Dimensional Reduction: some new results

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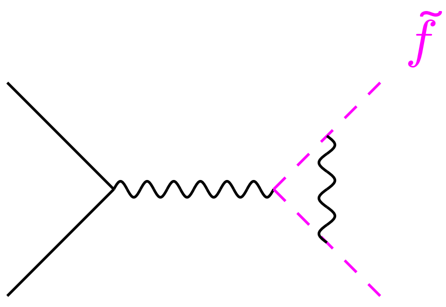
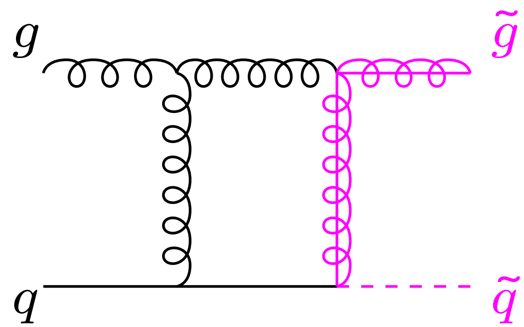
1. Motivation: Why consider regularization? Problems of DRED?
2. Factorization-Problem: Understand and eliminate “non-factorizing terms”
3. SUSY of DRED: New method, apply at 1-, 2-loop level

Motivation

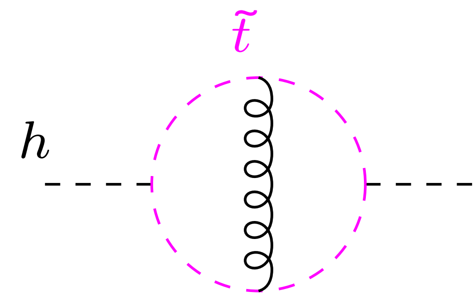
Precise measurement of SUSY observables



σ, $m_{\tilde{f}}$



$gq \to \tilde{g}\tilde{q}$



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justifies/necessitates SUSY loop calculations. ($\rightarrow$ SPA project)

These require

- Regularization
- Renormalization

Motivation

Regularization: intermediate steps in calculation

Dim. Regularization (DREG) : D dimensions, D Gluon-components, 4 Gluino-components

$\Rightarrow$ breaks SUSY $\Rightarrow$ complicated in practice

Dim. Reduction (DRED) : D dimensions, 4 Gluon-components, 4 Gluino-components

$\Rightarrow$ doesn't (?) break SUSY $\Rightarrow$ usually applied

$\overline{DR}$ renormalization scheme: common, directly related to DRED

$\overline{DR}$ scheme and DRED are central in SPA project and in many SUSY calculations

Problems of DRED

SUSY, Consistency

- Does DRED preserve SUSY?
- Mathematical inconsistency

  [Siegel'80]
- Symmetry-restoring countert-erms necessary in calculations?

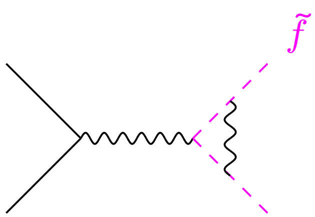
QCD-Factorization

- Hadron processes in DRED:

$$\sigma_{\text{had}} = f_{\text{parton}} \otimes \sigma_{\text{parton}} + \text{non-factorizing terms?}$$

[Beenakker,Kuijf,Neerven,Smith'88]

[Beenakker,Höpker,Spira,Zerwas'96]



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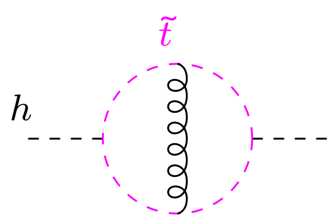

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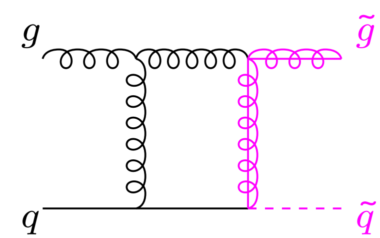


$gq \to \tilde{g}\tilde{q}$

Aims:

SUSY, Consistency

[DS '05]

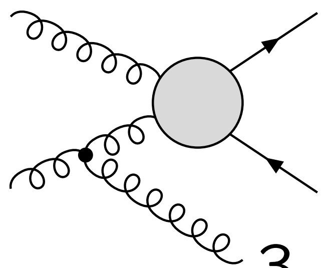
- Method to verify SUSY
- Perform verification in important cases

QCD-Factorization

[Signer, DS, in preparation]

- "non-factorizing terms": Origin?
- Can DRED be used for hadron processes?
- Reconcile DRED with factorization!

