Recent Higher-Order Corrections in the r/cMSSM Higgs Sector

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1. Introduction

2. Corrections of $\mathcal{O}(\alpha_b\alpha_s)$ in the rMSSM

3. Corrections of $\mathcal{O}(\alpha_t\alpha_s)$ in the cMSSM

4. Conclusions

1. Introduction

Enlarged Higgs sector: Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$

$$+ \underbrace{\frac{g'^2 + g^2}{8}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}} |H_1 \bar{H}_2|^2$$

gauge couplings, in contrast to SM

physical states: $h^0, H^0, A^0, H^\pm$

Goldstone bosons: $G^0, G^\pm$

Input parameters:

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2 (\tan\beta + \cot\beta)$$

$\tilde{t}/\tilde{b}$ sector of the MSSM: (scalar partner of the top/bottom quark)

Stop, sbottom mass matrices ($X_t = A_t - \mu^*/\tan\beta$, $X_b = A_b - \mu^*\tan\beta$):

$$\mathcal{M}^2_{\tilde{t}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m_t^2 + DT_{t_1} & m_t X_t^* \\ m_t X_t & M^2_{\tilde{t}_R} + m_t^2 + DT_{t_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{t}}} \begin{pmatrix} m^2_{\tilde{t}_1} & 0 \\ 0 & m^2_{\tilde{t}_2} \end{pmatrix}$$

$$\mathcal{M}^2_{\tilde{b}} = \begin{pmatrix} M^2_{\tilde{b}_L} + m_b^2 + DT_{b_1} & m_b X_b^* \\ m_b X_b & M^2_{\tilde{b}_R} + m_b^2 + DT_{b_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{b}}} \begin{pmatrix} m^2_{\tilde{b}_1} & 0 \\ 0 & m^2_{\tilde{b}_2} \end{pmatrix}$$

mixing important in stop sector (also in sbottom sector for large $\tan\beta$)

soft SUSY-breaking parameters A_t, A_b also appear in ϕ-$\tilde{t}/\tilde{b}$ couplings

$SU(2)$ relation $\Rightarrow M_{\tilde{t}_L} = M_{\tilde{b}_L}$

$\Rightarrow$ relation between $m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_{\tilde{b}_1}, m_{\tilde{b}_2}, \theta_{\tilde{b}}$

Contrary to the SM:

m_h is not a free parameter

MSSM tree-level bound: $m_h < M_Z$, excluded by LEP Higgs searches

Large radiative corrections:

Dominant one-loop corrections:

$$\Delta m_h^2 \sim G_\mu m_t^4 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$$

The MSSM Higgs sector is connected to all other sector via loop corrections (especially to the scalar top sector)

Measurement of m_h, Higgs couplings $\Rightarrow$ test of the theory

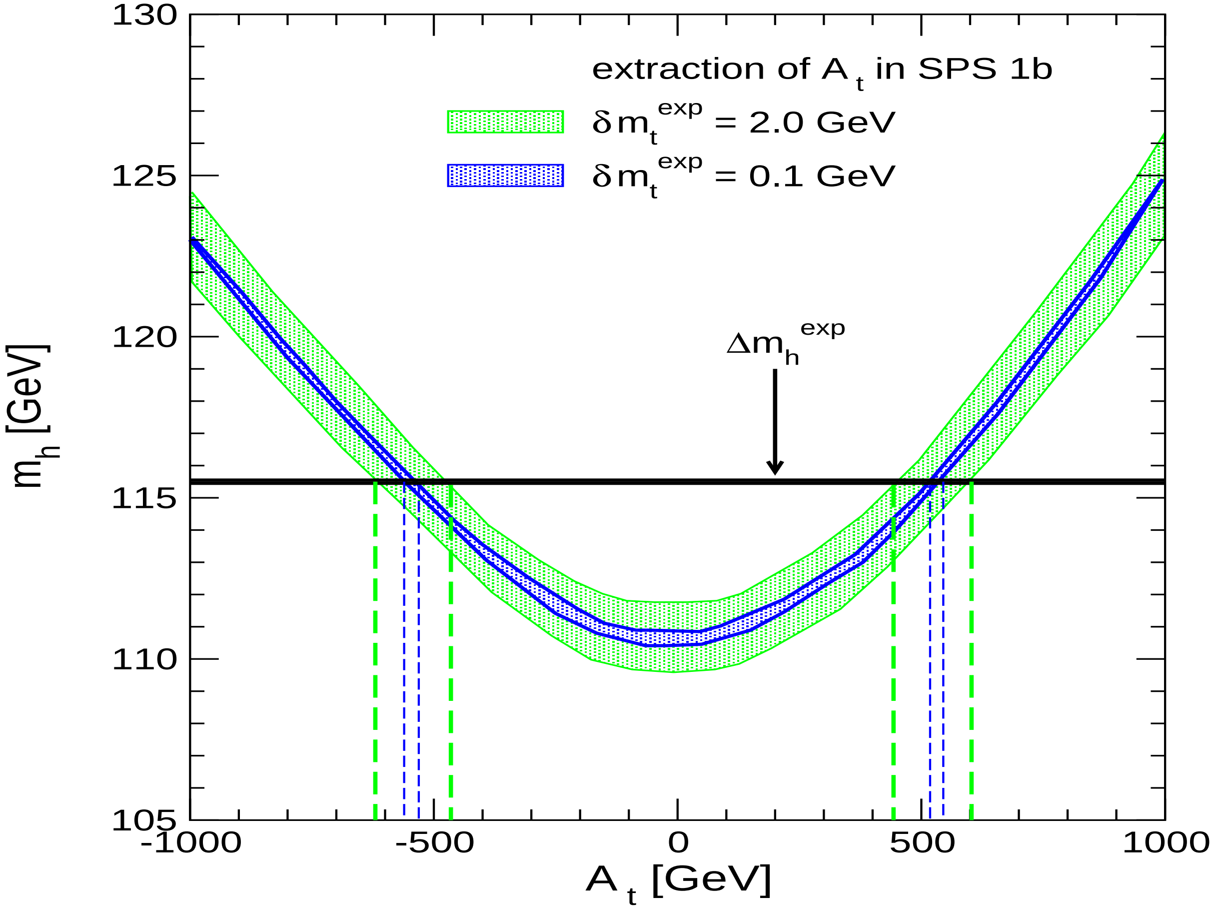
LC: $\Delta m_h \approx 0.05$ GeV

$\Rightarrow$ aim for theoretical precision!

($\Rightarrow m_h$ will be (the best?) electroweak precision observable)

Example of application: m_h prediction as a function of A_t

[S.H., S. Kraml, W. Porod, G. Weiglein '02]



SPS1b:

$m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}, m_{\tilde{b}_2}$ known,

A_t unknown

$\tan\beta, M_A$ known,

realistic parametric errors assumed (from SUSY exp. errors)

$\Rightarrow$ extraction of A_t possible

Theory error neglected

$\Rightarrow$ m_h is crucial input for SUSY fit programs (Fittino, Sfitter)

$\to$ see talk by Peter Wienemann

Experimental situation:

LC will provide high accuracy measurements !

Theory situation:

measured observables have to be compared with theoretical predictions (in the MSSM)

Measured data is only meaningful if it is matched with theoretical calculations (masses, couplings) at the same level of accuracy

> Theoretical calculations should be viewed as an essential part of all future High Energy Physics programs

⇒ concentrate on Higgs masses and couplings here

2. Correction of $\mathcal{O}(\alpha_b\alpha_s)$ in the rMSSM

Evaluation of Higgs boson masses in the MSSM with real parameters:

Two-point vertex function:

$$\Gamma(q^2) = \begin{pmatrix} q^2 - m^2_{H,\text{tree}} + \hat{\Sigma}_{HH}(q^2) & \hat{\Sigma}_{hH}(q^2) \\ \hat{\Sigma}_{hH}(q^2) & q^2 - m^2_{h,\text{tree}} + \hat{\Sigma}_{hh}(q^2) \end{pmatrix}$$

determination of $\det(\Gamma(q^2)) = 0 \Rightarrow M_h, M_H, \alpha_{\text{eff}}, \ldots$

