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B&K mixing and CP asymmetries in SUSY at NLO

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We derive bounds on the squark mass splittings comparing the full NLO computation of gluino-mediated contributions to $\Delta B=2$ and $\Delta S=2$ processes with the measured values of

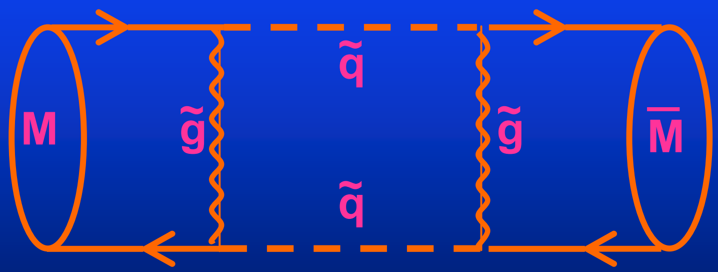
ΔM_K ε_K $a_{J/\psi K_S}$ ΔM_d ΔM_s

analysis in progress...

What's new

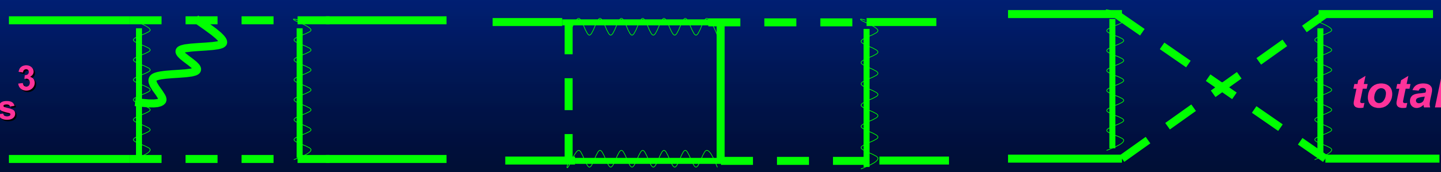
- gluino contributions calculated in the MSSM at NLO:

α_s^2



$M = K, B_d, B_s, D$

α_s^3



total = 279 graphs

- updated measures (especially ΔM_d, sin2β and γ) and parameters (CKM and quark masses values)

What was already there

- NLO evolution (2 loop anomalous dimension matrix)
- B parameters computed on the lattice

And what is (still) missing

- global analysis including also charginos, neutralinos, charged Higgs exchange
- unquenched $B_{3,4,5}$ parameters

CP Violation and FCNC in SUSY

Overview of the Calculation

Framework for a Model Independent Analysis

Numerical Analysis

The basic problem

It is an unexpected experimental fact that in charged current weak interactions the mass eigenstates are mixed.

The SM provides a minimal but very peculiar description

* *just one CP violating phase*
* *nearly diagonal CKM*
* *FCNC strongly suppressed*

It is difficult for New Physics to reproduce all this :

1) adding new particles requires more flavour rotations to reach the mass eigenstate basis
2) an extended Higgs sector is another source of CPviol and FCNC

we use FCNC and CP violating processes to constrain the huge SUSY parameter space

NP interactions are suppressed by some power of the NP scale Λ. Naturalness demands Λ low, FCNC Λ high → some extra assumptions are invoked to relax the tension, for example *soft-breaking universality*

focus on SUSY squark sector

Squared mass matrix for the squarks $(\tilde{d}_L, \tilde{s}_L, \tilde{b}_L, \tilde{d}_R^*, \tilde{s}_R^*, \tilde{b}_R^*)$ in the basis where the quark mass matrices $m_{u,d}$ are diagonal

$$\begin{pmatrix} M_{DLL}^{tree\ 2} + c_1 V m_u^{\dagger 2} V & |A|\ m_{3/2}\left(1+ \frac{c_2}{M_W^2} V m_u^{\dagger 2} V\right) m_d \\ |A|\ m_{3/2}\, m_d \left(1+ \frac{c_2}{M_W^2} V m_u^{\dagger 2} V\right) & M_{DLL}^{tree\ 2} \end{pmatrix}$$

- White terms are flavour diagonal and come from a flavour blind $\mathcal{L}_{soft}$

- Orange terms are driven by radiative corrections [c_i running coeff. contain $\log (M_{NP}/M_W)$] and are governed by $V=V_{CKM}$ → FCNC in the strong interactions $\tilde{q}\tilde{q}g$

Here FCNC arise radiatively and are naturally small; this is not true in eneral.

Every block is 3x3 in flavour space

$$\begin{pmatrix} m_1^2 & \Delta^{12} & \Delta^{13} \\ \Delta^{21} & m_2^2 & \Delta^{23} \\ \Delta^{31} & \Delta^{32} & m_3^2 \end{pmatrix}_{LR}$$

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FULL THEORY — **EFFECTIVE THEORY**

SUSY scale $M_s = \frac{m_{\tilde{q}} + m_{\tilde{g}}}{2}$

hadronic scale
(renormalization scale of the operators)

the matching equation is written at the high scale M_s

$$\left(\begin{array}{c}\text{1-loop + 2-loop diagrams} \\ \text{calculated in the MSSM}\end{array} \right) = \Sigma_i \, C_i(\mu) \, O_i(\mu)$$

then the Wilson coefficients $C_i(\mu)$ are evoluted with RGE [with the NLO anomalous dimension matrix (ADM) for $\Delta F=2$] to the low energy scale, where the operators are computed with some non perturbative method

In order to properly account for the scheme and scale dependence in the physical result *2 loop diagrams are needed*

Low energy part

Integrating out squarks and gluinos gives the operators (SUSY basis)

$$Q_1 = \bar{d}_L^{\alpha} \gamma_\mu b_L^{\alpha} \, \bar{d}_L^{\beta} \gamma_\mu b_L^{\beta} \leftarrow \text{SM} \qquad Q_4 = \bar{d}_R^{\alpha} b_L^{\alpha} \, \bar{d}_L^{\beta} b_R^{\beta}$$

$$Q_2 = \bar{d}_R^{\alpha} b_L^{\alpha} \, \bar{d}_R^{\beta} b_L^{\beta} \qquad Q_5 = \bar{d}_R^{\alpha} b_L^{\beta} \, \bar{d}_L^{\beta} b_R^{\alpha}$$

$$Q_3 = \bar{d}_R^{\alpha} b_L^{\beta} \, \bar{d}_R^{\beta} b_L^{\alpha} \qquad \tilde{Q}_{1,2,3} = Q_{1,2,3} \, (L \leftrightarrow R)$$

The matrix elements of the operators between the mesons M are given in the Vacuum Insertion Approximation (VIA) as functions of the quark and the meson masses and decay constant.
The effect of non-factorizable corrections is contained in the B parameters

$$\langle \bar{M} | Q_i(\mu) | M \rangle = f\,(m_q, m_M, f_M)\, B_i(\mu)$$

Now the best tool to calculate the B parameters is lattice QCD

- unquenched only $B_{1,2}$ (no surprises with respect to the unquenched case)
- the d quark is very light. Because of computer power, a direct simulation is not possible. Chiral extrapolation is problematic.

important: $Q_{2,3,4,5}$ can be enhanced with respect to Q_1 by $m_M^2/(m_{q_1}+m_{q_2})^2$

High energy part

2 loop diagrams in the MSSM ⟶ NLO Wilson coefficient

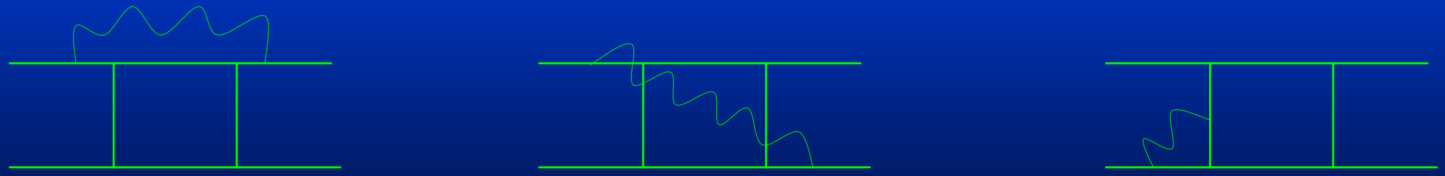
- integrals without external momenta, 2 masses ($m_{\tilde g}$, $m_{\tilde q}$), zero quark masses
- both Dimensional REDuction (γ in 4-dim) and NDR (γ and impulses in $d=4-2\varepsilon$ dim) regularizations. NDR breaks SUSY → redefinition of α_s is necessary.
- with and without mass insertion approximation

evanescent operators (EO)

for example:

fierz ⟶ $\bar{d}_L^{\alpha} \gamma_\mu b_L^{\alpha} \bar{d}_L^{\beta} \gamma_\mu b_L^{\beta} = \bar{d}_L^{\alpha} \gamma_\mu b_L^{\beta} \bar{d}_L^{\beta} \gamma_\mu b_L^{\alpha}$ **valid only in 4-dim**

While working in d-dim define EO = $Q_1^{\alpha\alpha\beta\beta} - Q_1^{\alpha\beta\beta\alpha} = O(\varepsilon)$.
It is dangerous because infrared divergences x EO → finite part to the result.
IR div. cancel in the matching equation (full and effective theories have the same IR behaviour), not before within the full theory like UV div.



