

# Two-loop SUSY QCD correction to the gluino pole mass

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- Gluino mass:  
precision measurement vs. radiative correction
- $O(\alpha_s^2)$  correction to the gluino pole mass
- Numerical results

Supersymmetry in the unified theory must be broken in our world.

## Unified Theory

spontaneous SUSY breaking



## Low-energy effective theory

(here assume MSSM)

soft SUSY breaking parameters

scalar masses,  $\phi^3$  couplings, gaugino masses  $M_{3,2,1}$

Important clue to the SUSY breaking mechanism

# Gaugino masses in the MSSM

## Gauginos in MSSM

SU(3) gluino  $\tilde{g}$ , SU(2) wino  $\tilde{W}$ , U(1) bino  $\tilde{B}$   
masses:  $(M_3, M_2, M_1)$

In many candidates of the unified theories, these gaugino masses unify at GUT/Planck scale

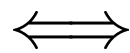
- SUSY GUT with gauge-symmetric SUSY breaking
- minimal SUGRA
- dilaton-dominated SUSY breaking                      etc.

In other theories,  $M_{3,2,1}$  do not unify but, instead, their ratios are predicted. (anomaly mediation ...)

To test these models, it is important to obtain precise values of  $(M_3, M_2, M_1)$  in future studies at LHC and ILC

**Physical observables**

masses, cross-sections. . .  
of SUSY particles



**Lagrangian parameters**

$M_{1,2,3}(Q \sim 1 \text{ TeV})$

To probe the SUSY breaking mechanism by precision measurements, we need precise formulas of the relations between physical observables (masses etc.) and SUSY-breaking parameters in the lagrangian.

Here we consider the relation between the physical mass of the gluino  $m_{\tilde{g}}$  and the tree-level mass  $M_3$  in the lagrangian, to the two-loop order.

## Two-loop correction to the gluino mass

Gluino  $\tilde{g}$  is expected to be copiously produced at the LHC, then produce decay chains such as

$$\tilde{g} \rightarrow q\tilde{q} \rightarrow qq\tilde{\chi}_2^0 \rightarrow qqll\tilde{\chi}_1^0$$

Combined analysis of various decay chains of SUSY particles may determine  $m_{\tilde{g}}$  ( $= M_3$  at tree-level) quite precisely.

A simulation: (Chiorboli et al.)

for the parameter set SPS1a with  $m_{\tilde{g}} \sim 600$  GeV

$\delta m_{\tilde{g}} = \pm 8$  GeV from the LHC ( $\mathcal{L} \sim 300$  fb $^{-1}$ )

→ 6.5 GeV by combining with the ILC data

[ $\sqrt{s} \leq 1$  TeV,  $\mathcal{L} \sim 1000$  fb $^{-1}$ ]

On the other hand,  $m_{\tilde{g}}$  receives large radiative corrections.

$O(\alpha_s)$  corr.  $\sim O(10)$  %

↓

Naive expectation:  $O(\alpha_s^2)$  corr.  $\sim O(1)$  %

Comparable to experimental uncertainty?

We need explicit calculation of the two-loop correction to  $m_{\tilde{g}}$ .

Glauino pole mass  $m_{\tilde{g}}$  at  $O(\alpha_s^2)$

Given by the complex pole of the gluino propagator  $s_p = (m_{\tilde{g}} - i\Gamma_{\tilde{g}}/2)^2$

$$m_{\tilde{g}} = M_3(Q) + \delta m_{\tilde{g}}^{(1)} + \delta m_{\tilde{g}}^{(2)}$$

$M_3(Q)$ : running mass in the lagrangian

$\delta m_{\tilde{g}}$ : Calculated from the self energy  $i(\Sigma_K(p^2)\not{p} + \Sigma_M(p^2))$

For simplicity, we assume

◇ degenerate squark mass  $m_{\tilde{q}}$

◇  $m_q \ll (m_{\tilde{g}}, m_{\tilde{q}})$

→ ignore  $m_q$  and  $\tilde{q}_L$ - $\tilde{q}_R$  mixing in the loops.