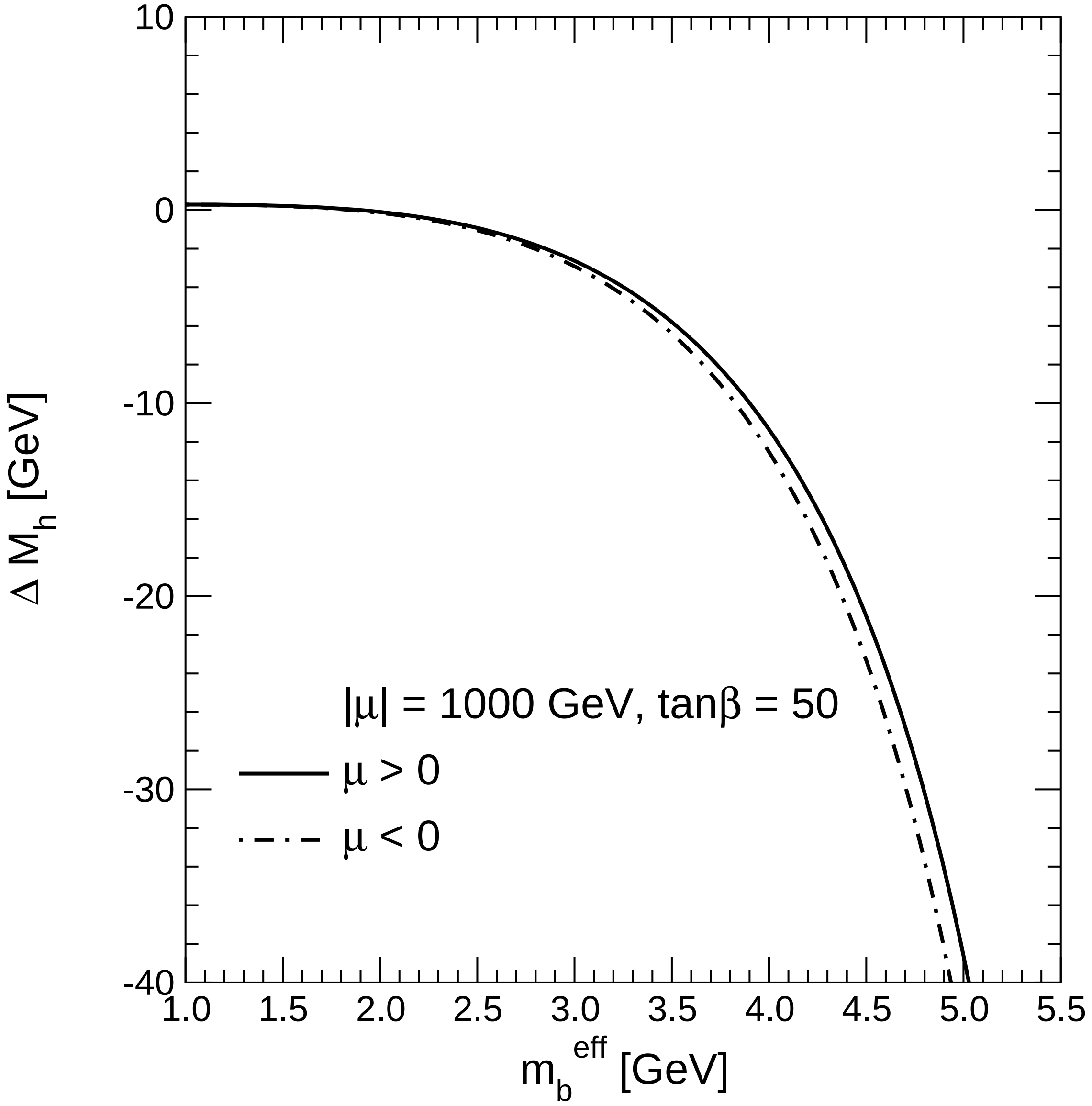
Main task: calculation of $\hat{\Sigma}(q^2)$, including renormalization

Here:

– evaluation of 2-loop corrections of $\mathcal{O}(\alpha_b\alpha_s)$

– comparison of 4 different renormalization schemes

Motivation: Why 2-loop corrections in the $b/\tilde{b}$ sector?



1-loop corrections $\mathcal{O}(\alpha_b)$ to M_h can be sizable

Precise M_h prediction
⇒ 2-loop corrections necessary

The Higgs self-energy at 2-loop:

$\rightarrow$ α_s correction to the leading 1-loop term $\sim m_b^4$

Approximations:

– only m_b^2 ($\sim y_b^2$) terms
– gauge couplings are set to 0
– external momentum is set to 0

$$\Rightarrow \hat{\Sigma}_{22}^{(2)}(q^2) \approx \Sigma_{22}^{(2)}(0) + \cos^2\beta\, \delta M_A^{2\,(2)} - \frac{e}{2\, M_W s_W}\left(\sin^2\beta\, \cos\beta\, \delta t_1^{(2)} - \sin\beta\, (1 + \cos^2\beta\,)\delta t_2^{(2)}\right)$$

in the $\phi_1\phi_2$ basis with

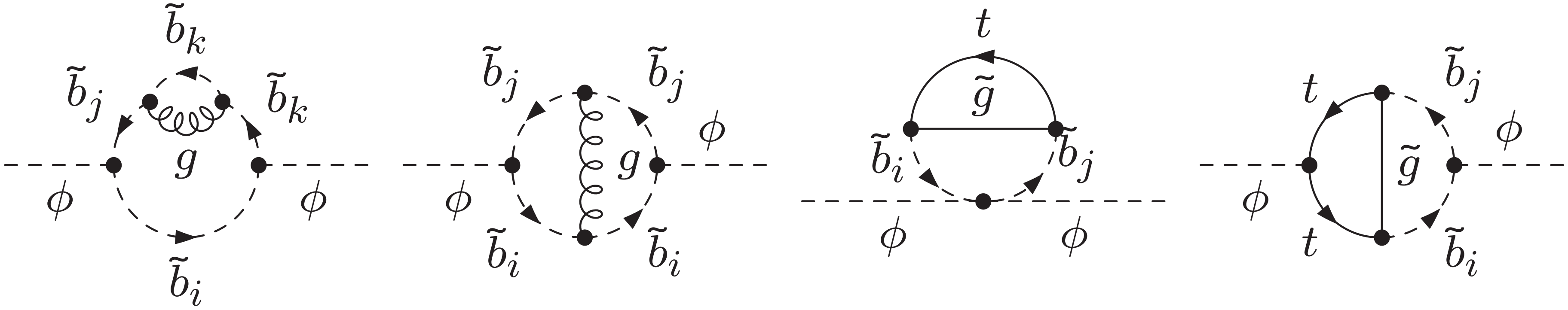
$\Sigma_{22}^{(2)}(0)$: unrenormalized 2-2 self-energy

$\delta M_A^{2\,(2)} = \Sigma_A^{(2)}(0)$: A mass counter term

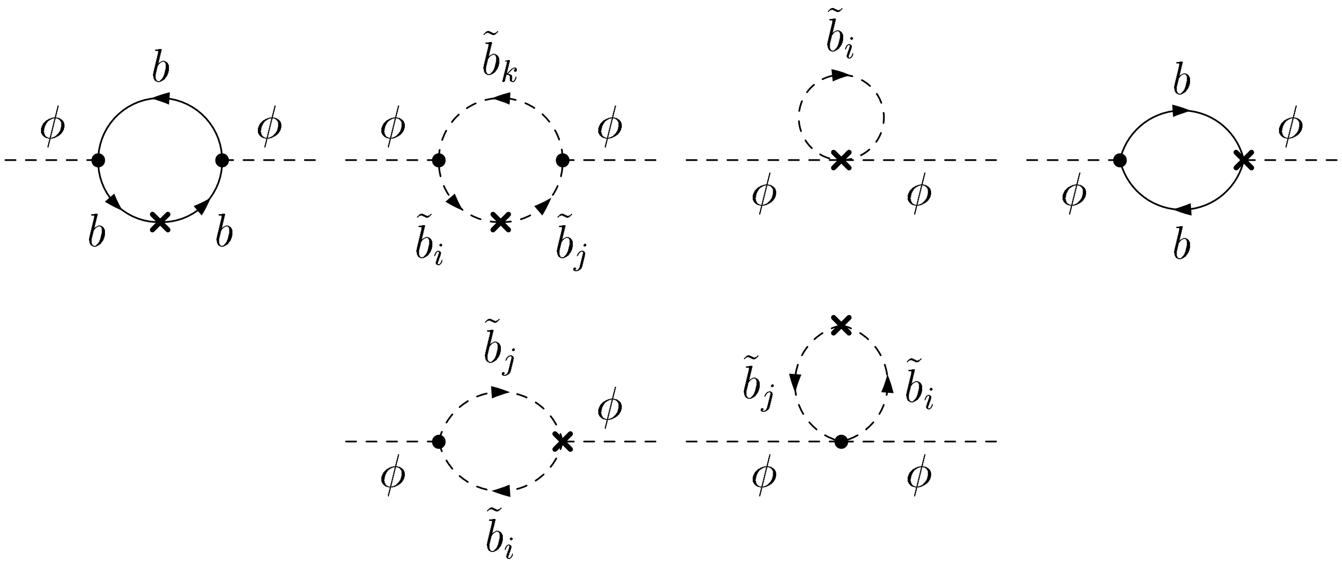
$\delta t_i^{(2)} = -T_i^{(2)}$: ϕ_i tad-pole

Contributions to the 2-loop self-energy:

2-loop self-energy diagrams:



diagrams with counter term insertion:



$\rightarrow$ different renormalization schemes enter

Renormalization:

Calculation of two-loop corrections of $\mathcal{O}(\alpha_t \alpha_s)$ and $\mathcal{O}(\alpha_b \alpha_s)$

$\Rightarrow$ parameters of the $t/\tilde{t}$ and $b/\tilde{b}$ are defined at the 1-loop level

$\Rightarrow$ different choices of renormalization possible

$t/\tilde{t}$ sector:

one renormalization scheme:
4 independent parameters:
$m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, $\theta_{\tilde{t}}$, m_t on-shell
$\rightarrow$ A_t given in terms of the others

$b/\tilde{b}$ sector: four schemes analyzed

Investigation of scheme dependence:

$\Rightarrow$ information about size of missing higher order corrections

$\Rightarrow$ estimate of theory uncertainty

Renormalization schemes in the $b/\tilde{b}$ sector:

$$SU(2) \text{ relation} \Rightarrow M_{\tilde{t}_L} = M_{\tilde{b}_L}$$

$\Rightarrow$ out of $m_{\tilde{b}_1}$, $m_{\tilde{b}_2}$, $\theta_{\tilde{b}}$, A_b, m_b only 3 are independent

$\Rightarrow$ two parameters (incl. CTs) are given in terms of the others

In all four schemes: $m_{\tilde{b}_1}$ dep. ($SU(2)$ relation), $m_{\tilde{b}_2}$ OS

scheme	b-mass m_b	A_b	mixing angle $\theta_{\tilde{b}}$
m_b $\overline{\text{DR}}$	$\overline{\text{DR}}$	$\overline{\text{DR}}$	dep.
A_b, $\theta_{\tilde{b}}$ OS	dep.	OS	OS
A_b, $\theta_{\tilde{b}}$ $\overline{\text{DR}}$	$\overline{\text{DR}}$	dep.	$\overline{\text{DR}}$
m_b OS	OS	dep.	OS

$\Rightarrow$ scheme m_b OS: analogous to the $t/\tilde{t}$ sector

$\rightarrow$ obvious choice ?