High energy part

2 loop diagrams in the MSSM ⟶ NLO Wilson coefficient

- integrals without external momenta, 2 masses ($m_{\tilde{g}}$, $m_{\tilde{q}}$), zero quark masses
- both Dimensional REDuction (γ in 4-dim) and NDR (γ and impulses in $d=4-2\varepsilon$ dim) regularizations. NDR breaks SUSY → redefinition of α_s is necessary.
- with and without mass insertion approximation (flavour/mass eigenstates basis)

evanescent operators (EO)

for example:

fierz ⟶ $d_L^\alpha \gamma_\mu b_L^\beta \, d_L^\beta \gamma_\mu b_L^\alpha = d_L^\alpha \gamma_\mu b_L^\beta \, d_L^\beta \gamma_\mu b_L^\alpha$ valid only in 4-dim

While working in d-dim define EO = $Q_1^{\alpha\alpha\beta\beta} - Q_1^{\alpha\beta\beta\alpha} = O(\varepsilon)$.
It is dangerous because infrared divergences x EO → finite part to the result.
IR div. cancel in the matching equation (full and effective theories have the same IR behaviour), not before within the full theory like UV div.

Solutions

- avoid IR div: we gave mass to the gluon (done NDR and DRED)
- do the matching in d-dim keeping also EO's (checked in DRED where EO's arise when a $g_{\mu\nu}$ in 4-dim from γ algebra meets a $g_{\mu\nu}$ in d-dim from impulses)

CP Violation and FCNC in SUSY

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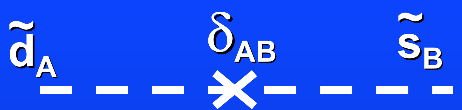
Now we have the NLO result for $\Delta S=2$ as function of the squark mass, the gluino mass and the squark mixing matrix *which is strongly model-dependent*

THE MASS INSERTION METHOD

1. Work in the superCKM basis where the vertices $q\tilde{q}\tilde{g}$ are flavour diagonal.
   The squark mass matrix $\tilde{M}_D$ is not diagonal.
2. Compute an average diagonal squark mass $<m> = (m_1 m_2 \dots m_n)^{1/2}$
3. Define δ = off diagonal elements of $\tilde{M}_D / <m>$
4. Expand the squark propagators in δ at the first (n-th) order

Or equivalently use the Feynman rule



A,B = L,R

→ the experimental bounds are now on the δ's

↑ the δ's are easily computed in every model, without diagonalizing the whole mass matrix M_D

But you are assuming that

↓ the off diagonal entries in M_D are small quantities

↓ the diagonal entries are nearly degenerate

↓ no interference effects occur

self-service analysis

CASE 1

Take the ratio off-diagonal / average diagonal terms of the squark mass matrix. Compare with the bounds we give on the δ's.

CASE 2

Because of possible interference/cancellation effects you might decide to trust only the order of magnitude of the bounds on the δ's. We give also the exact result: you can fill in directly the exact parameters of your model in our Wilson coefficients.

CASE 3

You compute the Wilson coefficients of whatever model that extends the SM adding new heavy particles.

In cases 2-3 **to get the physical amplitude you still have to**

1) evolve the Wilson coefficients to the low energy scale μ

We give the evolution equation (with NLO ADM) in the ready-to-use form

$$C_r(\mu) = \sum_i \sum_s \left(b_i^{(r,s)} + \eta\, c_i^{(r,s)}\right) \eta^{a_i}\, C_s(M_{high})$$

$b_i^{(r,s)}$, $c_i^{(r,s)}$, a_i magic numbers $\qquad \eta = \alpha_s(M_{high})/\alpha_s(m_t)$

2) multiply for the matrix elements of the operators computed at the scale μ

We list their analytic formulae and the numerical values of the parameters contained: quark and meson masses, decay constants, B-parameters.

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Contact Experiment-Theory

$K^o\bar{K}^o$ system $\rightarrow \boldsymbol{\delta^d_{12}}$

$$\Delta M_K = 2\,\mathrm{Re}\langle K|\mathcal{H}^{\Delta S=2}|\bar{K}\rangle$$

$$\varepsilon_K = \frac{\mathrm{Im}\langle K|\mathcal{H}^{\Delta S=2}|\bar{K}\rangle}{\sqrt{2}\,\Delta M_K}$$

$B_d\bar{B}_d$ system $\rightarrow \boldsymbol{\delta^d_{13}}$

$$\Delta m_d = 2|\langle B_d|\mathcal{H}^{\Delta B=2}|\bar{B}_d\rangle|$$

$$a_{J/\psi K_S} = \sin 2\beta_{eff} \sin(\Delta m_d t)$$

New physics can enter !

$\rightarrow \mathrm{Arg}\langle\bar{B}_d|\mathcal{H}^{\Delta B=2}|B_d\rangle$

measured ← → **calculated NLO in terms of**
$m_{\tilde{q}}$, $m_{\tilde{g}}$, δ's

$B_s\bar{B}_s$ system $\rightarrow \boldsymbol{\delta^d_{23}}$

interesting but only bounded:

- The SM amplitude is real
- $\left(\frac{\Delta m_d}{\Delta m_s}\right)_{SM} \sim \left(\frac{\Delta m_d}{\Delta m_s}\right)_{MSSM}$

The method

$$\langle \bar{M}|\mathcal{H}^{\Delta S=2}|M\rangle = \mathrm{Re}\, A_{SM} + i\, \mathrm{Im}\, A_{SM} + A_{SUSY}\, \mathrm{Re}(\delta^d_{ij})^2_{AB} + i\, A_{SUSY}\, \mathrm{Im}(\delta^d_{ij})^2_{AB}$$

The constraints are obtained imposing that the sum of SUSY contributions is proportional to a single δ and the SM contribution reproduce the measured value of the observable. The d's appear quadratically in the Wilson coefficients in the combinations

$$\delta_{LL}^2 \quad \delta_{RL}^2 \quad \delta_{LL}\delta_{RR} \quad \delta_{LR}\delta_{RL} \quad \delta_{RR}^2 \quad \delta_{LR}^2$$

the mixed products are bounded setting $\delta_{ij}=\delta_{kl}$

This is the main limit of the analysis → intereference effects possible order of magnitude bounds

The parameters x_i are extracted from flat distribution in $[x_i - \sigma_i, x_i + \sigma_i]$
The average observables O are calculated with the weighting factor

$$e^{-\frac{(O(x_i) - O_{exp})^2}{\sigma_{exp}^2}}$$

[if σ_{exp} <<, not efficient → we solve in the less constrained parameter]

Main input parameters used in the preliminary analysis

$$\sin 2\beta = 0.726 \pm 0.037$$

$$\Delta m_d \ (ps^{-1}) = 0.502 \pm 0.006$$

CKM parameters from the UTfit (www.utfit.org)

only measures New Physics free

$$\bar{\rho} = 0.25 \pm 0.06$$

$$\bar{\eta} = 0.33 \pm 0.03$$

$$\lambda_c = 0.2258 \pm 0.0014$$

$$V_{cb} = 0.0416 \pm 0.0007$$

B-parameters

from hep-lat/0110091 in RI-MOM

$$B_1(m_b) = 0.87(4)^{+5}_{-4}$$

$$B_2(m_b) = 0.82(3)(4)$$

$$B_3(m_b) = 1.02(6)(9)$$

$$B_4(m_b) = 1.16(3)^{+5}_{-7}$$

$$B_5(m_b) = 1.91(4)^{+22}_{-7}$$

The plots shown here are at fixed values of the squark and gluino masses:

$$m_{\tilde q} = m_{\tilde g} = 350 \text{ GeV}$$
$$x = m_{\tilde g}^2/m_{\tilde q}^2 = 1$$

How do the constraints on the δ's vary with x and $m_{\tilde g,q}$?

- The results of the analysis are reliable for $x \leq \mathcal{O}(1)$. However
  x > 1 disfavoured by the evolution from M_{PL} to M_W: $x_W = \frac{9\, x_{PL}}{1 + 7\, x_{PL}}$
  x << 1 only in specific models
- If $m_{\tilde q,\tilde g}$ or $x \rightarrow \infty$, the bounds on the δ's become loose

consequence of decoupling

The constraints on the δ's don't solve to the flavour problem:

