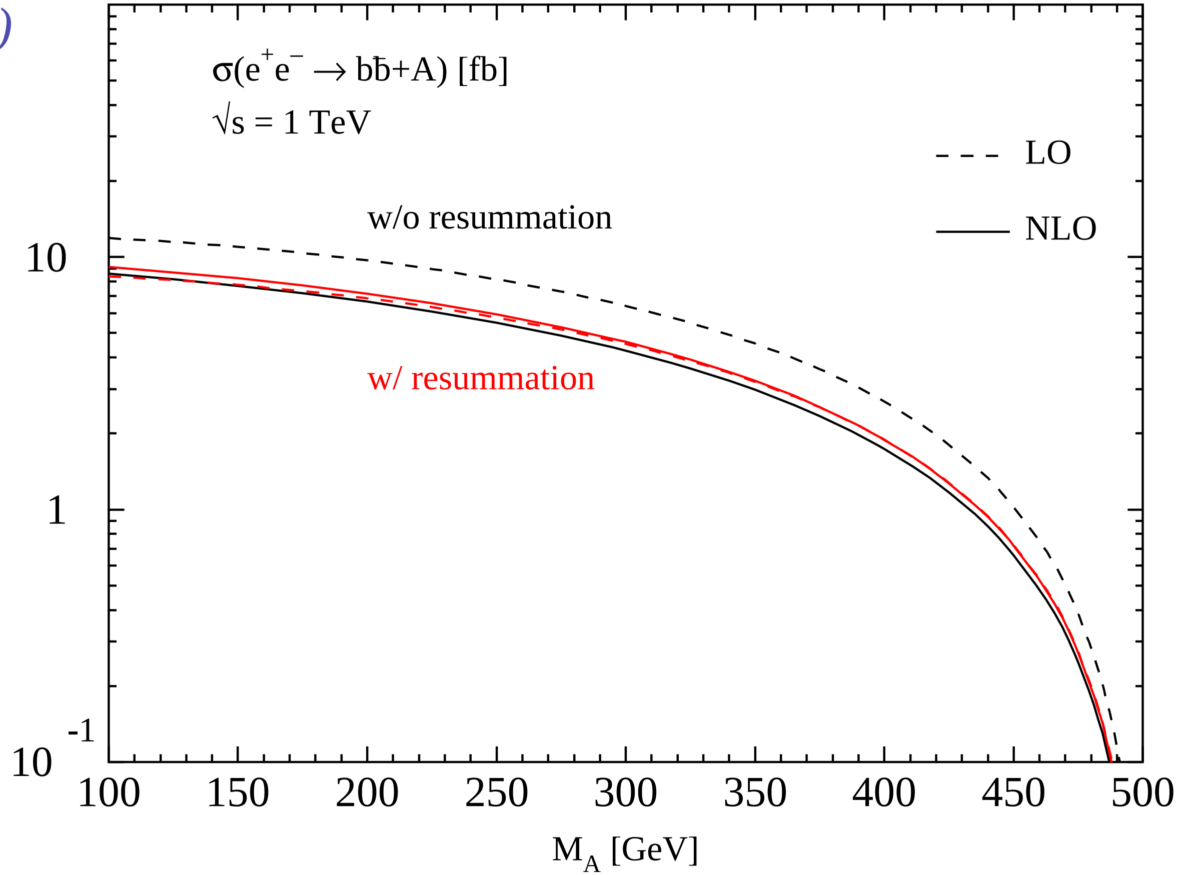
$m_{\tilde{g}} =$ 916.1 GeV

$m_{\tilde{q}_L} =$ 762.5 GeV

$m_{\tilde{b}_R} =$ 780.3 GeV

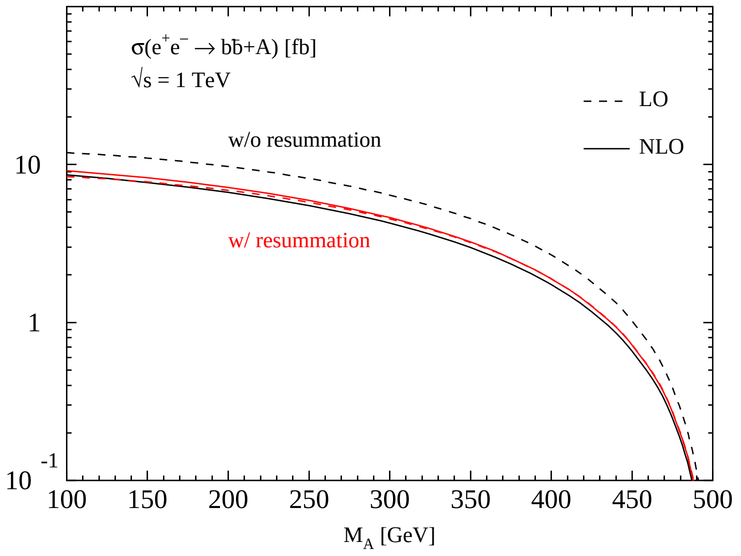