Resummed bottom quark mass:

$\longrightarrow$ absorb the leading corrections in a resummed form in the bottom quark mass at the 1-loop level

$$m_b^{\overline{\rm DR}} = \frac{\tilde{m}_b^{\rm pole} + \Sigma_b^{\tan\beta\,{\rm non-enh.}}{}_{|{\rm fin}}}{1 + \Delta m_b}$$

with

$$\Delta m_b = \frac{2\,\alpha_s}{3\,\pi} \tan\beta\,\mu\,m_{\tilde{g}}\, I(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, m^2_{\tilde{g}})$$

$$\Sigma_b^{\tan\beta\,{\rm non-enh.}}{}_{|{\rm fin}} = \tan\beta \text{ non-enhanced terms in } \Sigma_{b,s}$$

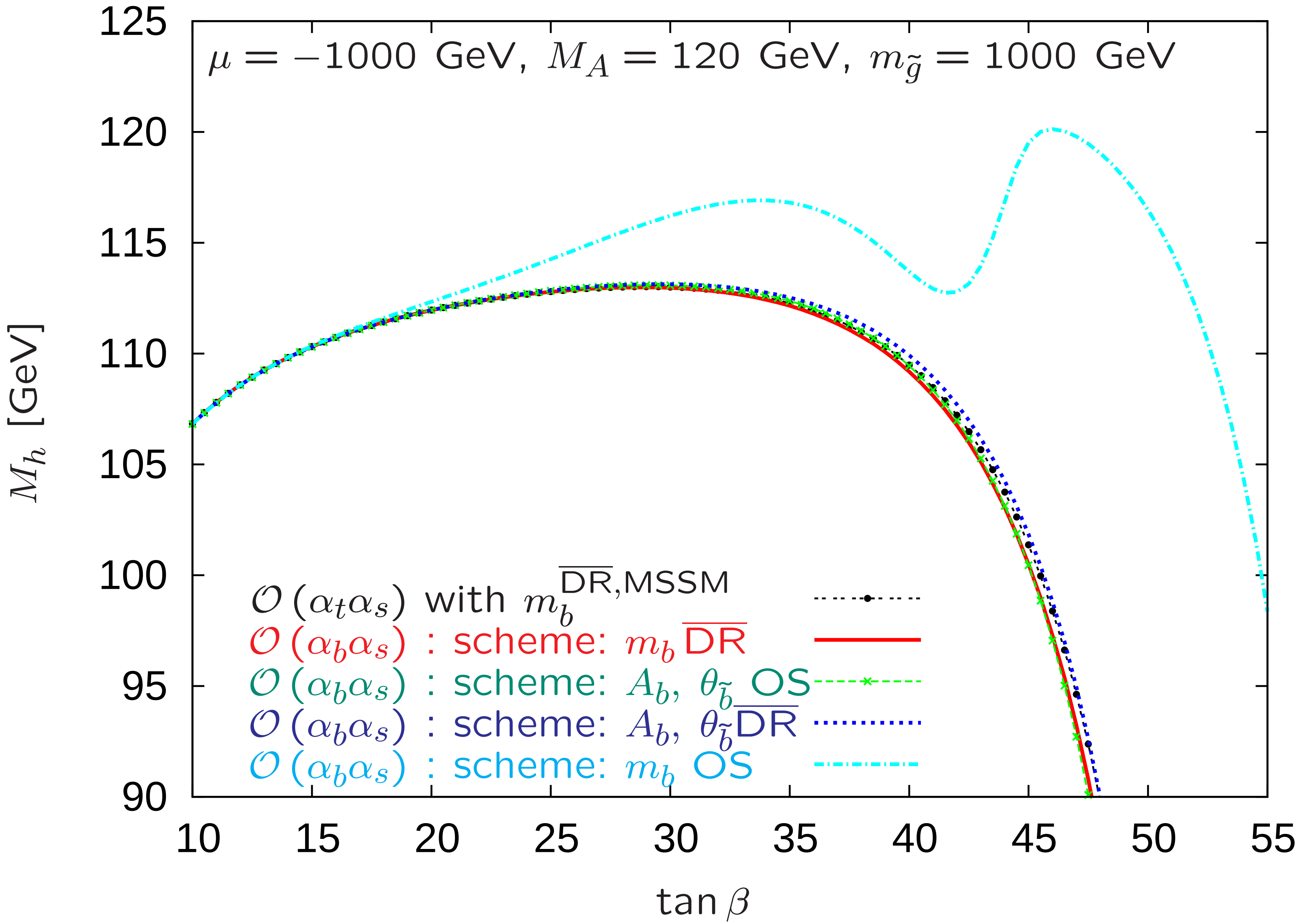
$$\tilde{m}_b^{\rm pole} = m_b^{\overline{\rm MS}}(M_Z) \times \left[1 + \frac{\alpha_s}{\pi}\left(\frac{4}{3} - \log\frac{(m_b^{\overline{\rm MS}})^2}{M_Z^2}\right)\right]$$

"formal" pole mass obtained from the $\overline{\rm MS}$ mass

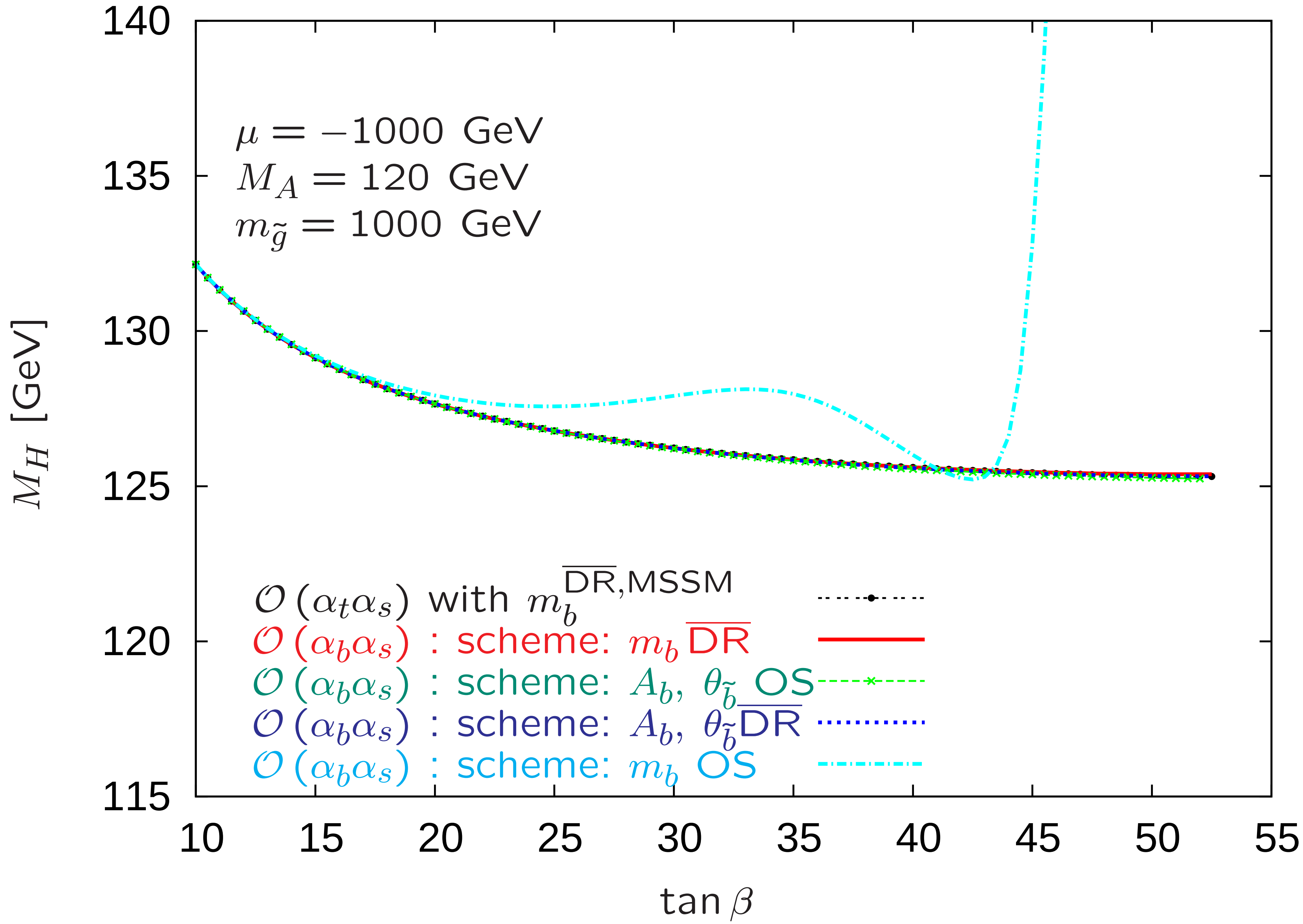
$\Rightarrow$ large higher-order corrections included at the 1-loop level

$\longrightarrow$ other renormalization schemes by finite shift

M_h as a function of $\tan\beta$, $\mu < 0$:



M_H as a function of $\tan\beta$, $\mu < 0$:



Observations:

- Scheme m_b OS gives very large corrections

  Reason: A_b is a dependent quantity $\Rightarrow$ large corrections via δA_b

$$\delta A_b = \frac{1}{m_b}\left[-\frac{\delta m_b}{2\,m_b}(m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2})\sin 2\theta_{\tilde{b}} + \ldots\right]$$
$$= \frac{1}{m_b}\left[-\delta m_b(A_b - \mu \tan\beta) + \ldots\right]$$

$$\hat{\Sigma}_{HH} \sim (\cos\alpha\, A_b)^2, \quad \hat{\Sigma}_{hh} \sim (\sin\alpha\, A_b)^2$$