very high scale for new physics

$m_{\tilde g,q}$ >> and loose δ's

or flavour symmetries + new physics at lower scale

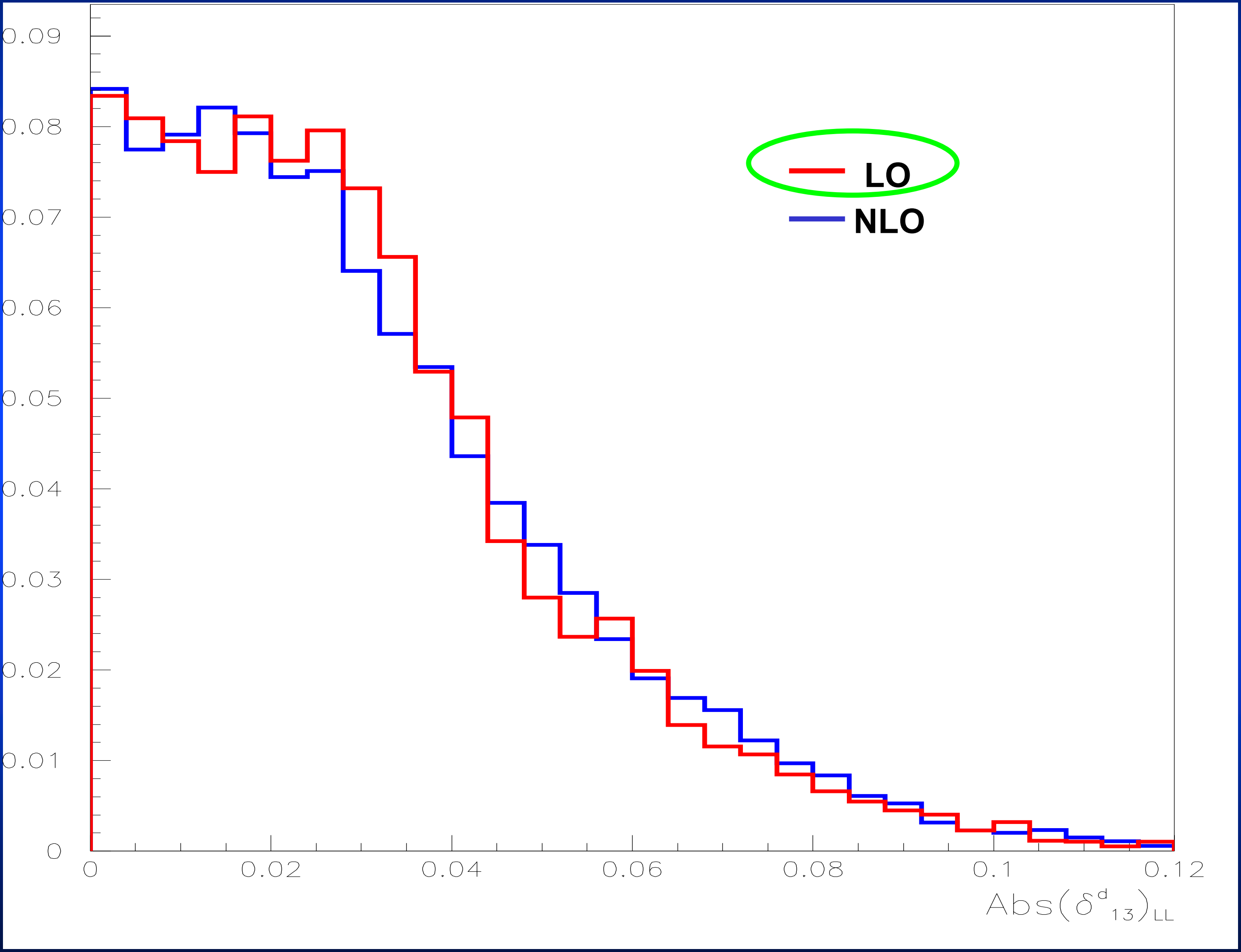
lower $m_{g,\tilde q}$ and stringent δ's

are both possible

What is the effect of the NLO coefficients ?

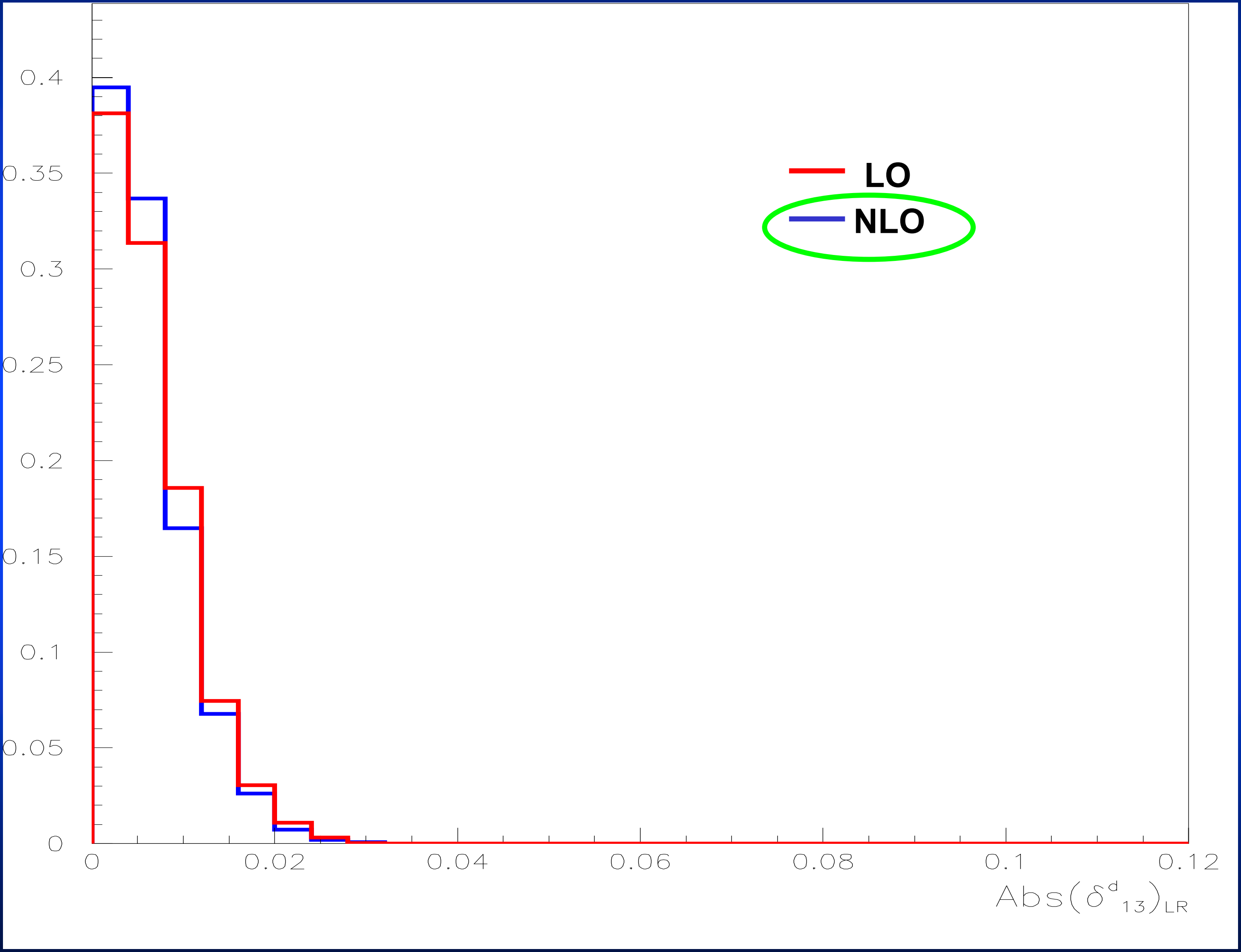
- Depends on the mass insertion considered
- Doesn’t alter the order of magnitude
- A direct comparison with previous analysises is not possible, but the improvements on the bounds appears mainly due to the higher precision of the recent measures.
- A major effect is due to the NLO evolution with respect to the LO evolution
- Weakens the dependence on the matching scale