$m_{\tilde{t}_R} =$ 670.7 GeV

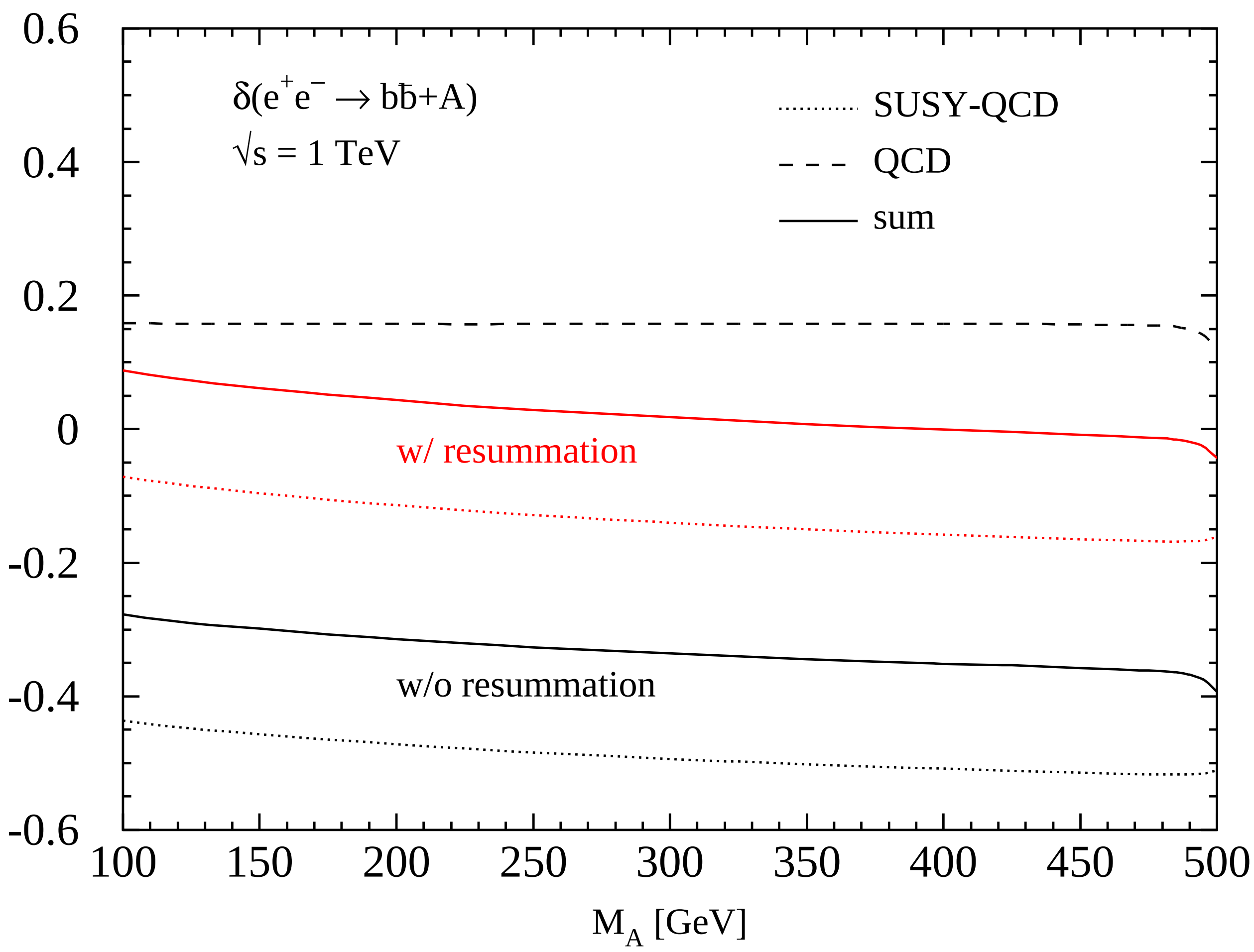
$\longrightarrow m_{\tilde{b}_1} = 100.6$ GeV
$m_{\tilde{b}_2} = 576.1$ GeV