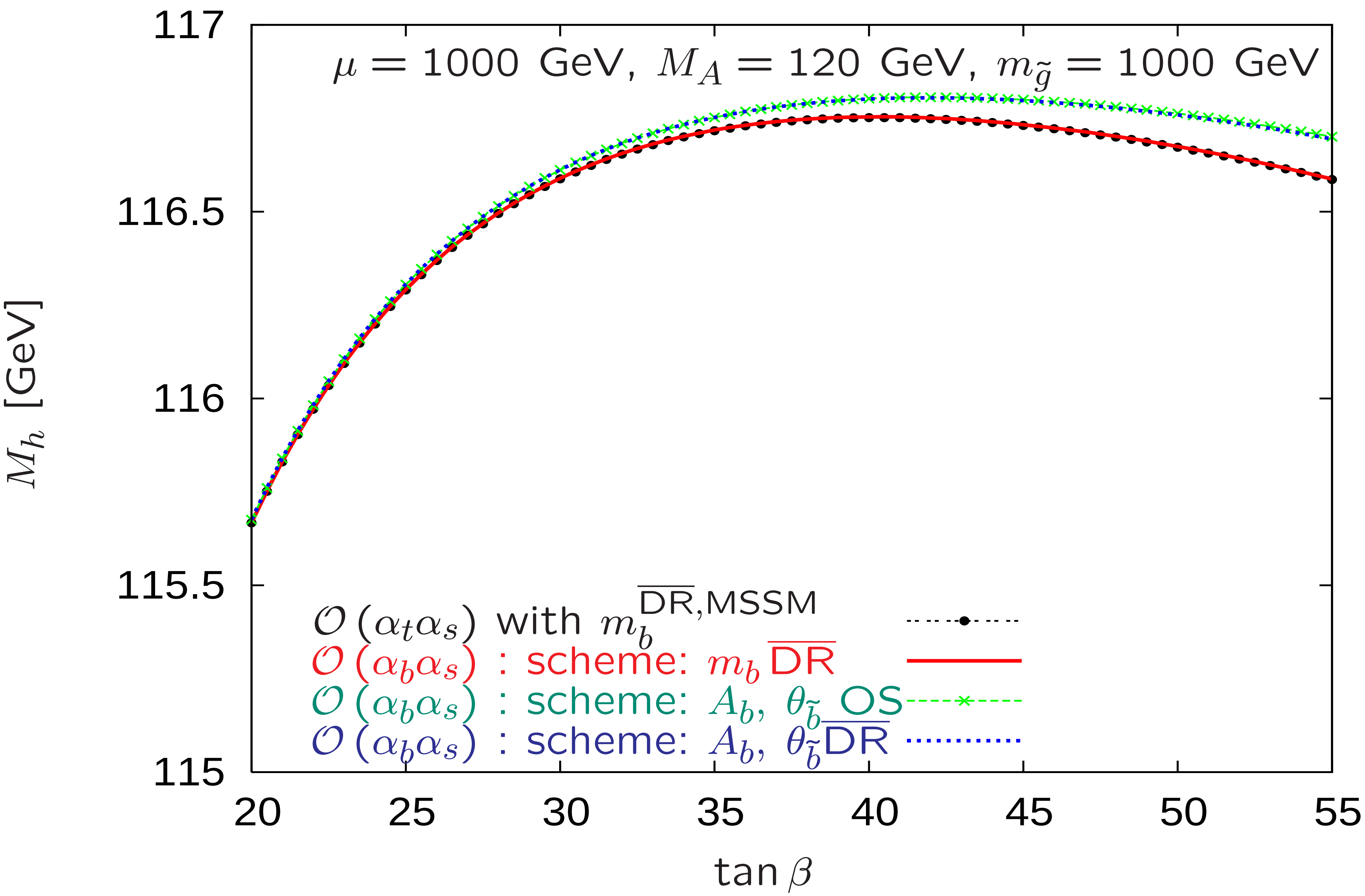
  $\Rightarrow$ effect more pronounced for M_H

  $\Rightarrow$ Scheme m_b OS is discarded as a useful renormalization scheme

- Other schemes: differences of $\mathcal{O}(1\ \text{GeV})$ for large $\tan\beta$

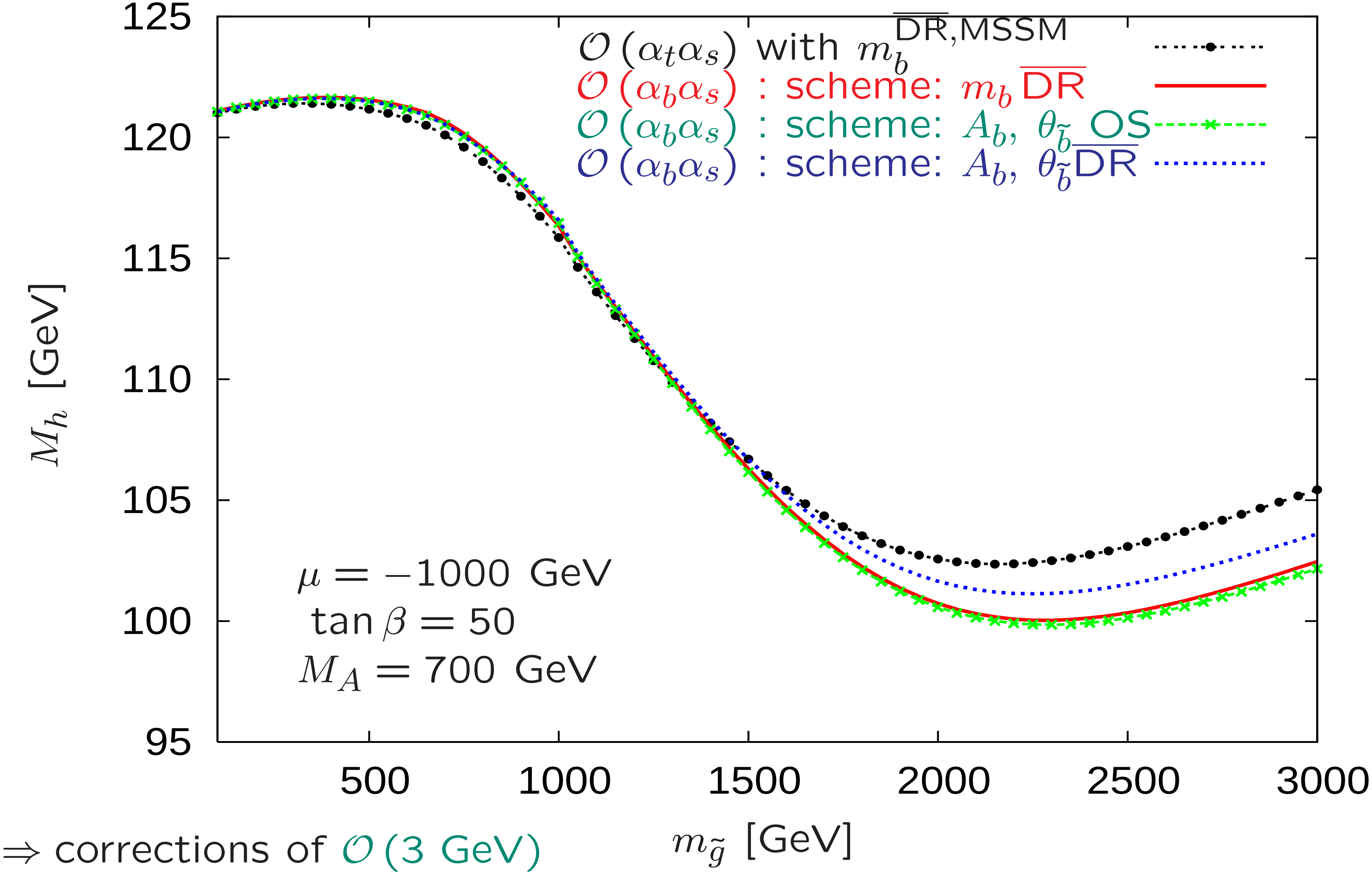
  $\Rightarrow$ non-negligible

M_h as a function of $\tan\beta$, $\mu > 0$:



$\Rightarrow$ small corrections, scheme m_b $\overline{\rm DR}$: "no" correction

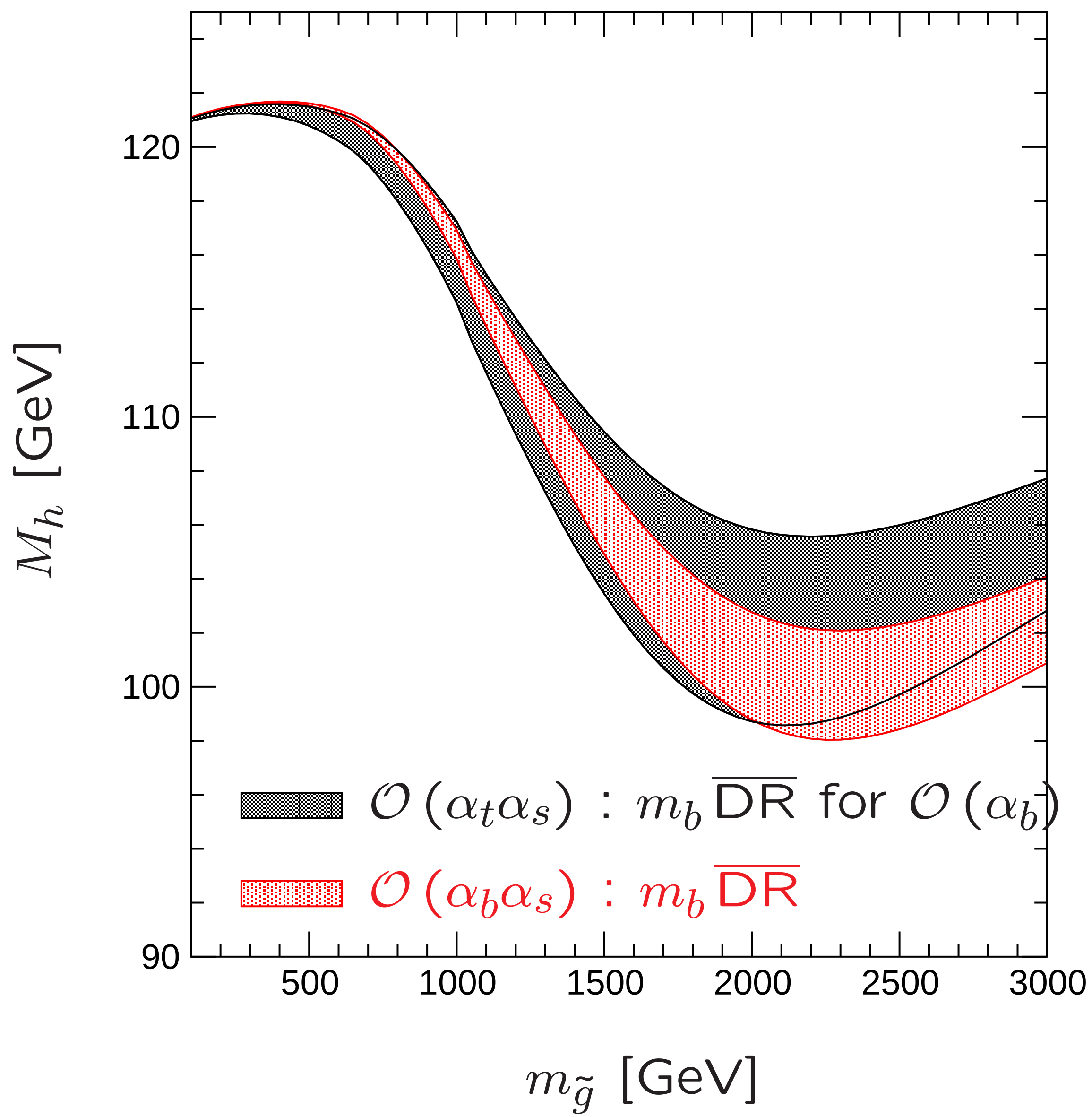
M_h as a function of $m_{\tilde{g}}$, $\mu < 0$:



$\Rightarrow$ corrections of $\mathcal{O}(3$ GeV$)$

scheme difference $\mathcal{O}(2$ GeV$)$

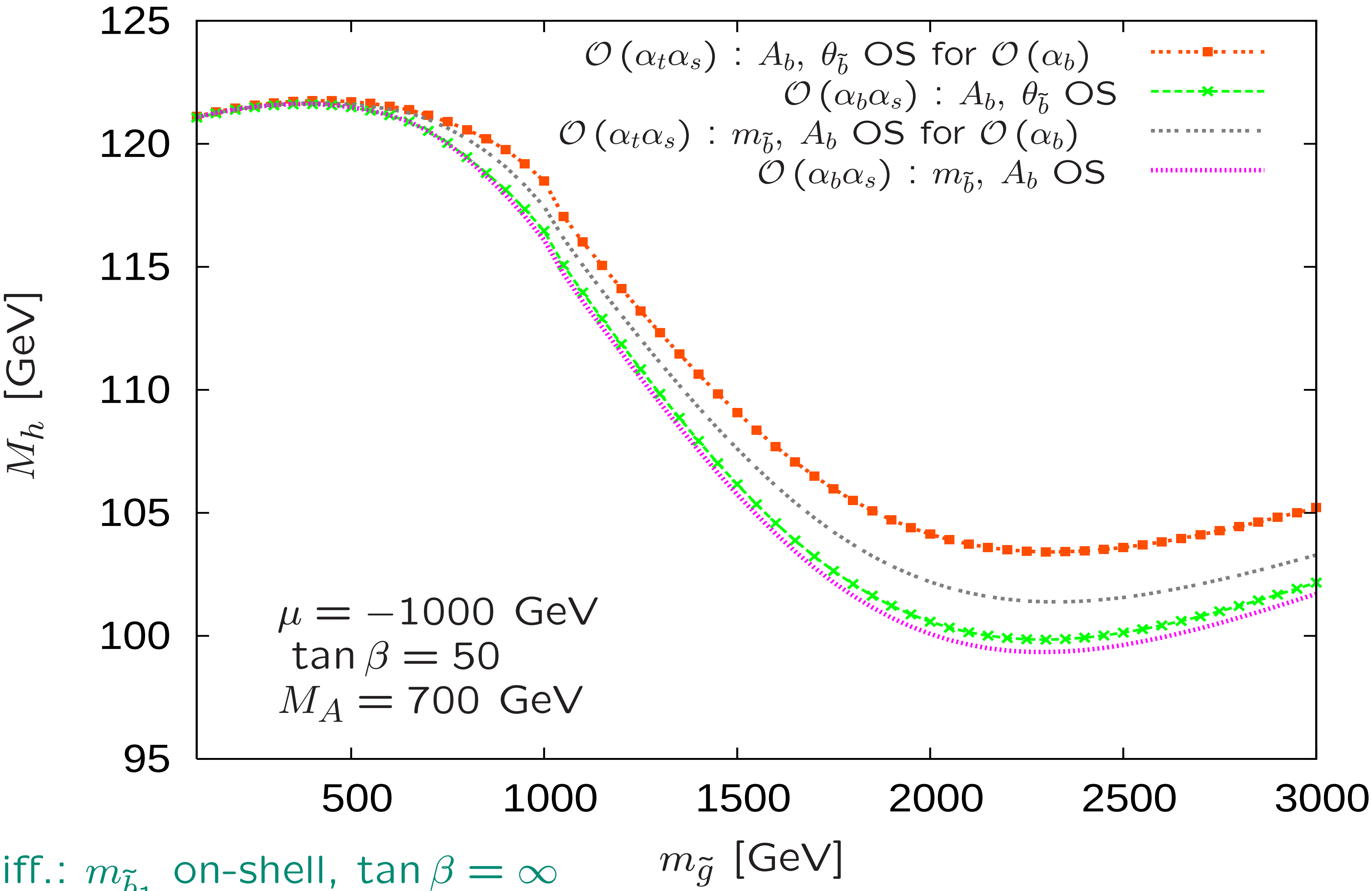
Dependence on renormalization scale $\mu^{\overline{\text{DR}}}$:



$M_A = 700$ GeV
$\mu = -1000$ GeV
$\tan\beta = 50$
$m_t/2 \leq \mu^{\overline{\text{DR}}} \leq 2\,m_t$

$\Rightarrow$ scale dependence
$\mathcal{O}(\pm 2$ GeV$)$ for large $m_{\tilde{g}}$

Comparison with existing calculation: [A. Brignole et al. '02]



diff.: $m_{\tilde{b}_1}$ on-shell, $\tan\beta = \infty$

$\Rightarrow$ 2-loop: differences $\mathcal{O}(0.5$ GeV$)$

3. Corrections of $\mathcal{O}(\alpha_t \alpha_s)$ in the cMSSM

Higgs potential of the cMSSM contains two Higgs doublets:

$$
\begin{aligned}
H_1 &= \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix} \\
H_2 &= \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = e^{i\xi} \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
V &= m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - (m_{12}^2 \epsilon_{ab} H_1^a H_2^b + \text{h.c.}) \\
&\quad + \underbrace{\frac{g'^2 + g^2}{8}}(H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}} |H_1 \bar{H}_2|^2
\end{aligned}
$$

gauge couplings, in contrast to SM

Five physical states: $h^0, H^0, A^0, H^\pm$ (no $\mathcal{CP}$V at tree-level)

2 $\mathcal{CP}$-violating phases: ξ, $\arg(m_{12})$ $\Rightarrow$ can compensate each other

Input parameters: $\tan\beta = \frac{v_2}{v_1}$, M_A or $M_{H^\pm}$

Effects of complex parameters in the Higgs sector:

Complex parameters enter via loop corrections:

– μ : Higgsino mass parameter

– $A_{t,b,\tau}$: trilinear couplings $\Rightarrow$ $X_{t,b,\tau} = A_{t,b,\tau} - \mu^* \{\cot\beta, \tan\beta\}$ complex

– $M_{1,2}$: gaugino mass parameter (one phase can be eliminated)

– M_3 : gluino mass parameter

$\Rightarrow$ can induce $\mathcal{CP}$-violating effects

Result:

$$(A, H, h) \to (h_3, h_2, h_1)$$

with

$$m_{h_3} > m_{h_2} > m_{h_1}$$

Inclusion of higher-order corrections:

($\rightarrow$ Feynman-diagrammatic approach)

Propagator / mass matrix with higher-order corrections:

$$\begin{pmatrix} q^2 - M_A^2 + \hat{\Sigma}_{AA}(q^2) & \hat{\Sigma}_{AH}(q^2) & \hat{\Sigma}_{Ah}(q^2) \\ \hat{\Sigma}_{HA}(q^2) & q^2 - m_H^2 + \hat{\Sigma}_{HH}(q^2) & \hat{\Sigma}_{Hh}(q^2) \\ \hat{\Sigma}_{hA}(q^2) & \hat{\Sigma}_{hH}(q^2) & q^2 - m_h^2 + \hat{\Sigma}_{hh}(q^2) \end{pmatrix}$$

$\hat{\Sigma}_{ij}(q^2)$ $(i, j = h, H, A)$: renormalized Higgs self-energies

$\hat{\Sigma}_{Ah}, \hat{\Sigma}_{AH} \neq 0 \;\Rightarrow\; \mathcal{CP}$V, $\mathcal{CP}$-even and $\mathcal{CP}$-odd fields can mix

Our result for $\hat{\Sigma}_{ij}$:

– full 1-loop evaluation: dependence on all possible phases included

– New: $\mathcal{O}(\alpha_t\alpha_s)$ corrections in the FD approach

rMSSM: difference between FD and RGiEP approach $\mathcal{O}$(few GeV)

Differences to the real case:

- use $M_{H^\pm}$ as on-shell mass, since A mixes with h, H in higher orders
  $\Rightarrow \tilde{b}$ sector enters via $\Sigma_{H^\pm}$
  $\Rightarrow$ renormalization of the $\tilde{b}$ sector