$|(\delta^d_{13})_{LL}|$

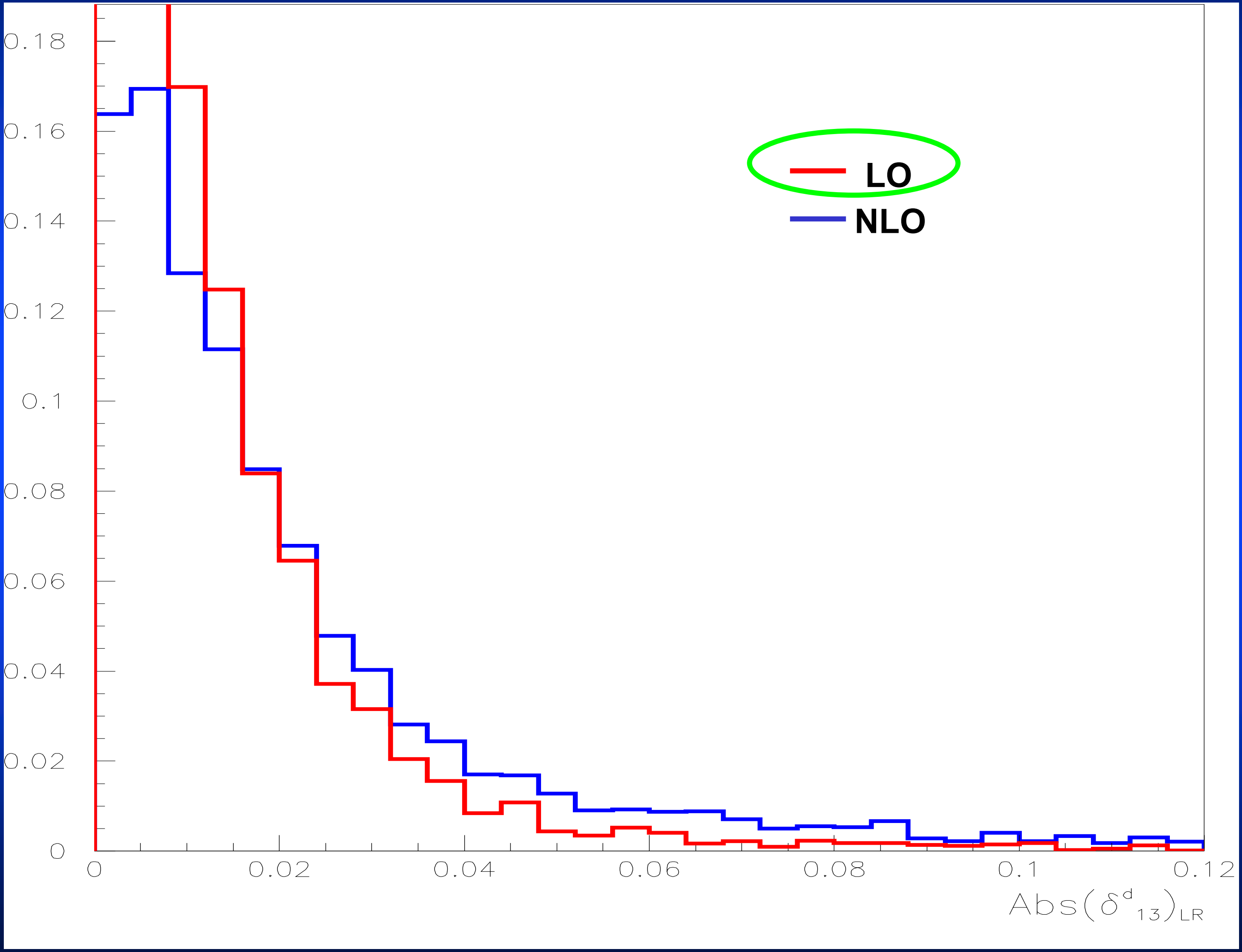


0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.02
0.04
0.06
0.08
0.1
0.12
Abs(δ^d_{13})$_{LL}$
LO
NLO

$\left|(\delta^d_{13})_{LR}\right|$

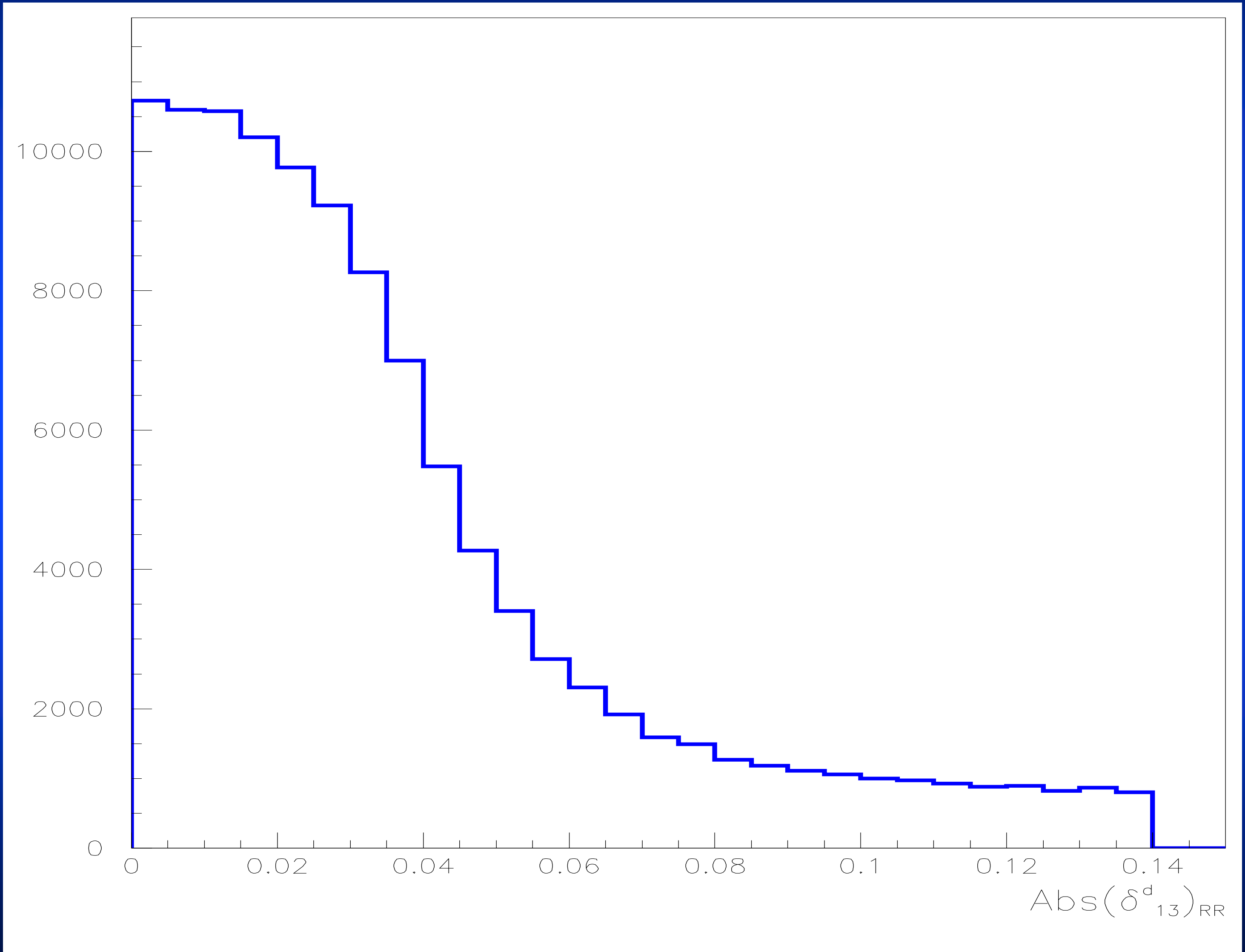


$\left|(\delta^d_{13})_{LR=RL}\right|$

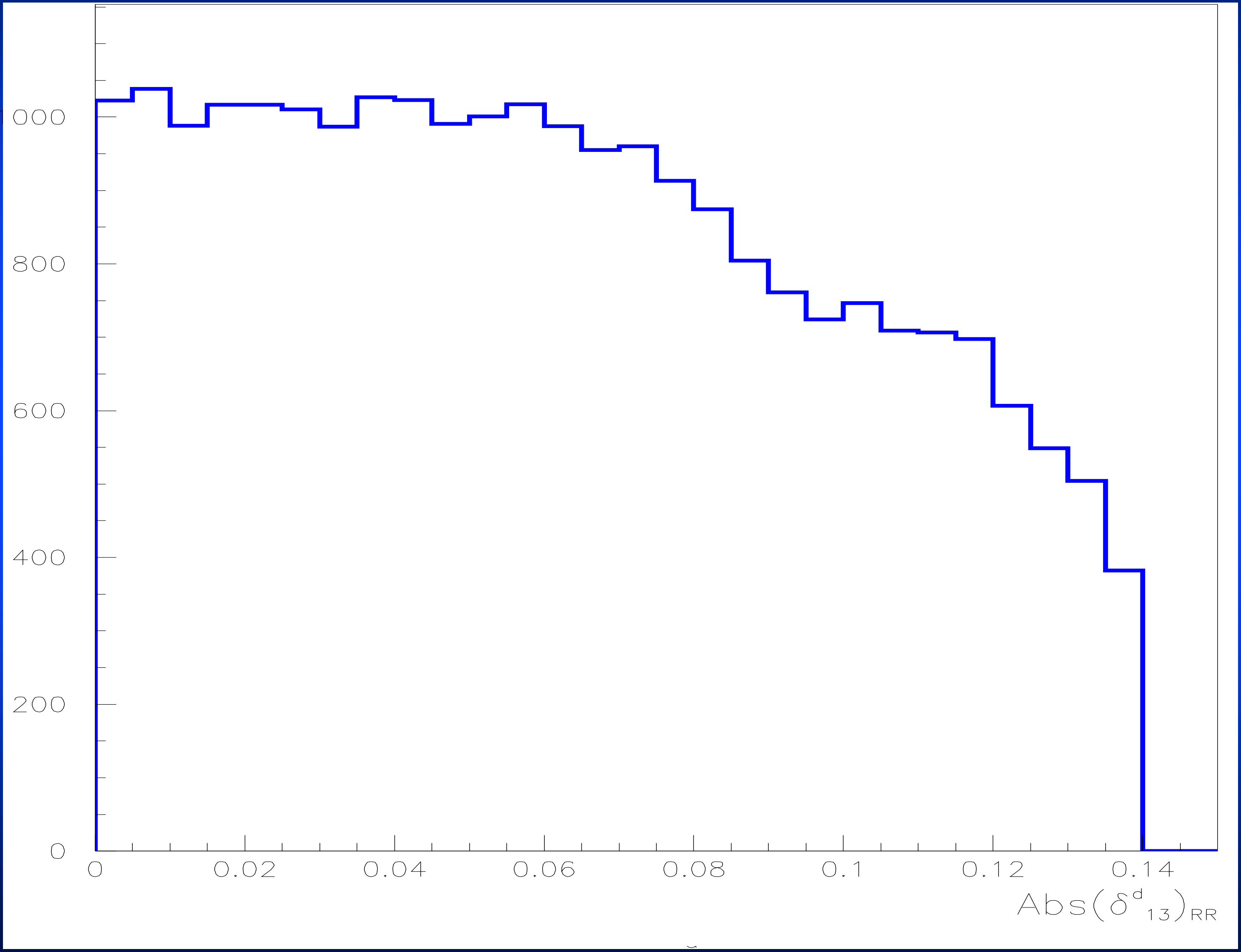


What is the effect of every experimental bound on the δ's?