bulk of SUSY-QCD corrections is absorbed by the resummed bottom Yukawa couplings

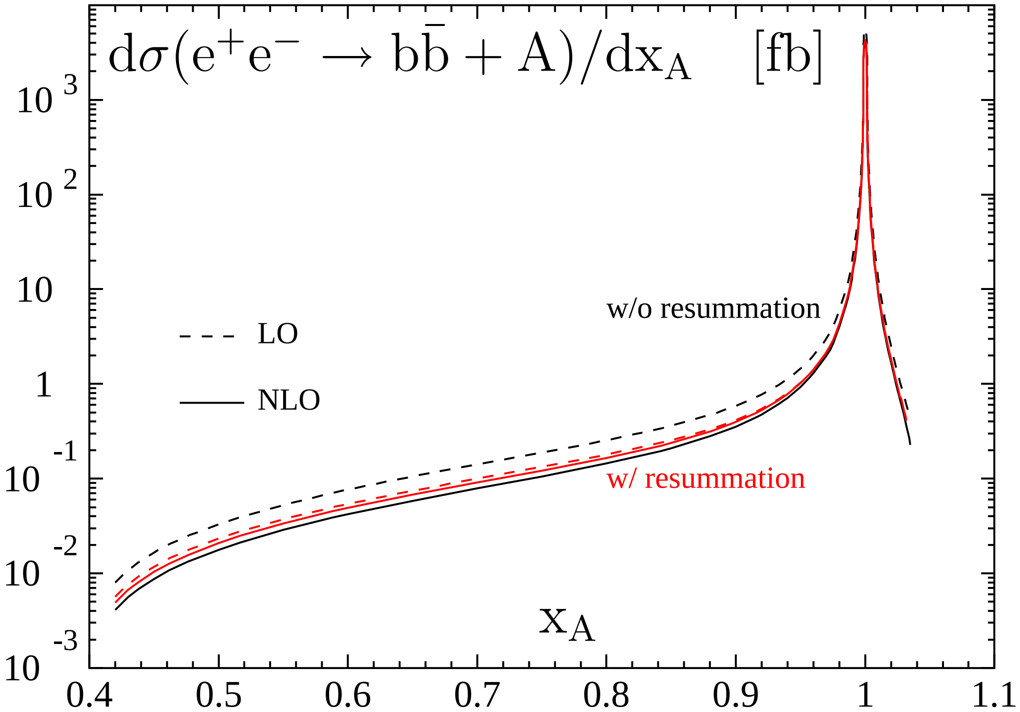
IV. $e^+e^- \to b\bar{b}A$: Relative Corrections w/ and w/o Resummation