Consider $gg \to t\bar{t}$ (massive quark) at NLO: $gg \to t\bar{t}g$

Collinear limit $2\|3$:  $\Rightarrow$ divergence $\sim \frac{1}{k_2 k_3}$ should factorize

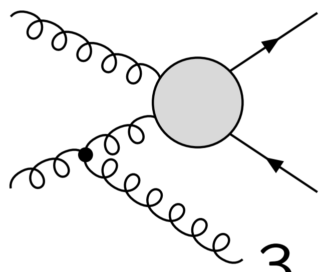
Result DREG, $m = 0$:

$$\sigma^{\mathsf{DREG}}(gg \to t\bar{t}g) \xrightarrow{2\|3} \sim \frac{1}{k_2 k_3} P_{g\to gg}\ \sigma^{\mathsf{DREG}}(gg \to t\bar{t})$$

$$\sigma_{\mathsf{had}} \sim f^{\mathsf{DREG}} \otimes \sigma^{\mathsf{DREG}}_{\mathsf{parton}}$$

Consider $gg \to t\bar{t}$ (massive quark) at NLO: $gg \to t\bar{t}g$

Collinear limit 2‖3:



$\Rightarrow$ divergence $\sim \frac{1}{k_2 k_3}$ should factorize

Result DRED , $m = 0$:

$$\sigma^{\text{DRED}}(GG \to t\bar{t}G) \xrightarrow{2\|3} \sim \frac{1}{k_2 k_3} P_{g\to gg}\ \sigma^{\text{DRED}}(GG \to t\bar{t})$$

$$\sigma_{\text{had}} \sim f^{\text{DRED}} \otimes \sigma^{\text{DRED}}_{\text{parton}}$$

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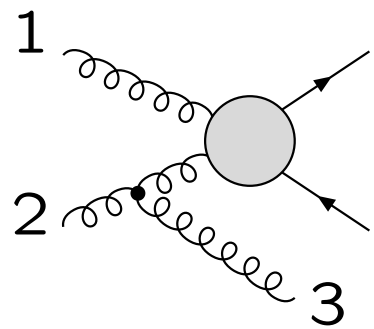

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Result DREG, $m \neq 0$:

$$\sigma^{\text{DREG}}(gg \to t\bar{t}g) \xrightarrow{2\|3} \sim \frac{1}{k_2 k_3} P_{g\to gg}\ \sigma^{\text{DREG}}(gg \to t\bar{t})$$

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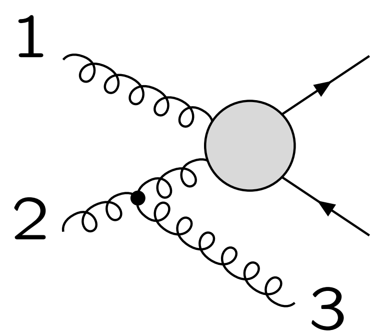


$\Rightarrow$ divergence $\sim \frac{1}{k_2 k_3}$ should factorize

Result DRED, $m \neq 0$:

$$\sigma^{\text{DRED}}(GG \to t\bar{t}G) \xrightarrow{2\|3} \sim \frac{1}{k_2 k_3} P_{g\to gg}\ \sigma^{\text{DRED}}(GG \to t\bar{t})$$
$$+ \frac{1}{k_2 k_3} K_g\ \sigma^{\text{DRED}}(GG \to t\bar{t}, m\text{-terms only})$$

$$\sigma_{\text{had}} \sim f^{\text{DREG}} \otimes \sigma^{\text{DRED}}_{\text{parton}}$$
$$+ \text{non-factorizing terms?}$$

Factorization-Problem of DRED

Problem discovered in *[Beenakker, Kuijf, van Neerven, Smith '88]*
[van Neerven, Smith '04]

Does this mean that DRED cannot be used for hadron processes?

Resort to DREG $\Rightarrow$ SUSY-restoring cts necessary $\Rightarrow$ complication
[Beenakker, Höpker, Zerwas '96]
[Beenakker, Höpker, Spira, Zerwas '96]

Aims:

[Signer, DS, in preparation]

- Identify origin of "non-factorizing terms"
- Rewrite them in factorized form
- Reconcile DRED with factorization

What have we done?