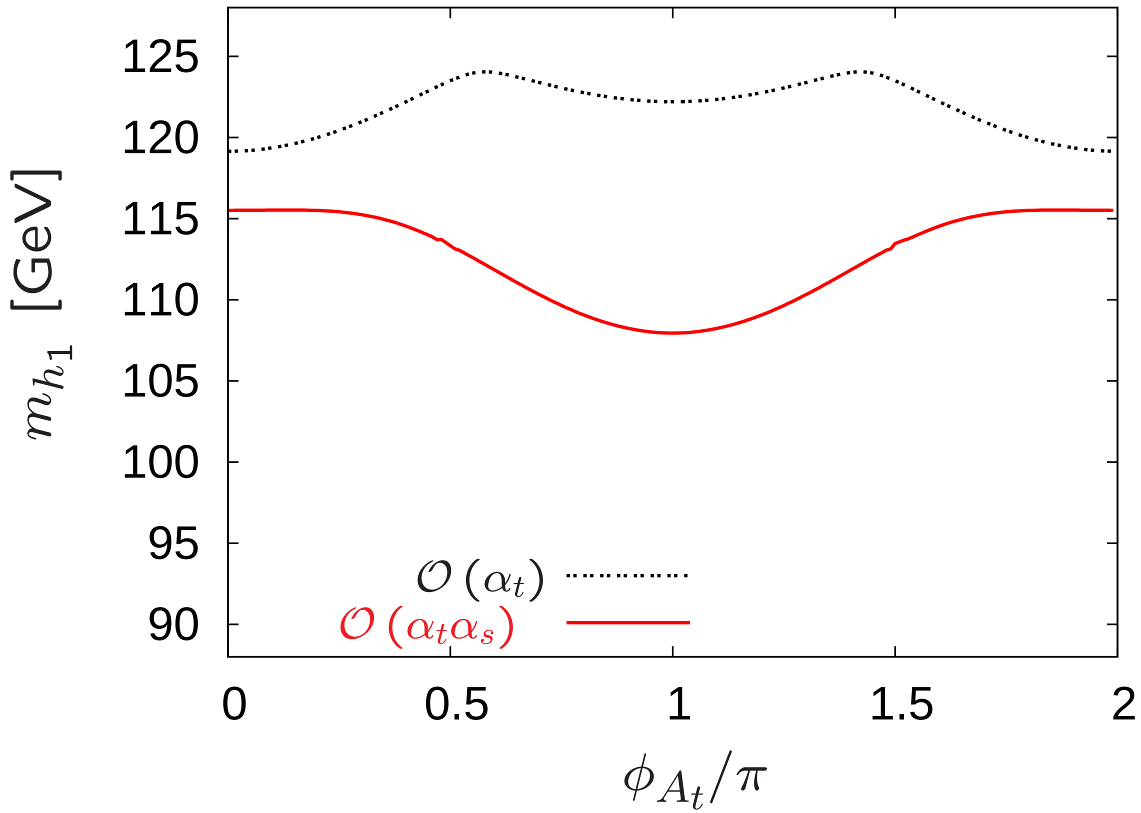
- A_t complex $\Rightarrow$ renormalization of $|A_t|$ and ϕ_{A_t}
  (no renormalization of μ, no $\mathcal{O}(\alpha_s)$ corrections)

- M_3 complex, but $m_{\tilde{g}}$ is real (and positive)
  $\Rightarrow$ phase of M_3 enters gluino vertices

- $T_A \neq 0 \Rightarrow$ renormalized to zero
  $\Rightarrow \delta t_A$ enters renormalized self-energies $\hat{\Sigma}_{hA}$, $\hat{\Sigma}_{HA}$

$\rightarrow$ so far all results preliminary!

m_{h_1} as a function of ϕ_{A_t}:

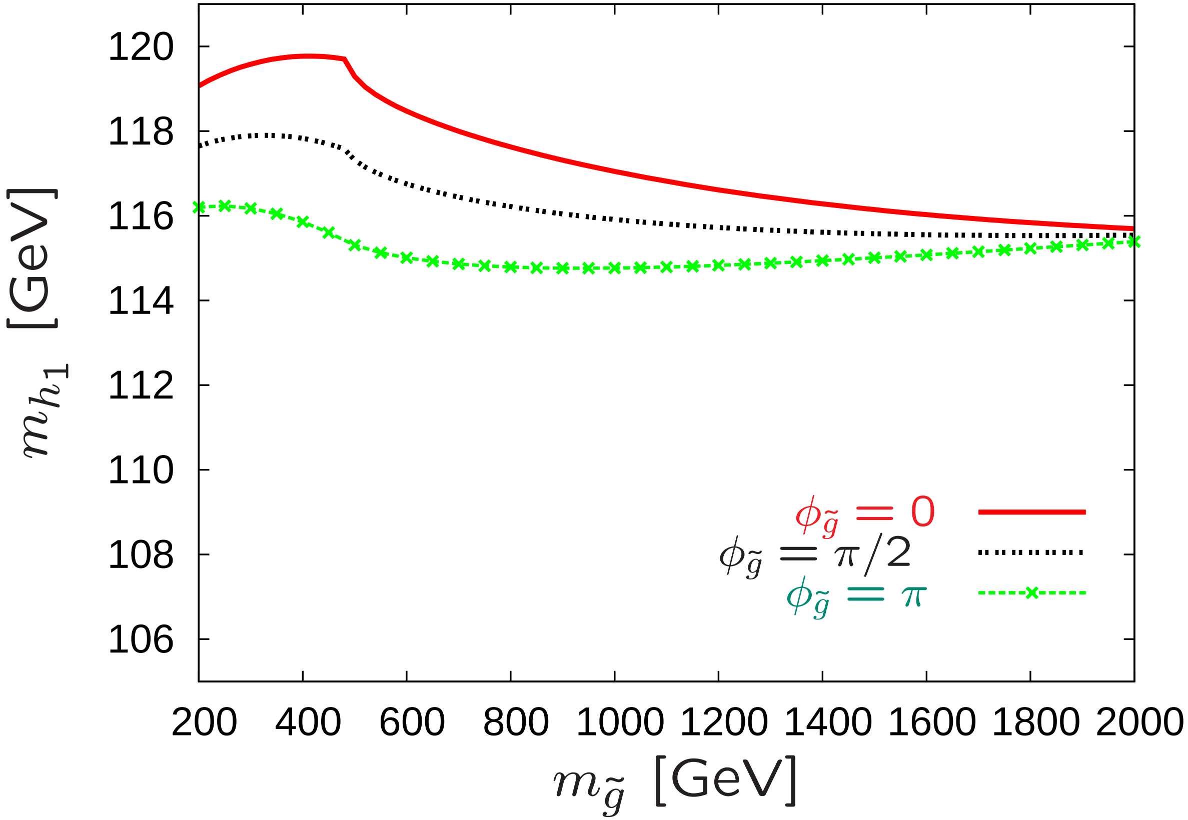


$M_{\text{SUSY}} = 1000$ GeV
$|A_t| = 2000$ GeV
$\tan\beta = 10$
$M_{H^\pm} = 150$ GeV

OS renormalization

$\Rightarrow$ modified dependence on ϕ_{A_t} at the 2-loop level

m_{h_1} as a function of $\phi_{\tilde{g}}$:



$M_{\text{SUSY}} = 500$ GeV
$A_t = 1000$ GeV
$\tan\beta = 10$
$M_{H^\pm} = 500$ GeV