after resummation partial cancellation of QCD and SUSY-QCD

IV. $e^+e^- \to b\bar{b}A$: Differential Cross Sections $d\sigma/dx_A$



continuum

resonant $h, H \to b\bar{b}$ decay

$[M_{b\bar{b}}^2 \approx M_{h,H}^2]$

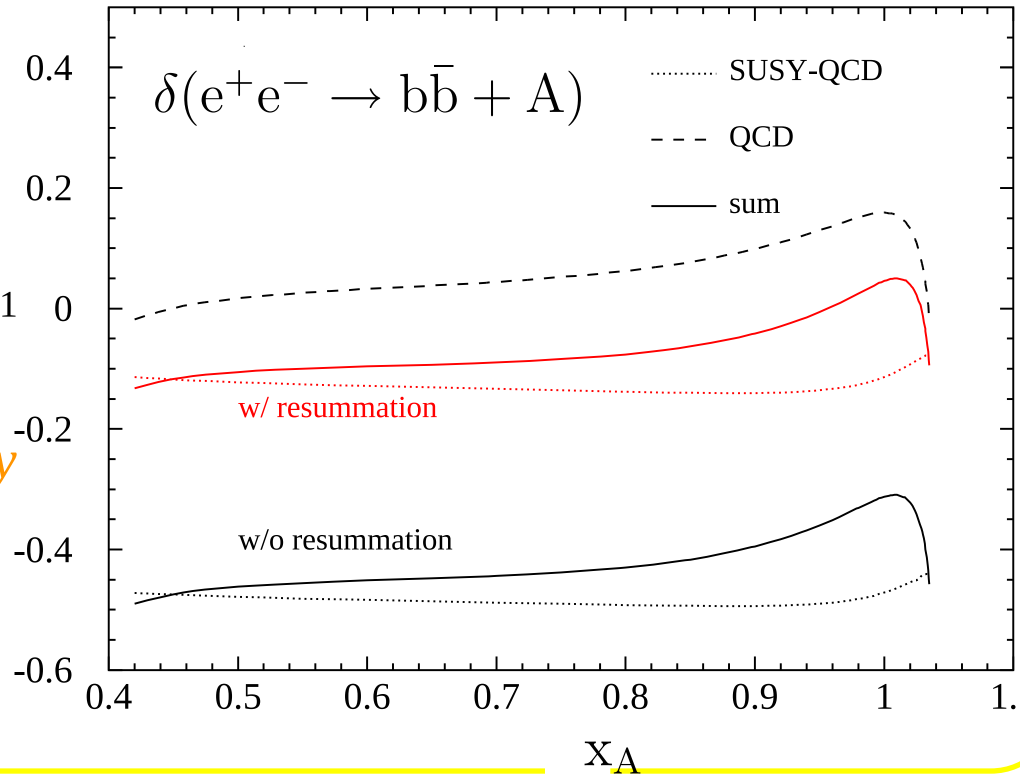
$x_A = 2E_A/\sqrt{s}$

$M_A = 200$ GeV

$\sqrt{s} = 1$ TeV

resonant $h, H \to b\bar{b}$ decay

continuum



after resummation partial cancellation of QCD and SUSY-QCD

V. Conclusions

$e^+e^- \to t\bar{t}\phi$

- *measurement of top Yukawa coupling*
- *QCD corrections:* $\sim -10\% \cdots +30\%$ *outside the* $t\bar{t}$*−threshold*
- *genuine SUSY-QCD corrections:* $\sim -10\% \cdots +10\%$