One-loop correction ( Martin, Vaughn; Pierce, Papadopoulos; ... )

$$\begin{aligned}\delta m_{\tilde{g}}^{(1)} &= -\text{Re}[M_3 \Sigma_K^{(1)}(M_3^2) + \Sigma_M^{(1)}(M_3^2)] \\ &= \frac{C_V \alpha_s(Q)}{4\pi} M_3(Q) \left( 5 - 6 \log \frac{M_3(Q)}{Q} \right) \\ &\quad + \frac{\alpha_s(Q)}{\pi} N_q T_F M_3(Q) B_1(M_3(Q)^2, 0, m_{\tilde{q}}(Q)) + O(\alpha_s m_q^2 / m_{\tilde{q}}^2)\end{aligned}$$

$$C_V = 3, \quad T_F = 1/2, \quad N_q = 6$$

typically  $\delta m_{\tilde{g}}^{(1)} / m_{\tilde{g}} = O(10) \%$

Enhanced by

- large  $\alpha_s$
- large SU(3) representation ( $C_V(\text{octet}) \gg T_F(\text{doublet})$  )

$(\alpha_s, M_3, m_{\tilde{q}})$  in  $\delta m_{\tilde{g}}^{(1)}$ : precise definition needed to give  $\delta m_{\tilde{g}}^{(2)}$

Here  $\overline{\text{DR}}'$  parameters at  $Q \sim M_3$  are used.

Two-loop  $O(\alpha_s^2)$  correction to  $m_{\tilde{g}}$

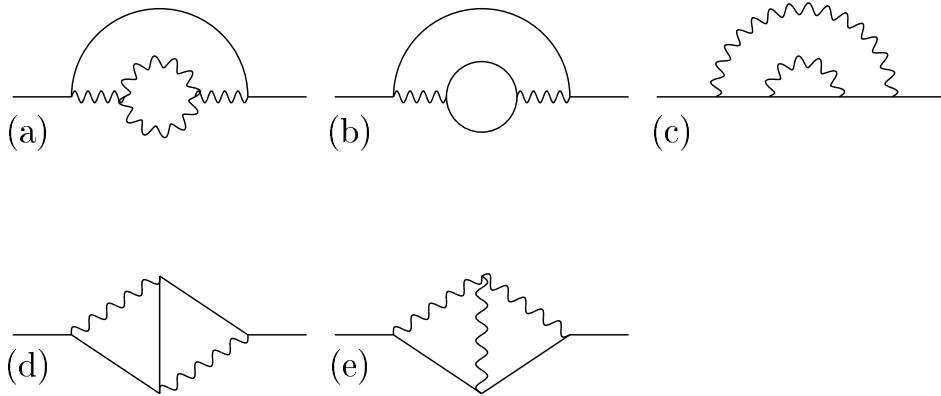
$$\begin{aligned} \delta m_{\tilde{g}}^{(2)} = & -\text{Re}[M_3 \Sigma_K^{(2)}(M_3^2) + \Sigma_M^{(2)}(M_3^2)] \\ & + \text{Re} \left[ \{M_3 \Sigma_K^{(1)}(M_3^2) + \Sigma_M^{(1)}(M_3^2)\} \{ \Sigma_K^{(1)}(M_3^2) + 2M_3^2 \dot{\Sigma}_K^{(1)}(M_3^2) + 2M_3 \dot{\Sigma}_M^{(1)}(M_3^2) \} \right] \end{aligned}$$

$$\delta m_{\tilde{g}}^{(2)} = \delta m_{\tilde{g}}^{(2,1)} + \delta m_{\tilde{g}}^{(2,2)} : \text{function of } (M_3, \alpha_s, m_{\tilde{q}})$$

$\delta m_{\tilde{g}}^{(2,1)}$ : loops with only gluons and gluinos ( $m_{\tilde{q}}$  indep.)