Consider 4-dim QCD reduced to D space-time dimensions, e.g. $D=3$

$$A^\mu = \begin{pmatrix} \left.\begin{matrix} A^0 \\ \vdots \\ A^2 \end{matrix}\right\} & D=3 \\ A^3 \} & 4-D=1 \end{pmatrix} \qquad \delta_{\text{gauge}} A^\mu = \begin{pmatrix} \begin{pmatrix} \partial^0 \\ \vdots \\ \partial^2 \end{pmatrix} \Lambda \\ 0 \end{pmatrix}$$

Resulting theory:

$$\left.\begin{array}{l} \text{3 space-time dimensions} \\ \text{3-component gluon } g \equiv (A^0, A^1, A^2) \\ \text{scalar } \phi \equiv A^3 \end{array}\right\} \text{3-dim QCD with extra scalar } \phi$$

Factorization certainly holds in such a theory
$\Rightarrow$ Previous result *must* be in agreement with factorization

Factorization-Problem of DRED

There are two independent partons g and ϕ

different partonic processes $\sigma(gg \to t\bar{t}),\ \sigma(g\phi \to t\bar{t}), \ldots$

What did we calculate?

$$
\begin{array}{ccccc}
\sigma(G,\ \ldots) & = & \sigma(g,\ \ldots) & + & \sigma(\phi,\ \ldots) \\
\downarrow & & \downarrow & & \downarrow \\
\text{4-dim gluon} & & D\text{-dim gluon} & & 4-D \text{ scalars} \\
\downarrow & & \downarrow & & \downarrow \\
\sum_{\text{pols}} \epsilon_\mu \epsilon^*_\nu = -g^{(4)}_{\mu\nu} + \ldots & = & -g^{(D)}_{\mu\nu} + \ldots & & -g^{(4-D)}_{\mu\nu}
\end{array}
$$

So far, we always took the sums $\sigma(G \ldots) = \sigma(g \ldots) + \sigma(\phi \ldots)$

Factorization-Problem of DRED

Earlier result:

$$\sigma(GG \to t\bar{t}G) \xrightarrow{2\|3} \sim \sigma(GG \to t\bar{t}) + \text{extra terms}$$

$$\overbrace{\Big(\ \sigma(Gg \to t\bar{t}) \ \ + \sigma(G\phi \to t\bar{t})\ \Big)}$$

We should have expected

$$\sigma(GG \to t\bar{t}G) \xrightarrow{2\|3} \sim \sigma(Gg \to t\bar{t}) \ + \sim \sigma(G\phi \to t\bar{t})$$

Check new expectation!

Factorization-Problem of DRED

Result:

$$\sigma(GG \to t\bar{t}G) \xrightarrow{2\|3} P_{G\to gG}\ \sigma(Gg \to t\bar{t}) + P_{G\to\phi G}\ \sigma(G\phi \to t\bar{t})$$

has factorized form as expected

Rearrange $\Rightarrow$ Reconcile "non-factorizing terms" with factorization:

$$K_g\, \sigma(GG \to t\bar{t}, m\text{-terms only}) \to P_{\phi\to g\phi}\ [\sigma(Gg \to t\bar{t}) - \sigma(G\phi \to t\bar{t})]$$

What happens for $m = 0$?

$$[\sigma(Gg \to t\bar{t}) - \sigma(G\phi \to t\bar{t})] = 0 \quad \Rightarrow \quad \text{no problem!}$$

Results:

- Actual NLO calculation $\sigma(GG \to t\bar{t}G)$ unaffected, only structure of factorization
- Consider g, ϕ as partons $\Rightarrow$ Factorization $\sim P_{j\to lk}\ \sigma(il \to t\bar{t})$
- Seemingly non-factorizing terms $\Leftrightarrow$ linear combination of $\sigma(il \to t\bar{t})$

Final comment: Results can be transferred to hadronic cross sections:

$$\sigma_{\text{had}} = f \otimes \sigma_{\text{parton}}$$

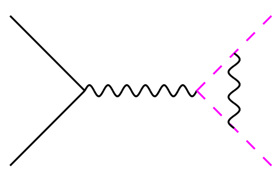
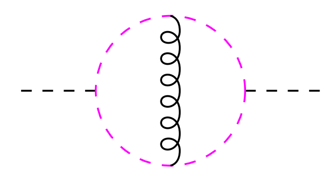
σ_{parton} can be evaluated in either DRED or DREG

Mathematical Inconsistency of DRED ⇒ 1=0!? *[Siegel'80]*

- No general proofs possible

To what extent is DRED SUSY-preserving?

- many SUSY Identities checked:
  - 1-Loop Ward identities *[Capper,Jones,van Nieuvenhuizen'80]*
  - β-functions *[Martin, Vaughn '93] [Jack, Jones, North '96]*
  - 1-Loop S-matrix relation *[Beenakker,Höpker,Zerwas'96]*
  - 1-Loop Slavnov-Taylor identities *[Hollik,Kraus,DS'99] [Hollik,DS'01]* *[Fischer,Hollik,Roth,DS'03]*

- sufficient for e.g.  but not for e.g. 