OS renormalization

$\Rightarrow$ threshold at $m_{\tilde{g}} = m_{\tilde{t}} + m_t$

$\Rightarrow$ large effects around threshold

$\Rightarrow$ phase dependence has to be taken into account

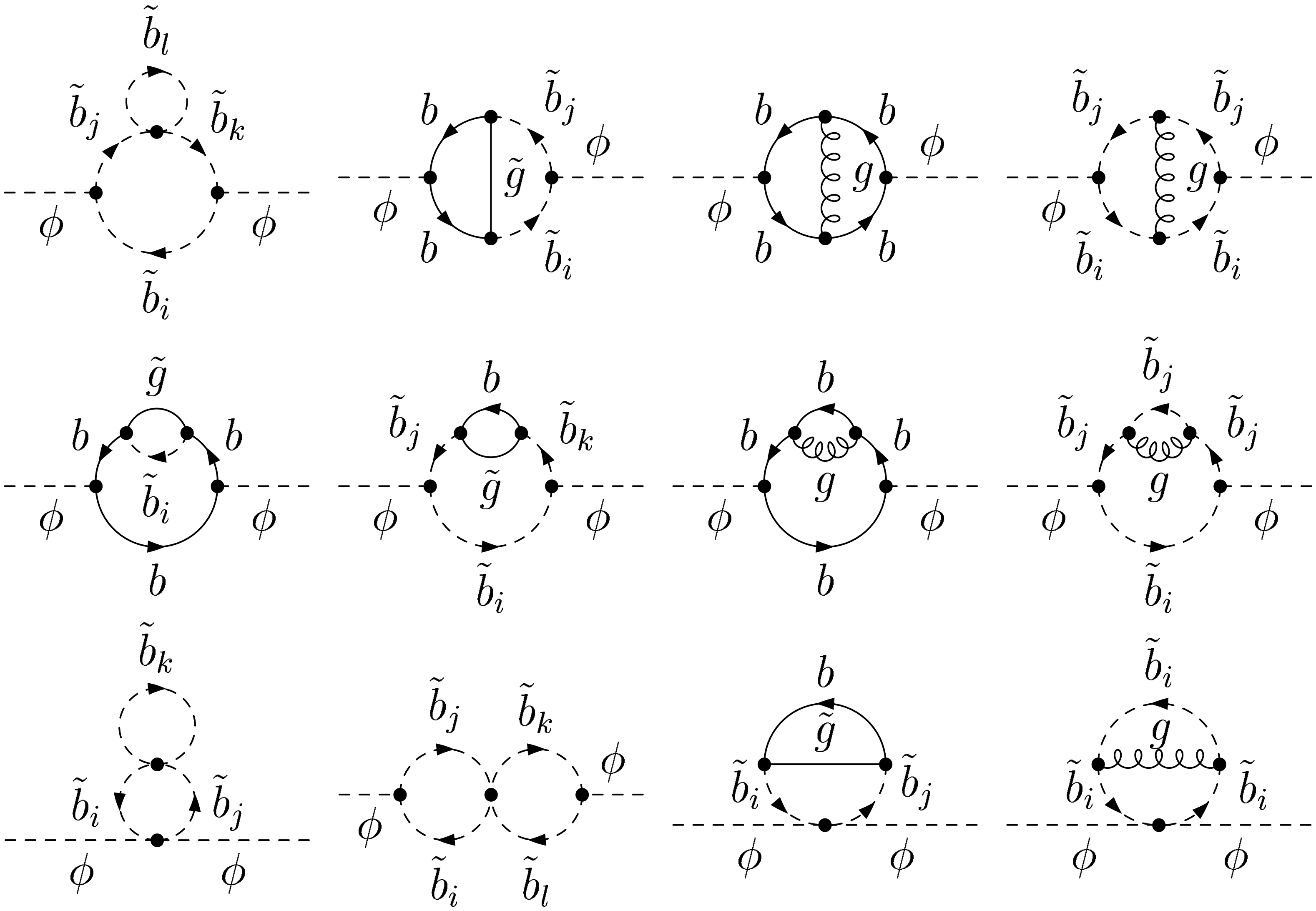
4. Conclusinos

- The LC will provide high precision results for a light r/cMSSM Higgs
- MSSM Higgs masses and couplings is connected via radiative corrections to all other sectors
- Evaluation of $\mathcal{O}(\alpha_b \alpha_s)$ corrections in the rMSSM:
  - new result for $\tan\beta \neq \infty$
  - investigation of different renormalization schemes
    $\Rightarrow$ error estimate from scheme and scale dependence
  - $\mu > 0$: corrections $\mathcal{O}(100$ MeV$)$ $\Rightarrow$ under control
  - $\mu < 0$: corrections $\mathcal{O}(2 - 3$ GeV$)$
    error estimate $\mathcal{O}(2$ GeV$)$ $\Rightarrow$ not under control
- Evaluation of $\mathcal{O}(\alpha_t \alpha_s)$ corrections in the cMSSM:
  - new renormalization for complex parameters
  - $\tilde{b}$ sector enters
  - ϕ_{A_t} dependence modified
  - $\phi_{\tilde{g}}$ dependence $\mathcal{O}(2$ GeV$)$ possible
- Results will be implemented into *FeynHiggs* (www.feynhiggs.de)

Backup slides

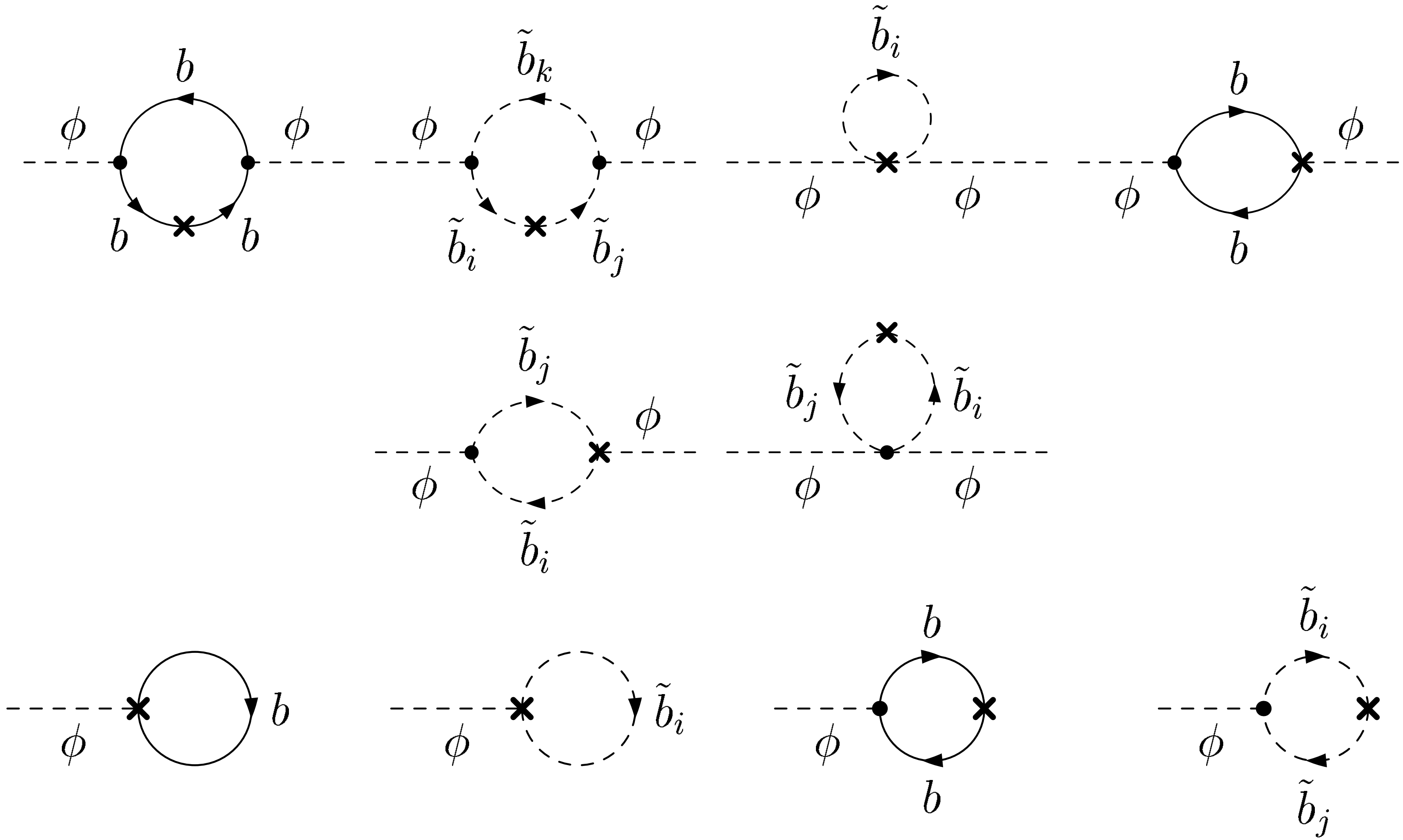
Contributions to the 2-loop self-energy:

2-loop self-energy diagrams:



Contributions to the 2-loop self-energy:

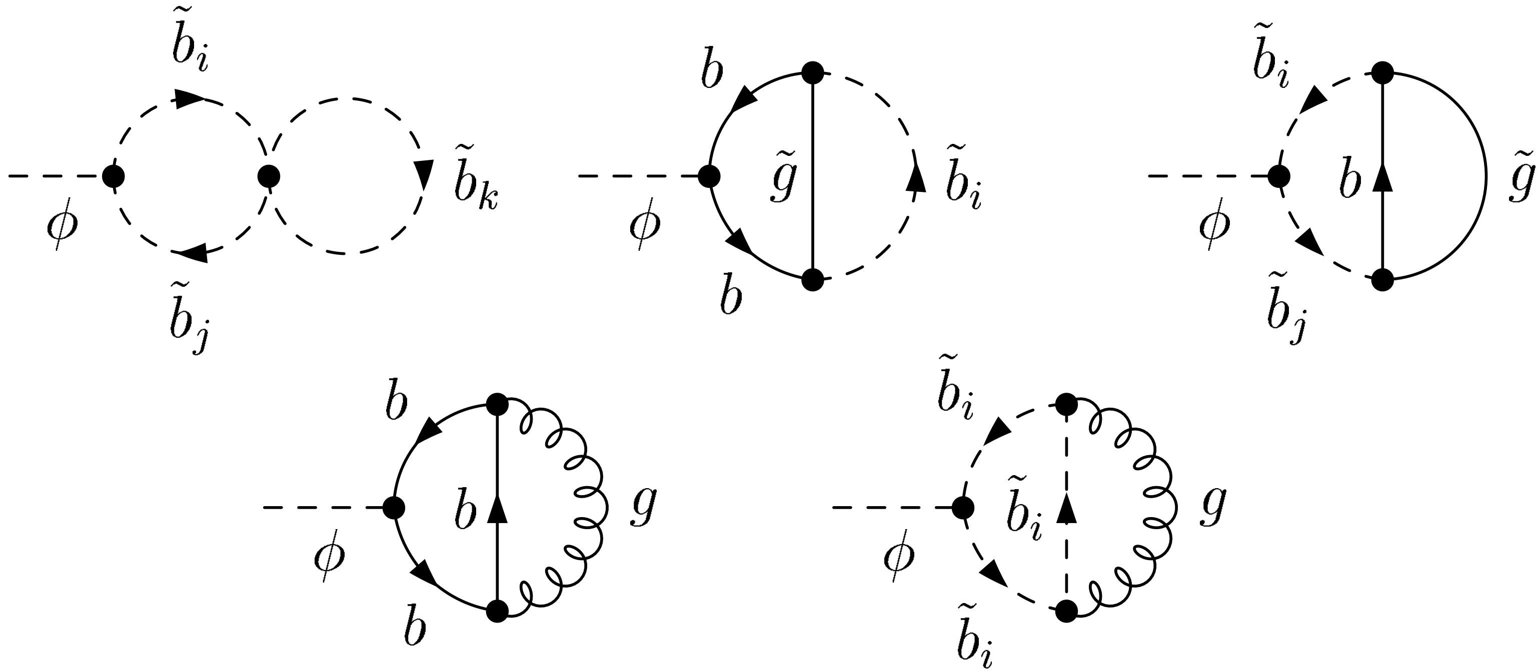
diagrams with counter term insertion:



$\rightarrow$ different renormalization schemes enter

Contributions to the 2-loop self-energy:

2-loop tad-pole diagrams:



Evaluation of 2-loop diagrams:

1. Generation of diagrams and amplitudes with FeynArts
   [*Küblbeck, Böhm, Denner '90*] [*Hahn '00 - '05*]

2. Algebraic evaluation and tensor integral reduction to scalar integrals: TwoCalc

   (works for two-loop self-energies)
   [*G. Weiglein '92*] [*G. Weiglein, R. Scharf, M. Böhm '94*]

3. Further evaluation: insertion of integrals, expansion in $\delta = \frac{1}{2}(4 - D)$
   $\rightarrow$ algebraical check: cancellation of divergencies

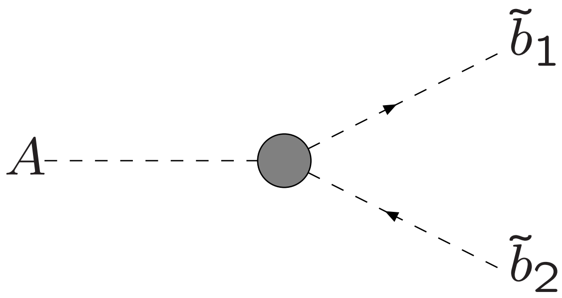
4. Result:
   - algebraic Mathematica code
   - Fortran code (planned: implementation into *FeynHiggs*)

Some more details:

– scheme m_b OS: analogous to the $t/\tilde{t}$ sector

$\rightarrow$ obvious choice ?