$\delta m_{\tilde{g}}^{(2,2)}$ : loops including quarks and squarks

## Correction with only gluinos and gluons

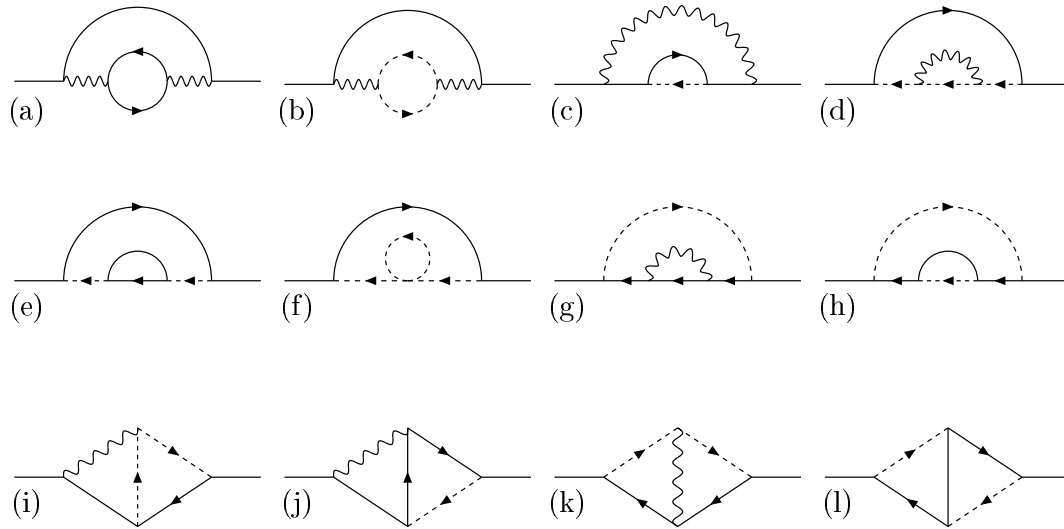


$$\delta m_{\tilde{g}}^{(2,1)} = \left( \frac{C_V \alpha_s}{4\pi} \right)^2 M_3 \left( -48 \log \frac{M_3}{Q} + 36 \log^2 \frac{M_3}{Q} + 26 + 5\pi^2 - 4\pi^2 \log 2 + 6\zeta_3 \right)$$

At  $Q = M_3$ ,  $\delta m_{\tilde{g}}^{(2,1)}/M_3 \sim 31(\alpha_s/\pi)^2 \sim 0.03$ .

cf.  $\delta m_{\tilde{g}}(\text{exp.})/m_{\tilde{g}} \sim 1.3\%$  for  $m_{\tilde{g}} \sim 600$  GeV (SPS1a)

# Correction including quarks/squarks



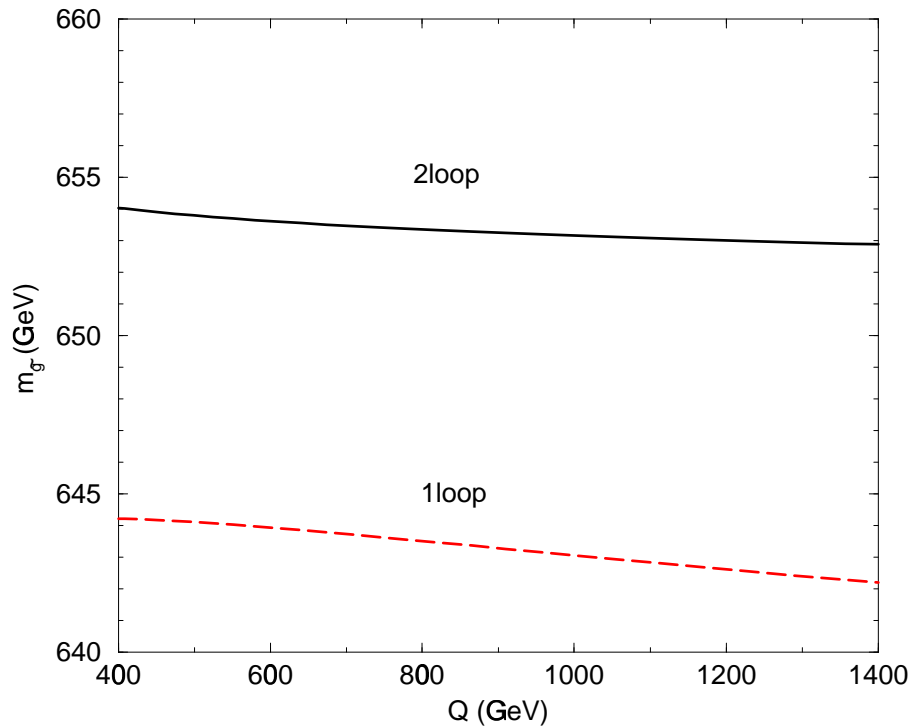
solid line with an arrow: quark, dashed line with an arrow: squark

$\delta m_{\tilde{g}}^{(2,2)}$ : complicated function of  $(M_3, \alpha_s, m_{\tilde{q}})$   
 $< 0$  for  $m_{\tilde{q}} \sim m_{\tilde{g}}$

# Residual dependence of $m_{\tilde{g}}$ on the renormalization scale

$M_3(580\text{GeV}) = 580 \text{ GeV}$ ,  $m_{\tilde{q}}(580\text{GeV}) = 800 \text{ GeV}$ ,

cf. tree-level mass:  $M_3(400) = 589 \text{ GeV} \rightarrow M_3(1400) = 559 \text{ GeV}$