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- further work necessary; traditional method tedious

DRED can be reformulated without inconsistency

$\Rightarrow$ general proof of "Quantum action principle"

SUSY Ward/ST identities

$$i\,\delta_{\mathsf{SUSY}}\langle T\phi_1\dots\phi_n\rangle \stackrel{?}{=} 0$$

Traditionally:
eval. LHS completely

equal by
Q.A.P.

$\Updownarrow$

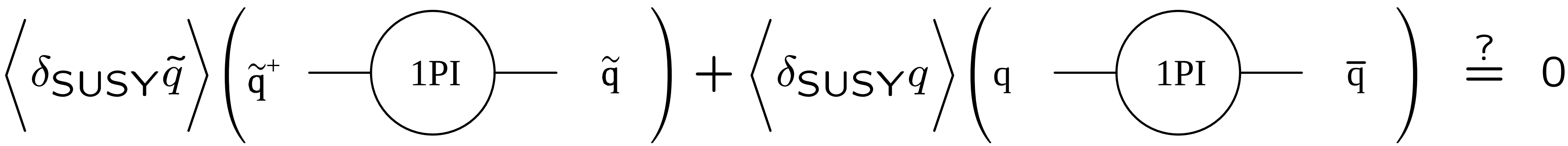
$$\langle T\phi_1\dots\phi_n\Delta\rangle \stackrel{?}{=} 0$$

New strategy:
eval. only insertion of Δ

$\Delta \equiv \delta_{\mathsf{SUSY}}\mathcal{L}$ in D dimensions $\longrightarrow$ insertion with simple Feynman rules

Example Slavnov-Taylor identity for propagators:

$$\delta_{\mathsf{SUSY}}\langle T\tilde{q}^{\dagger}q\rangle \stackrel{?}{=} 0$$



Studied and verified at one-loop in *[Hollik, DS '01]*

$\rightarrow$ full evaluation of all contributing diagrams necessary

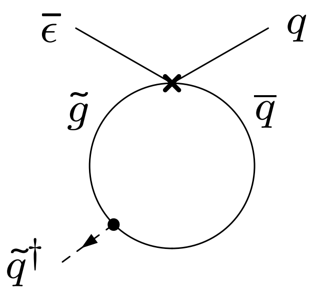
Now: Q.A.P. $\Rightarrow$ identity equivalent to

$$\langle T\tilde{q}^{\dagger}q\boldsymbol{\Delta}\rangle \stackrel{?}{=} 0$$

$\rightarrow$ evaluation of only one very simple Green function necessary

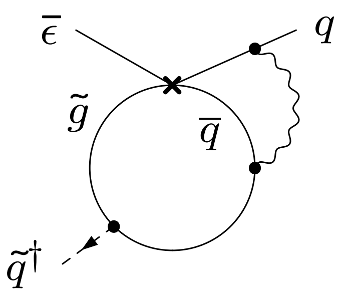
Propagator identity valid $\Leftrightarrow \langle T\tilde{q}^\dagger q \Delta \rangle = 0$

$\langle T\tilde{q}^\dagger q \Delta \rangle^{(1L)} =$



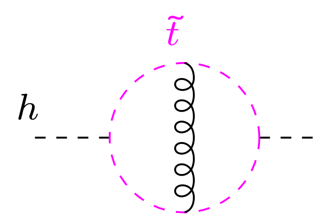
$=0 \longrightarrow$ Agrees with old result. Only one Feynman diagram; evaluation trivial

$\langle T\tilde{q}^\dagger q \Delta \rangle^{(2L)} =$



$=0 \longrightarrow$ New result, relevant for e.g. sfermion masses at 2-Loops

Several new results. Outlook: apply new method to e.g.



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Summary

DRED and related $\overline{DR}$ scheme: important schemes for SUSY-calculations, very common and proposed as standard "SPA-convention"

Improvements concerning two aspects of DRED

1. So far, no general proof that DRED preserves SUSY

   ($\leftrightarrow$ mathematical inconsistency [Siegel'80])

2. Factorization Problem of DRED [Beenakker,Kuijf,v Neerven,Smith ;88]

Aims/Results:

SUSY, Consistency

[DS '05]

- Method to verify SUSY
  Based on consistent reformulation, proof of Q.A.P., much simpler

- Perform verification in important cases
  several 1-,2-Loop identities

QCD-Factorization

[Signer, DS, in preparation]

- "non-factorizing terms": Origin?
  g, ϕ not treated independently

- Reconcile DRED with factorization!
  linear combination of $\sigma(g)$,$\sigma(\phi)$

- Can DRED be used for hadron processes? Yes