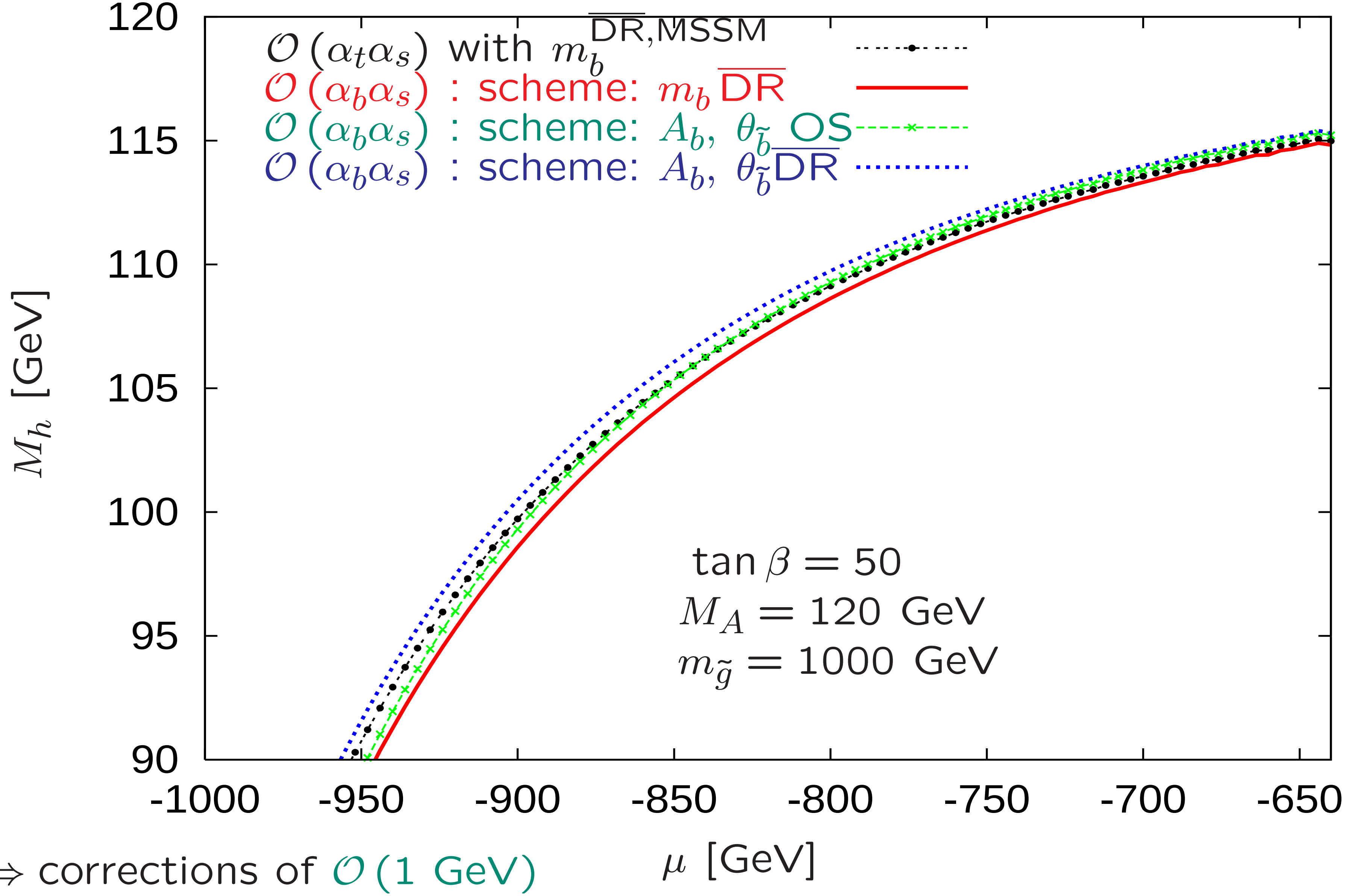
– A_b OS: determined via



analogous to [A. Brignole, G. Degrassi, P. Slavich and F. Zwirner '02]

– $\theta_{\tilde{t}}$ OS: $\delta\theta_{\tilde{b}} = \dfrac{\text{Re}\,\Sigma_{\tilde{b}_{12}}(m^2_{\tilde{b}_1}) + \text{Re}\,\Sigma_{\tilde{b}_{12}}(m^2_{\tilde{b}_2})}{m^2_{\tilde{b}_1} + m^2_{\tilde{b}_2}}$

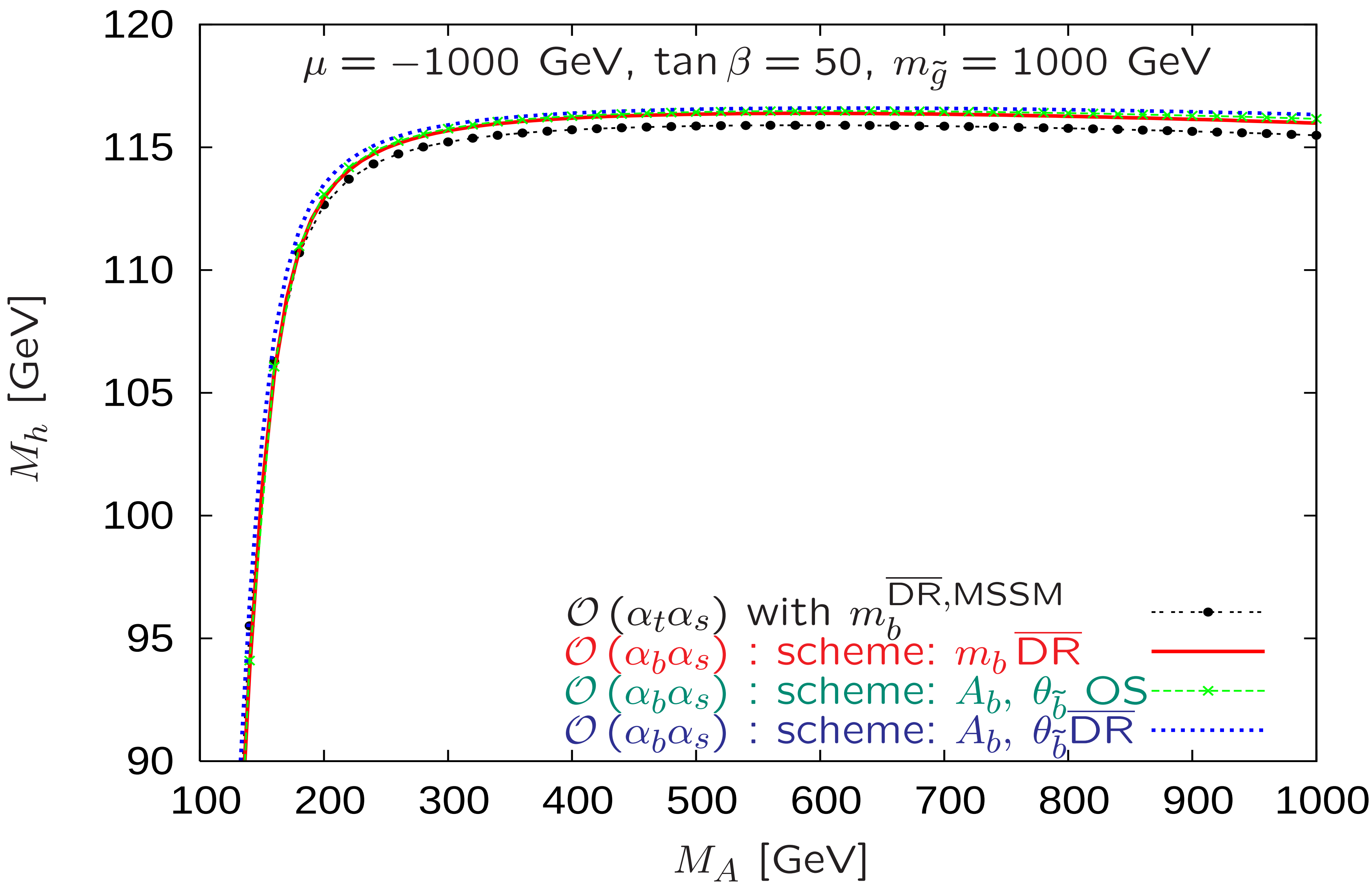
M_h as a function of μ, $\mu < 0$:



$\Rightarrow$ corrections of $\mathcal{O}(1$ GeV$)$

scheme difference similar

M_h as a function of M_A, $\mu < 0$:



$\Rightarrow$ subleading corrections of $\mathcal{O}$(1 GeV)