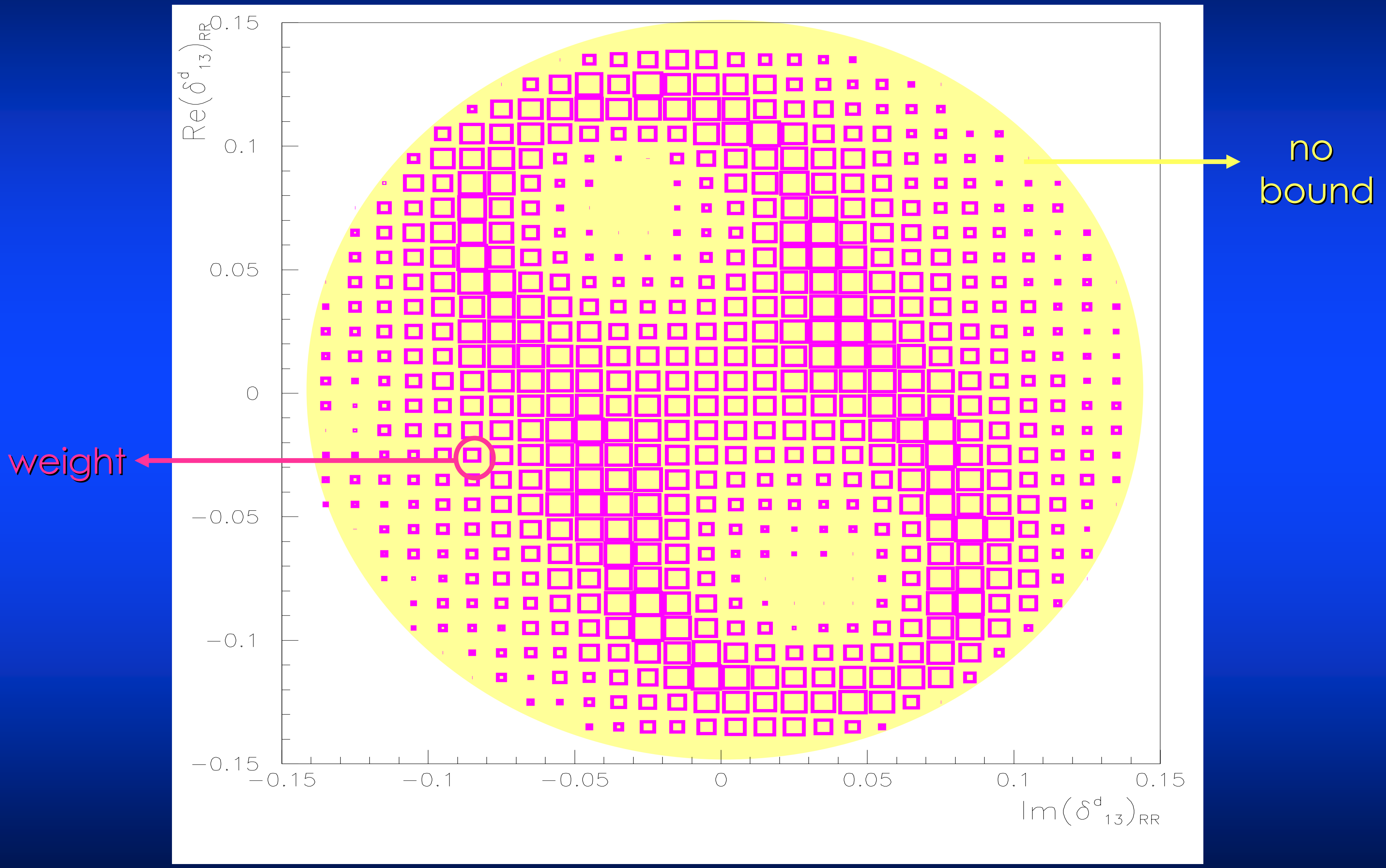
sin 2β on $(\delta^d_{13})_{RR}$



Δm_d on $(\delta^d_{13})_{RR}$



Δm_d

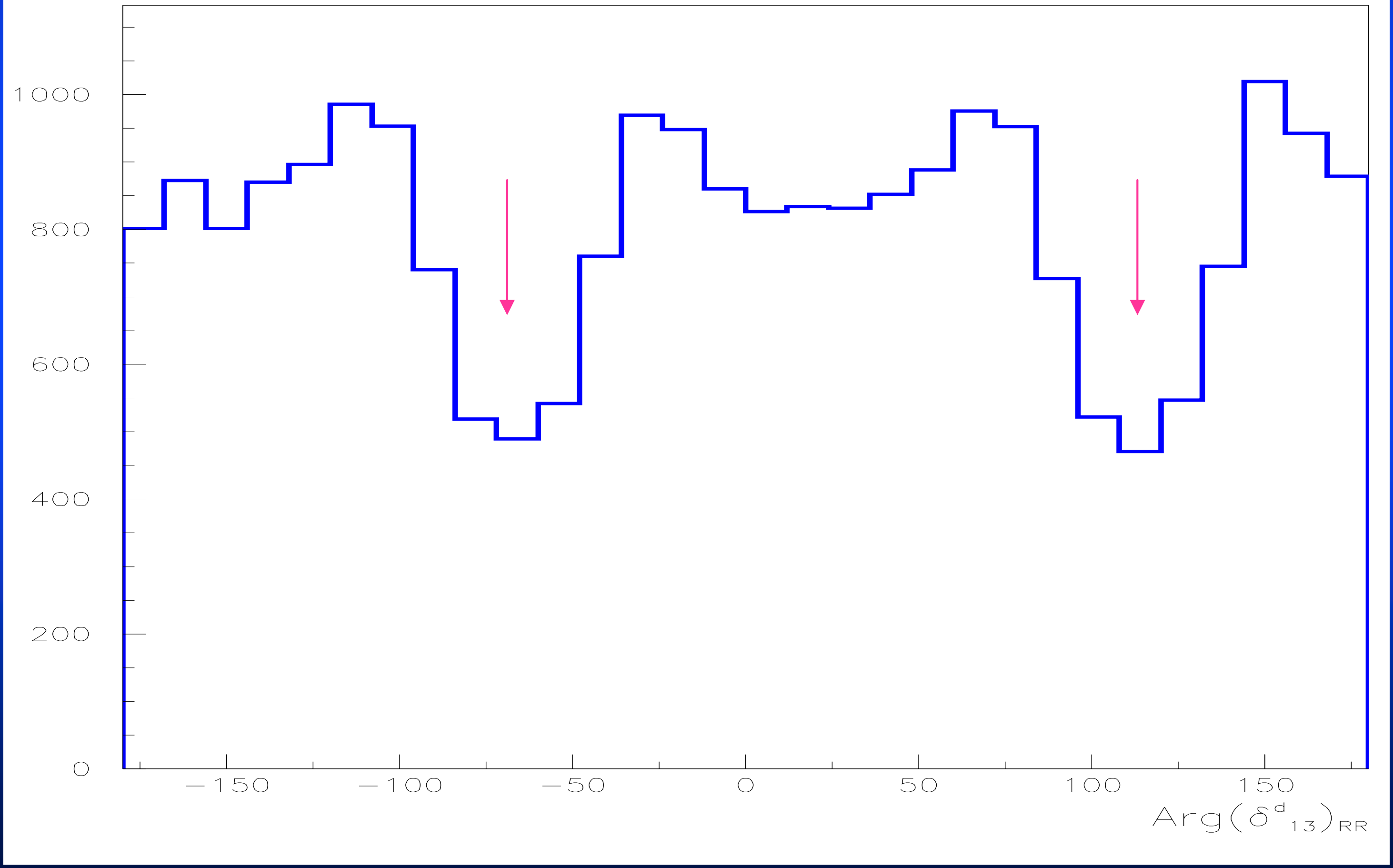


0.15
0.1
0.05
0
−0.05
−0.1
−0.15
$Re(\delta^d_{13})_{RR}$
−0.15
−0.1
−0.05
0
0.05
0.1
0.15
$Im(\delta^d_{13})_{RR}$
no bound
weight

projected on Arg $(\delta^d{}_{13})_{RR}$

$$\Delta m_d = 2|\langle \bar{B}_d|\mathcal{H}^{\Delta B=2}|B_d\rangle| = 2|A_{SM}\, e^{i2\beta} + A_{SUSY}\, e^{i2\varphi}|$$