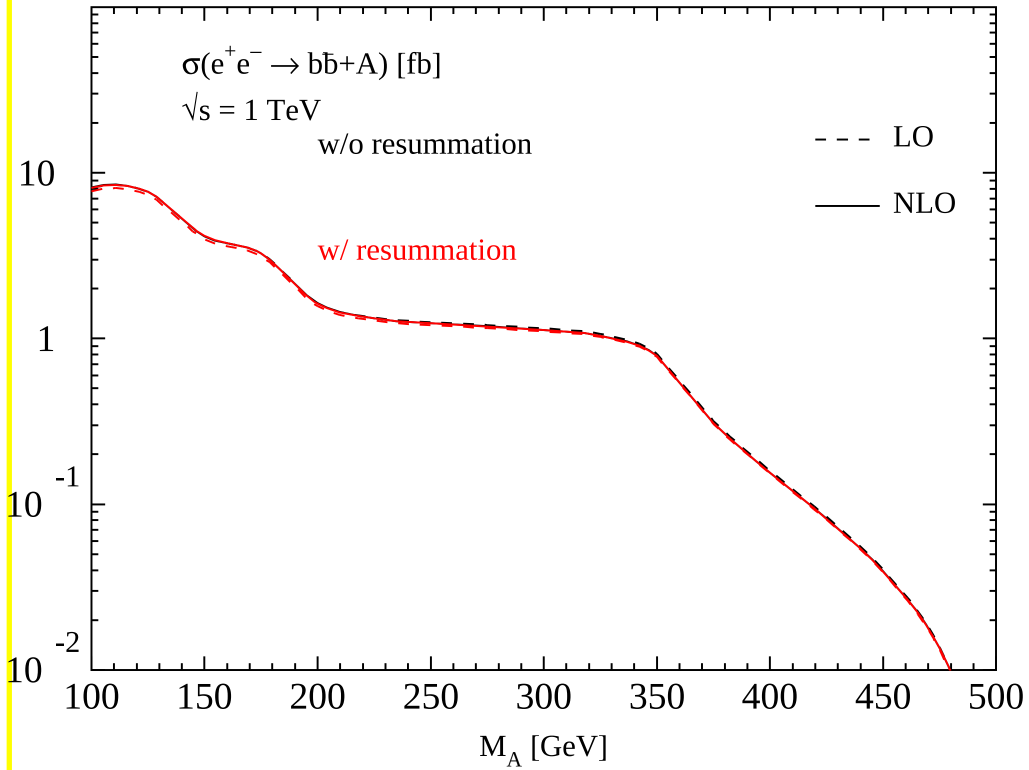
$e^+e^- \to b\bar{b}\phi$

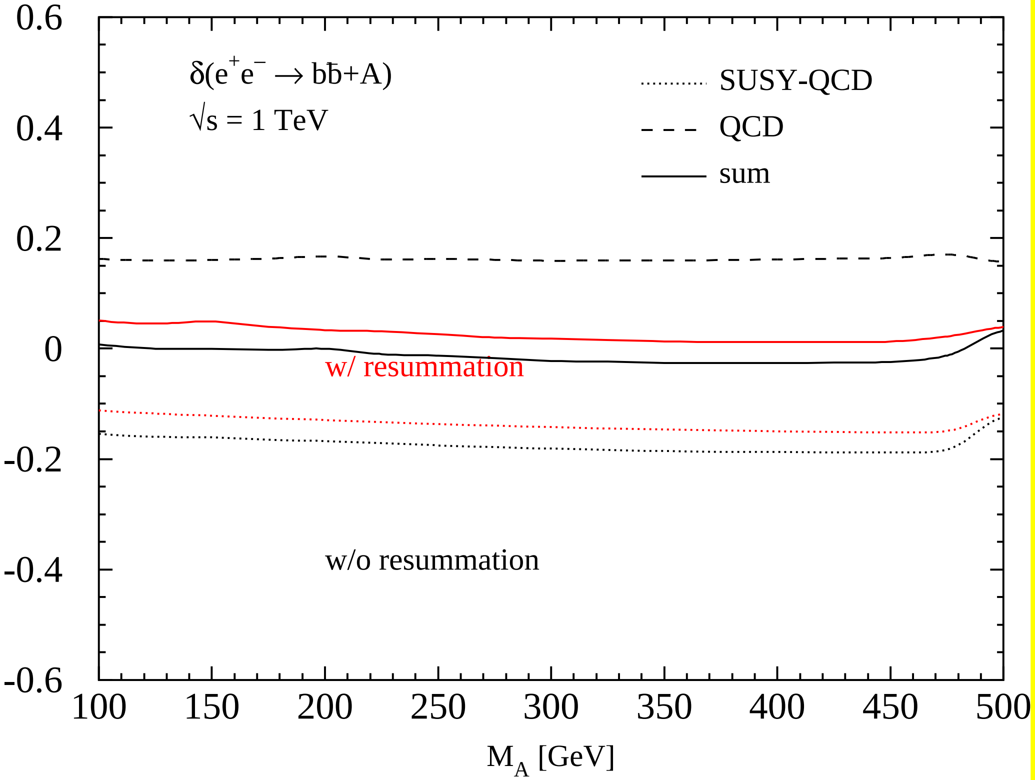
- *determination of* $\tan\beta$ *for large* $\tan\beta$
- *QCD corrections:* $\sim 5\% \ldots 20\%$
- *genuine SUSY-QCD corrections:* $\sim -5\% \cdots -20\%$
- *resummation effects important*

arXiv: hep-ph/0501164, NPB 719: 35-52 (2005)