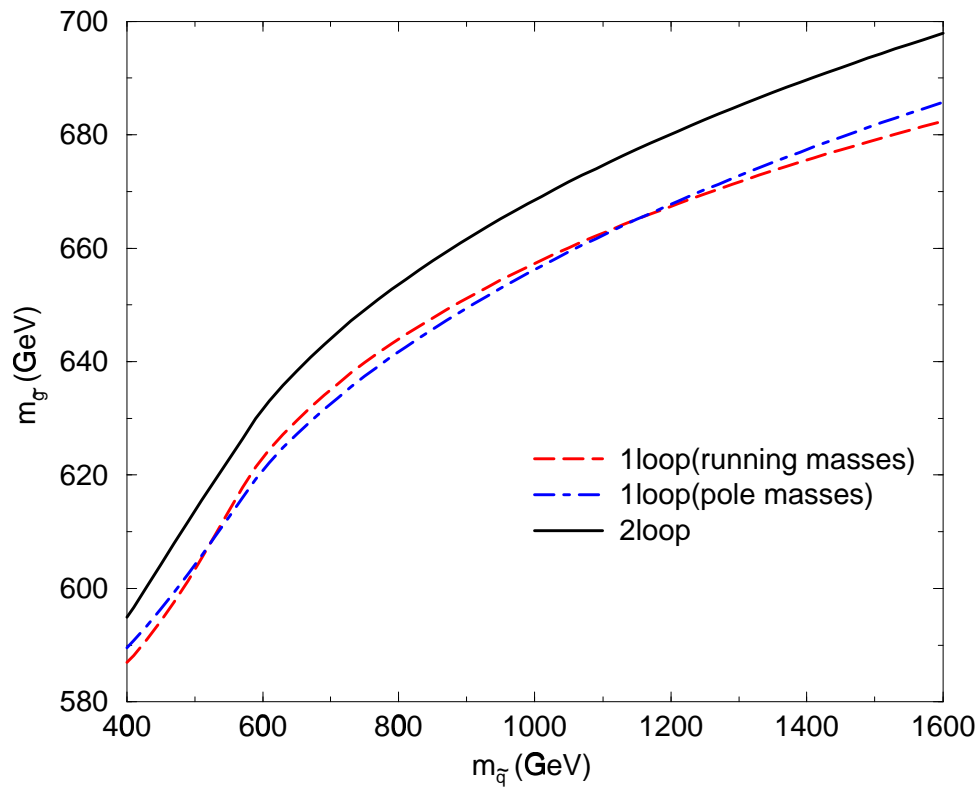


slight improvement by  $\delta m_{\tilde{g}}^{(2)}$

$|\delta m_{\tilde{g}}^{(2)}| \gg (Q \text{ dependence of } m_{\tilde{g}}^{(1)})$

# Glino pole mass at one- and two-loops

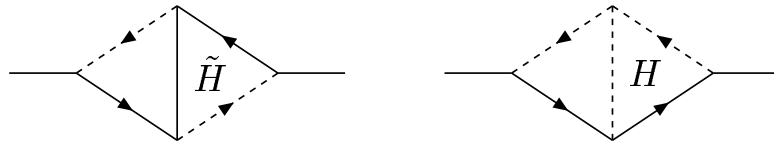
( tree:  $M_3(M_3) = 580$  GeV)



1 – 2 % increase of  $m_{\tilde{g}}$  by  $O(\alpha_s^2)$  corr.  $\geq \delta m_{\tilde{g}} \sim 1$  % at LHC/ILC(expected)

## Things to do

- \* realistic cases with nondegenerate squark masses
- \* contribution of  $m_q$  and  $\tilde{q}_L - \tilde{q}_R$  mixing to  $\delta m_{\tilde{g}}^{(2)}$   
suppressed by  $m_q^2 / (m_{\tilde{g}}^2, m_{\tilde{q}}^2)$   
may be important for light gluino/squarks
- \*  $O(\alpha_s h_q^2)$  corrections involving Higgs bosons/higgsinos



(related to renorm. of  $m_q$  and  $\tilde{q}_L - \tilde{q}_R$  mixing in  $\delta m_{\tilde{g}}^{(1)}$ )

Work in progress

## Conclusion

- \* The pole mass of the gluino  $m_{\tilde{g}}$  has been calculated as a function of the lagrangian parameters ( $M_3(Q) \dots$ ) to  $O(\alpha_s^2)$ .
- \* The two-loop correction to  $m_{\tilde{g}}$  for a given  $M_3(Q)$  is typically 1–2 %, which may be larger than the expected uncertainty in precision mass determination at future colliders.