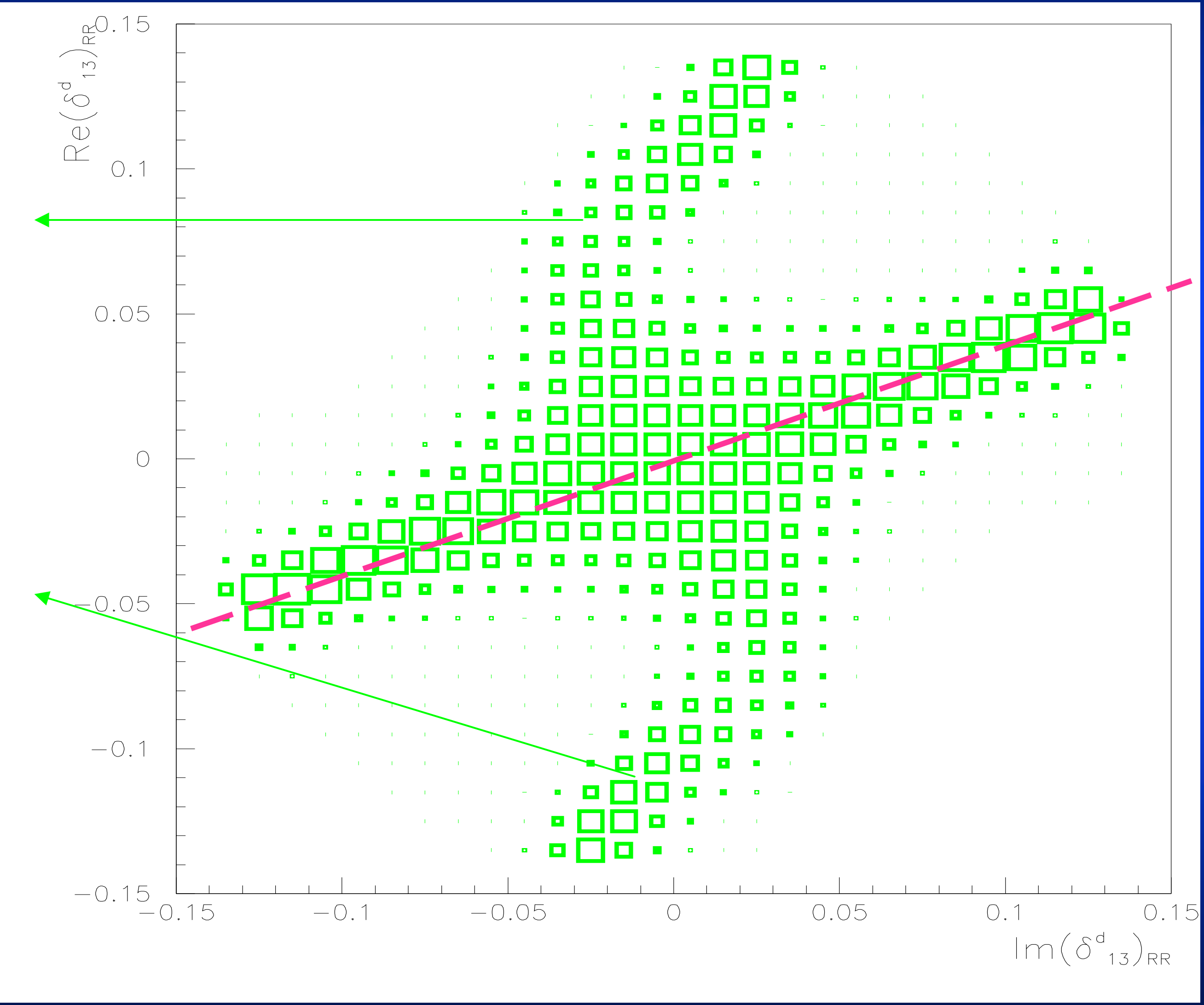
if $\varphi = \pm\beta$ SUSY contribution is quadratic, otherwise linear