IV. $e^+e^- \to b\bar{b}A$: Total Cross Sections and Relative Corrections

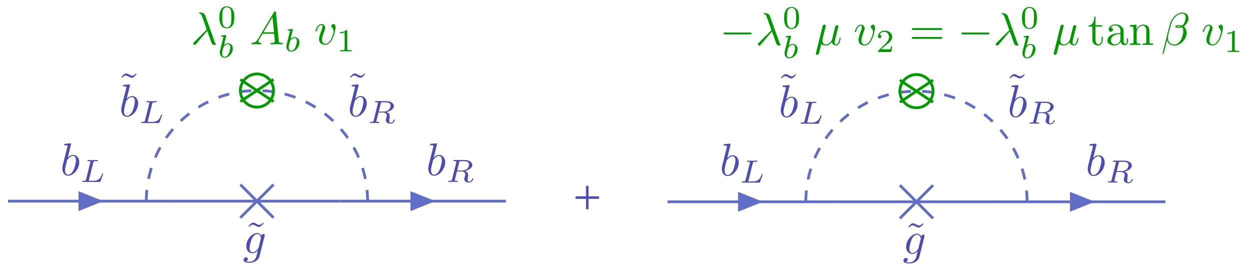
SPS1b *with* $\tan\beta =$ *3*





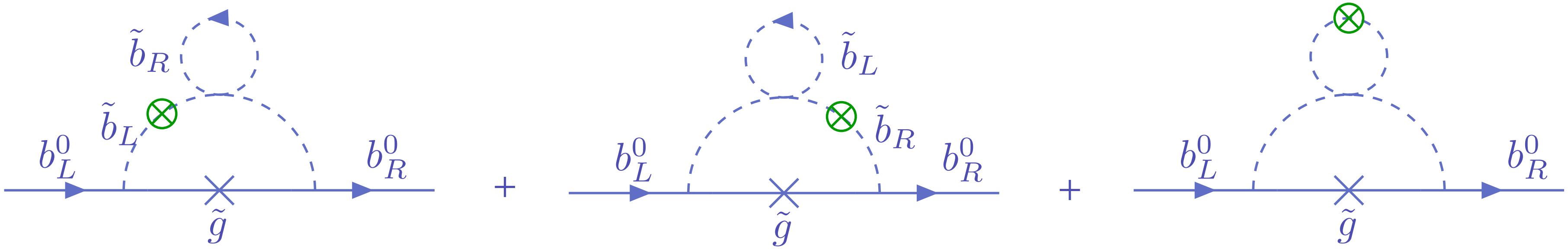
$$m_b = \lambda_b v_1 \implies m_b \equiv \lambda_b^0 v_1 + \Sigma_b^{NLO}(m_b) = \lambda_b^0 v_1 \left[1 + \Delta_1 + \Delta_2 \tan\beta\right]$$

▷ *self-energy at NLO:*



$$\alpha_s \lambda_b^0 \,(A_b - \mu \tan\beta)\, v_1 m_{\tilde{g}} \times C_0(0,0; m_{\tilde{b}_1}, m_{\tilde{b}_2}, m_{\tilde{g}}) \sim \alpha_s m_b (A_b - \mu \tan\beta)/M_{SUSY}$$

▷ *self-energy NNLO (irreducible):*



$$\left.\begin{array}{l} \alpha_s^2 \lambda_b^0 \,(A_b - \mu \tan\beta)\, v_1 m_{\tilde{g}} \times A_0(m_{\tilde{b}_i}) D_0(0,0,0; m_{\tilde{b}_1}, m_{\tilde{b}_2}, m_{\tilde{b}_j}, m_{\tilde{g}}) \\ \alpha_s^2 \lambda_b^0 \,(A_b - \mu \tan\beta)\, v_1 m_{\tilde{g}} \times B_0(0; m_{\tilde{b}_1}, m_{\tilde{b}_2}) C_0(0,0; m_{\tilde{b}_i}, m_{\tilde{b}_j}, m_{\tilde{g}}) \end{array}\right\} \sim \alpha_s{}^2 m_b \frac{A_b - \mu \tan\beta}{M_{SUSY}}$$

any further mass-insertion in the sbottom propagators:

$$\frac{1}{q^2 - m_{\tilde{b}_i}^2} \Rightarrow \frac{1}{q^2 - m_{\tilde{b}_1}^2} m_b (A_b - \mu \tan\beta) \frac{1}{q^2 - m_{\tilde{b}_2}^2} \sim -\frac{m_b (A_b - \mu \tan\beta)}{M_{SUSY}^2} \frac{1}{q^2 - m_{\tilde{b}_i}^2}$$

suppressed by another power of $m_b/M_{SUSY}^2 \implies$ *non-leading terms*

▷ *extention to any perturbative order:*

KLN theorem $\Rightarrow$ *irreducible diagrams do not develop power-like divergences in the bottom mass for* $m_b \to 0 \Rightarrow$ *no* $1/m_b$ *terms*

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$\Rightarrow$ *no leading irreducible terms of* $\mathcal{O}\left[(\alpha_s A_b)^n\right]$, $\mathcal{O}\left[(\alpha_s \mu \tan\beta)^n\right]$ *nor* $\mathcal{O}\left[\alpha_s^n A_b^{n-m} (\mu \tan\beta)^m\right]$

▷ *reducible higher order self-energy :* $\propto$ *(NLO self-energy)*$^n \propto \left(\alpha_s \frac{A_b - \mu\tan\beta}{M_{SUSY}}\right)^n \Rightarrow$ *leading*

Conclusion:

▷ *leading terms in NLO:* $\alpha_s \frac{A_b - \mu\tan\beta}{M_{SUSY}} = NLO$

▷ *leading terms in NNLO:* $\left(\alpha_s \frac{A_b - \mu\tan\beta}{M_{SUSY}}\right)^2 = (NLO)^2$

▷ *leading terms in NNNLO:* $\left(\alpha_s \frac{A_b - \mu\tan\beta}{M_{SUSY}}\right)^3 = (NLO)^3$

▷ ...

$$\Longrightarrow \boxed{1 + NLO + (NLO)^2 + \cdots = \frac{1}{1 - NLO}} \Longleftrightarrow \boxed{\frac{1}{1 + \Delta_b \tan\beta}}$$

$$\sigma_{w/}^{NLO} = \tilde{g}_b \sigma_0 \left[\tilde{g}_b + g_b(\delta_{NLO} - \delta_{NLO}^{LE}) \right]$$

$$g_b^h = -\frac{s_\alpha}{c_\beta}, \; \tilde{g}_b^h = g_b^h \left(\frac{1 - \Delta_b / t_\alpha}{1 + \Delta_b t_\beta} \right), \quad \delta_{NLO}^{LE} = -\frac{\Delta_2}{C_F \alpha_s / (2\pi)} \left(1 + \frac{1}{t_\alpha} \right) = k \left(1 + \frac{1}{t_\alpha} \right)$$

and $\delta_{NLO} = k$

$\alpha \to 0 \quad \Longrightarrow \quad s_\alpha \to 0, \; c_\alpha \to 1, \; t_\alpha \to s_\alpha$:

- $$\tilde{g}_b^h(\delta_{NLO} - \delta_{NLO}^{LE}) \to \frac{-s_\alpha / c_\beta + \Delta_b / (c_\alpha c_\beta)}{1 + \Delta_b t_\beta} \left[k + k \left(1 + \frac{1}{t_\alpha} \right) \right]$$
  $$\to \frac{1}{1 + \Delta_b t_\beta} \left[-k \underbrace{\frac{1}{c_\alpha c_\beta}}_{1/c_\beta} + k \frac{\Delta_b}{c_\beta} + k \left(\frac{1}{c_\beta} + \underbrace{\frac{1}{s_\alpha c_\beta}}_{\infty} \right) \right] \to \infty$$

- $$g_b^h(\delta_{NLO} - \delta_{NLO}^{LE}) \to -k \underbrace{\frac{s_\alpha}{c_\beta}}_{1/c_\beta} + k \left(\underbrace{\frac{s_\alpha}{c_\beta}}_{0} + \underbrace{\frac{1}{c_\alpha c_\beta}}_{1/c_\beta} \right) \to k/c_\beta \text{ is finite}$$