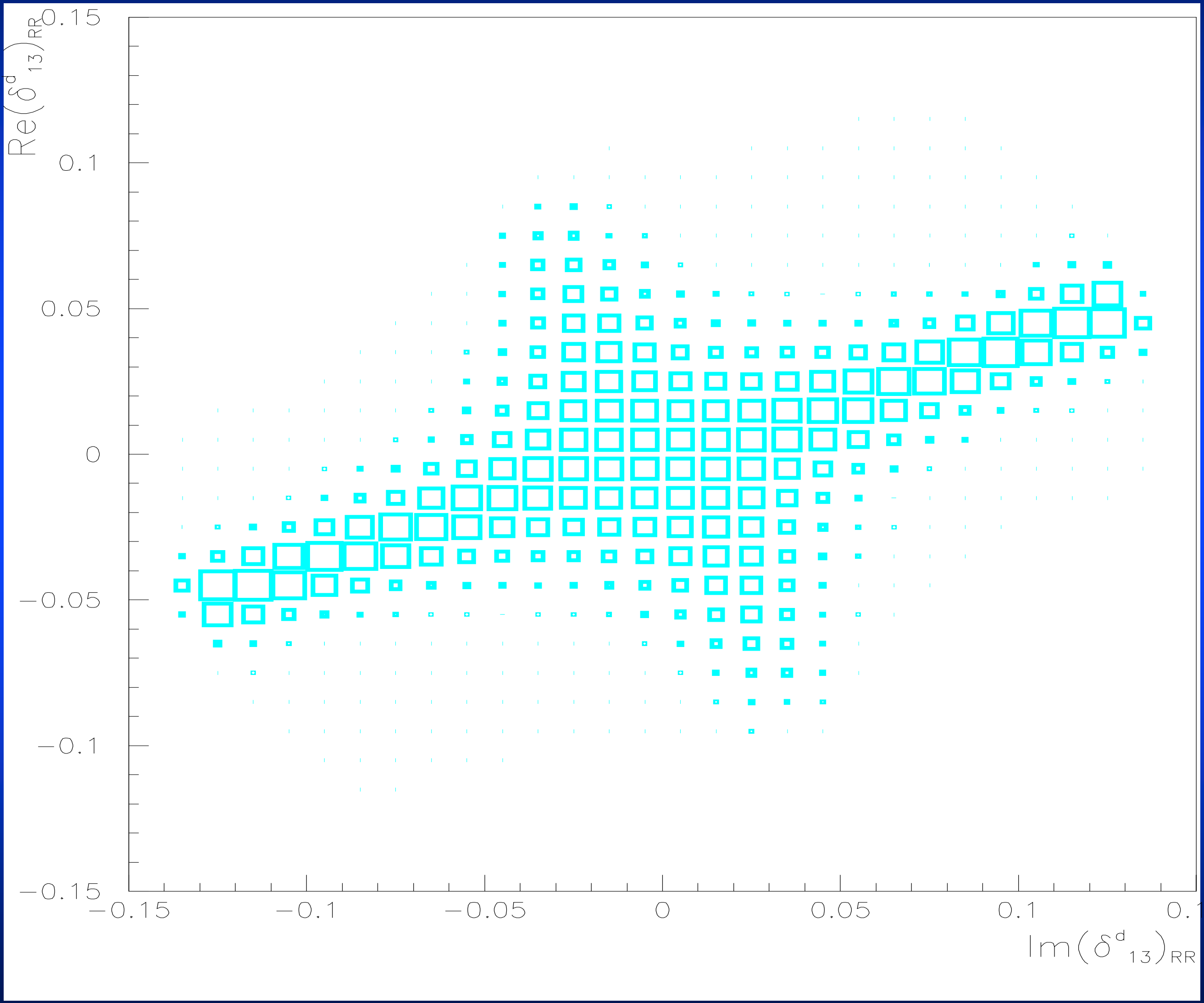
sin 2β

Wrong sign branches

Note the non uniform weights



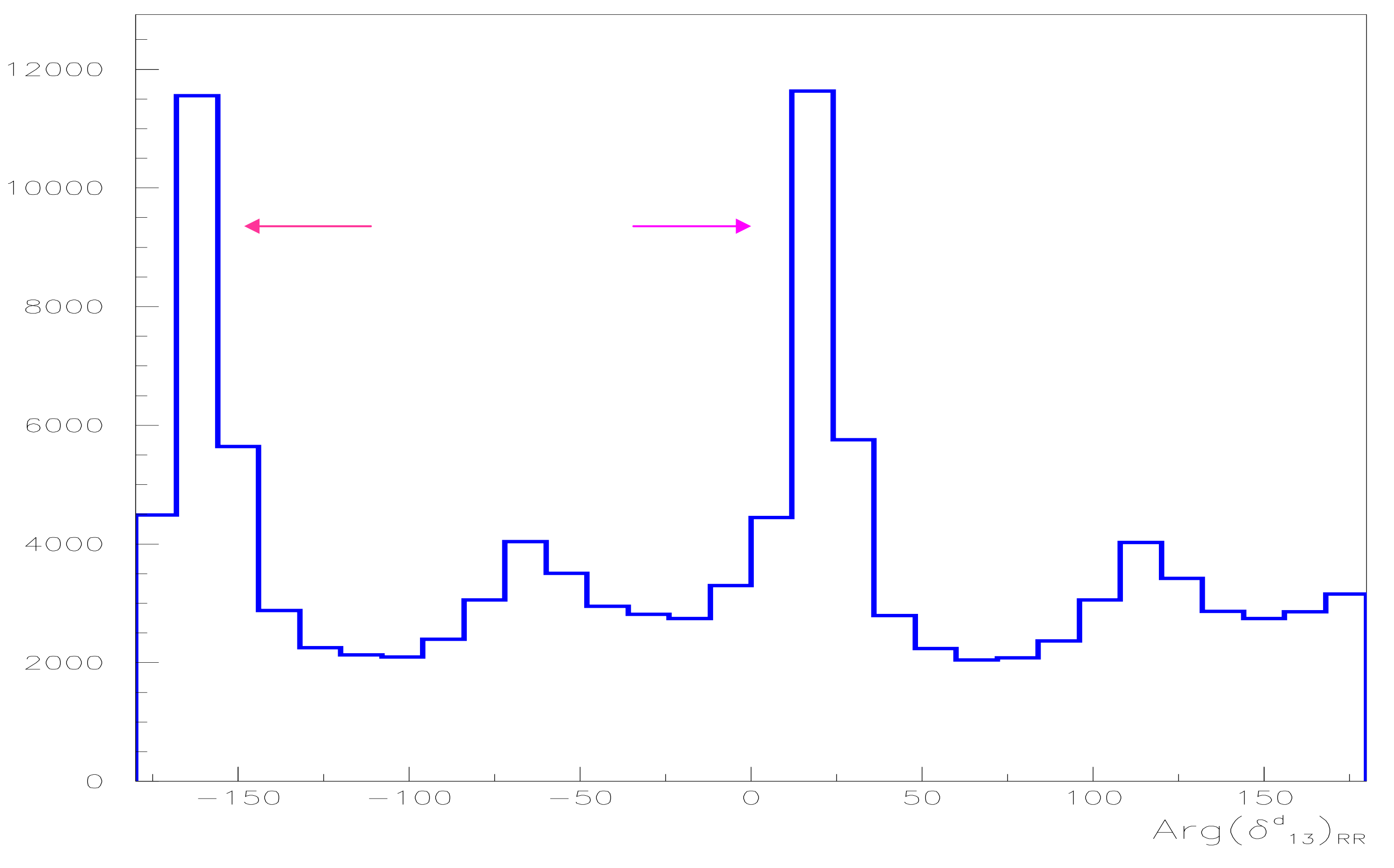
sin 2β and cos 2β



projected on Arg $(\delta^d_{13})_{RR}$

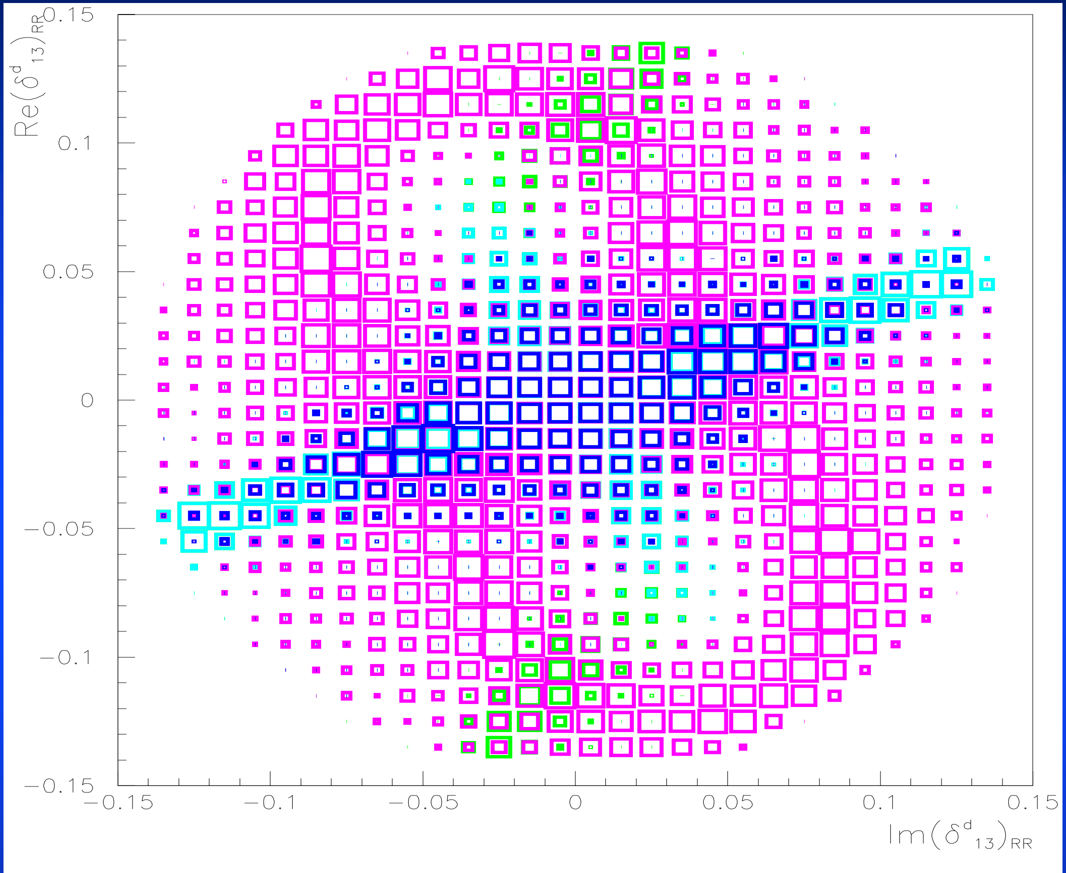
$$2\beta = \text{Arg}|\langle \bar{B}_d|\mathcal{H}^{\Delta B=2}|B_d\rangle| = \text{Arg}|A_{SM}\, e^{i2\beta} + A_{SUSY}\, e^{i2\varphi}|$$

if $\varphi = \beta$ the bound is satisfied for every A_{SUSY}

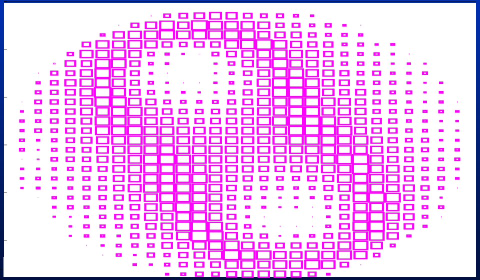


All the bounds together:

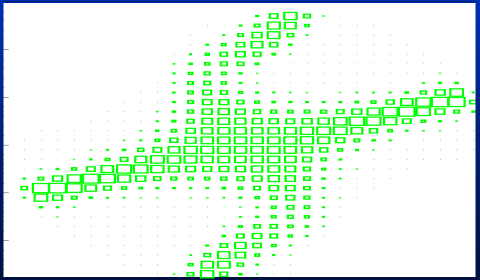
allowed region: dark blue squares



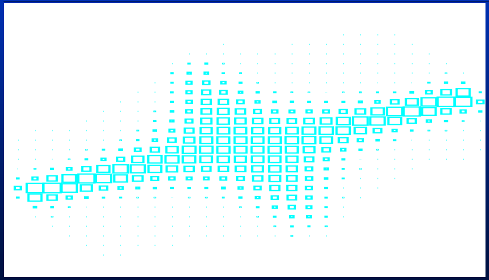
Δm_d



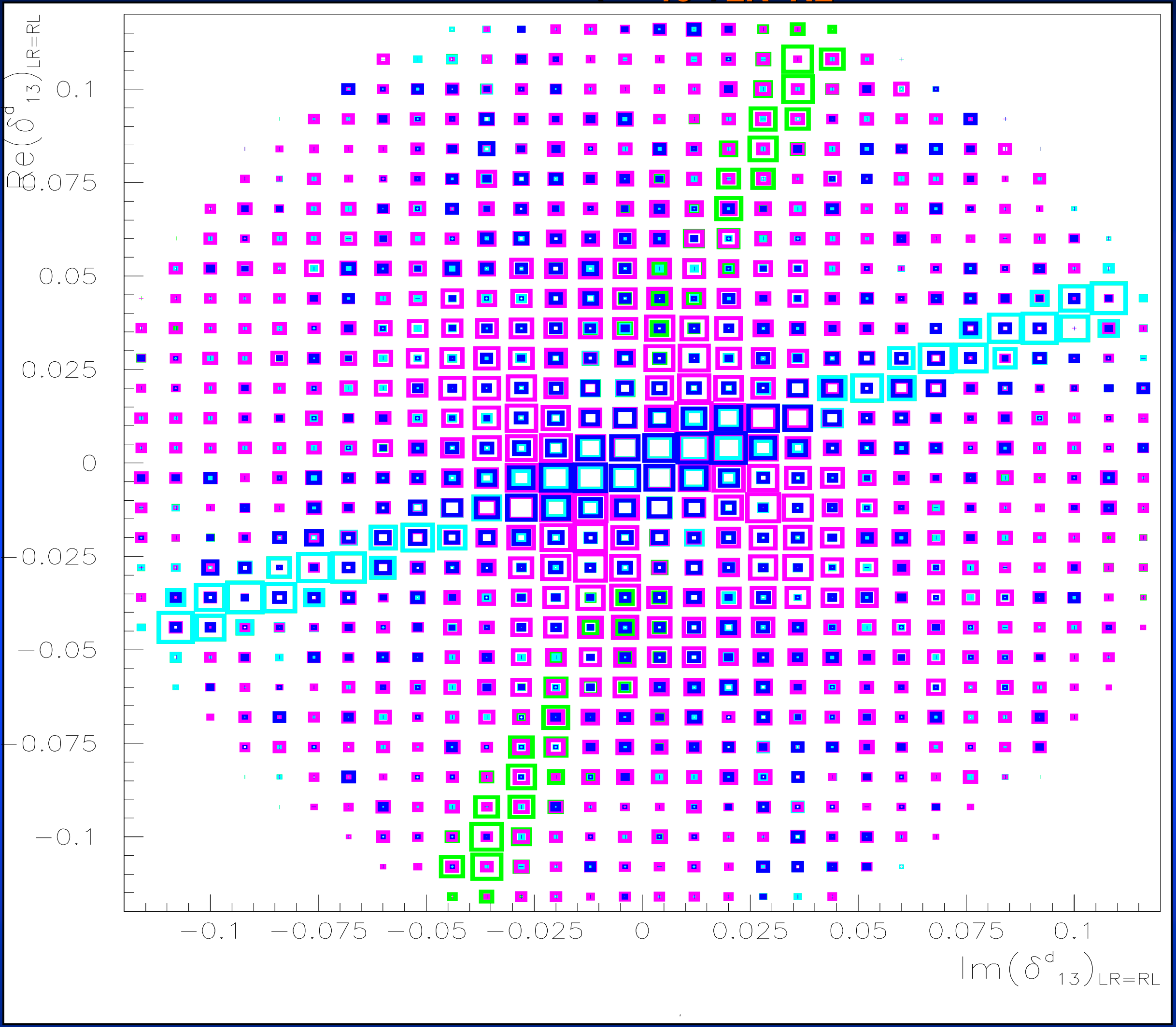
sin 2β



cos 2β

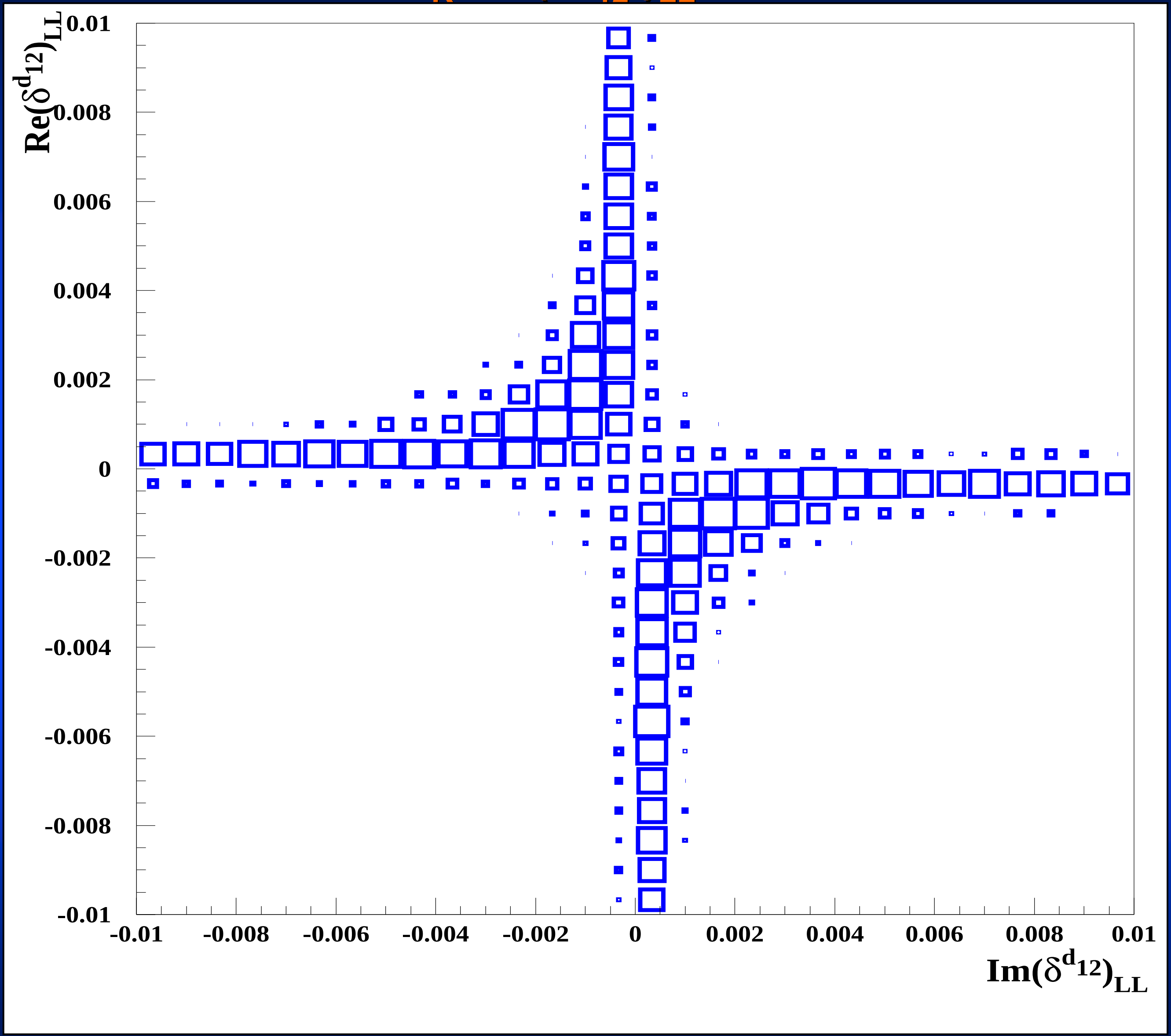


The same for $(\delta^d_{13})_{LR=RL}$

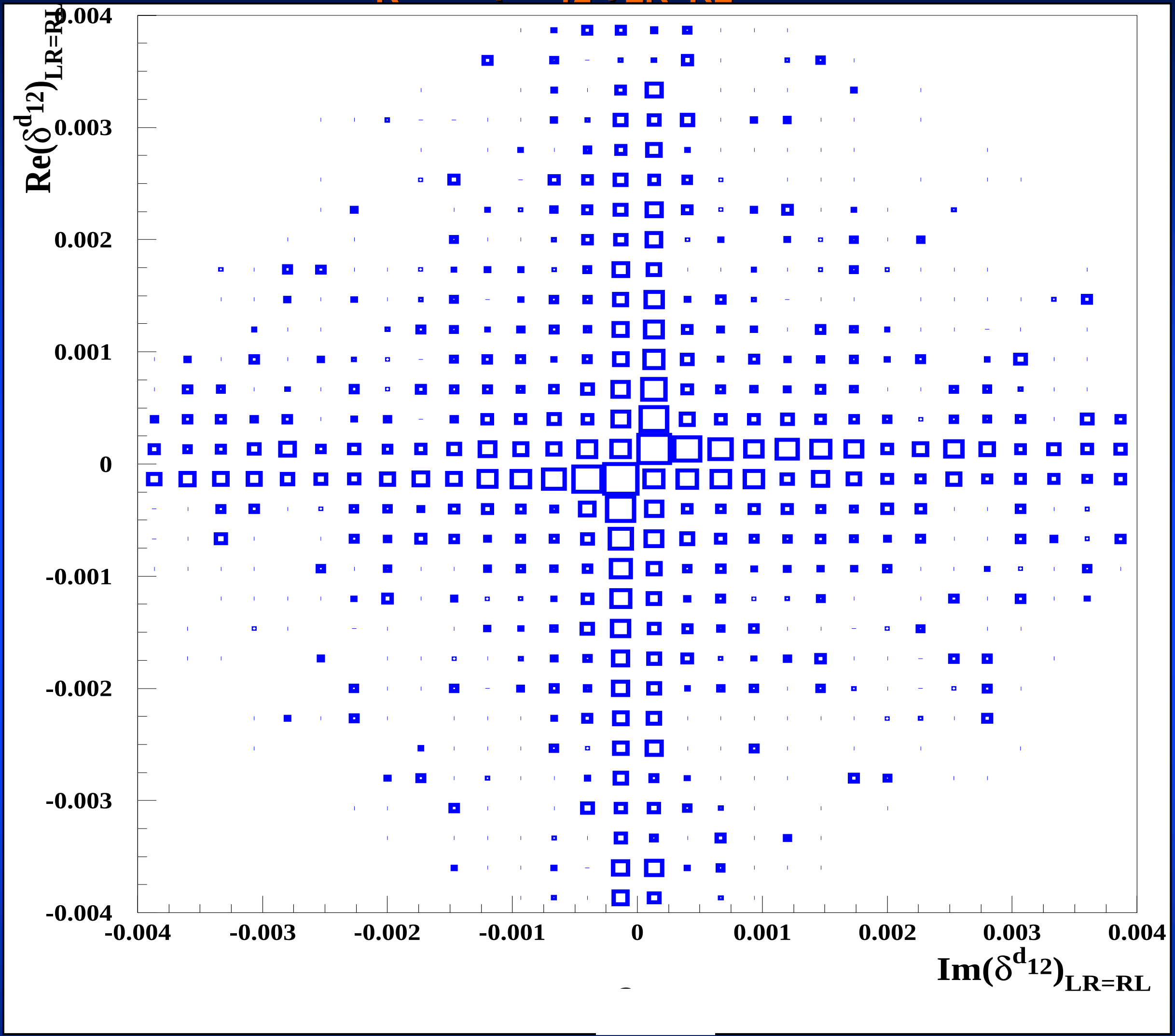


A partial cancellation in the Wilson coefficients occurs: the bounds are less effective, the plot is more scattered

ε_K on $(\delta^d_{12})_{LL}$



ε_K on $(\delta^d_{12})_{LR=RL}$



Again an interference effect . Usually SUSY models with dominant LR/RL mass insertions can contribute more to ε'_K