From transition magnetic moments to majorana neutrino masses

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based on hep-ph/0506085 [Davidson, M.G., Santamaria]

- Motivation
- Transition Magnetic Moments
- Operator Mixing
- Majorana Masses
- Dirac Scenario
- Conclusions

SUSY 2005

Motivation

- Δm_ν^2 and two mixing angles have been measured
  - $\theta_{23} \simeq \pi/4$, $\sin^2\theta_{12} \simeq .29$, $\sin^2\theta_{13} < 0.035$,
    $\Delta_{@} m^2 \simeq (0.049eV)^2$, $\Delta_{\odot} m^2 \simeq (0.0089eV)^2$
  - hierarchical ($m_3^2 \simeq \Delta m_{atm}^2, > m_2^2, m_1^2$),
    inverted ($m_3^2 \ll m_{1,2}^2, \;\; m_{1,2}^2 \simeq \Delta m_{atm}^2$) or degenerate
- In the future $0\nu 2\beta$ might tell us if ν is Majorana
- The Majorana mass term

$$[m_\nu]_{\alpha\beta}\, \overline{\nu^c}_\alpha \nu_\beta$$

  might give us some clue about high scale physics
  - Seesaw: $[m_\nu]_{\alpha\beta} = \left[Y_\nu^T\right]_{\alpha K} M_K^{-1} \left[Y_\nu\right]_{K\beta} \langle H_u^0\rangle^2 \;\rightarrow\; M_{NP} \sim 10^{14}\mathrm{GeV}$
  - SUSY Seesaw and universal soft masses[Davidson, Iberra]:
    Extract M_K and Yukawa Couplings

Beyond the Mass Operator

- Only $D=5$ Operator is the Mass Operator

$$[O_M]_{\alpha\beta} = (\overline{l^c_\alpha}\epsilon H)(H\epsilon P_L l_\beta)$$

- For more information about high scale physics study higher dimensional Operators
  - Next important $\Delta L = 2$ operators are $D=7$ e.g. [Babu, Leung '01]
  - Magnetic moment operators have three "massless" particles $\longrightarrow$ important consequences for astrophysics [Raffelt '96, Friedland '05]
  - There are already bounds from experiments and astrophysics
  - Try to constrain them further and find contributions to low energy physics

Transition Magnetic Moments

- Magnetic Moment

$$\frac{\mu_{\alpha\beta}}{2}\overline{\nu^c_\alpha}\sigma^{\mu\nu}P_L\nu_\beta F_{\mu\nu} + \text{h.c.}$$

  is antisymmetric in flavour space

- $\mathrm{SU}(2) \times \mathrm{U}(1)$ invariance $\rightarrow$ two $D = 7$ Operators:

$$[O_B]_{\alpha\beta} = g'(\overline{l^c_\alpha}\epsilon H)\sigma^{\mu\nu}(H\epsilon P_L l_\beta)B_{\mu\nu}$$

$$[O_W]_{\alpha\beta} = ig\epsilon_{abd}(\overline{l^c_\alpha}\epsilon\tau^a P_L l_\beta)(H\epsilon\tau^b H)W^d_{\mu\nu}$$

- They will mix into the $D = 7$ Mass Operator

$$[O_M]_{\alpha\beta} = (\overline{l^c_\alpha}\epsilon H)(H\epsilon P_L l_\beta)(H^\dagger H)$$

Bounds on the Magnetic Moments

- Strongest bounds from the "decay" of photons in a stellar plasma:
  - Observed cooling rate of globular cluster stars [Raffelt '99]:

$$2\,[\mu]_{\alpha\beta} \leq 3 \times 10^{-12}\mu_B, \qquad \mu_B = \frac{e}{2m_e}$$

- From ν scattering [Eidelman et. al. '04, Daraktchieva et. al. '05, Schwienhorst et al. '01]

$$2\mu_{e\beta} \leq 0.9{\times}10^{-10}\mu_B, \qquad 2\mu_{\mu\beta} \leq 6.8{\times}10^{-10}\mu_B, \qquad 2\mu_{\tau\beta} \leq 3.9{\times}10^{-7}\mu_B$$

- For transition magnetic moments

$$[\mu]_{\tau\beta} \leq [\mu]_{e\tau} \text{ or } [\mu]_{\mu\tau} \quad \rightarrow \quad 2\mu_{\tau\beta} \leq 6.8 \times 10^{-10}\mu_B$$

- For our numerical estimates we use

$$[\mu]_{\alpha\beta} \leq 10^{-12}\mu_B$$

Generate Magnetic Moments

- Dimensional Analysis
  - Neutrino masses are small:

$$m_\nu \sim 0.1\mathrm{eV} \sim \frac{v^2}{M} \rightarrow M \sim 10^{14}\mathrm{GeV}$$

  - while Magnetic Moments are large:

$$\mu \sim 10^{-12}\mu_B \sim \frac{Bv^2}{M^3} \sim \frac{m_W^2}{8\pi^2 M^3} \rightarrow M \sim 10\mathrm{TeV}$$

- Generate large μ and small m_ν
  - [Voloshin '88]: Use antisymmetry to cancel contribution to symmetric $[m_\nu]_{\alpha\beta}$,

    . . . [Babu, Mohapatra; Georgi, Randall, Cheng et. al. '90]
  - Use angular momentum conservation [Barr et. al.] '90

Mixing of the Operators

ν magnetic moments can be large

They are related to the ν Mass

Study the mixing of magnetic moments into masses

- Fine tuning arguments could give bounds for the magnetic moments
- Could lead to interesting mass matrix structure
- Use the framework of effective field theories to study the contribution of magnetic moment operators in a model independent way

Theoretical Framework: Effective Field Theories

At high scales $\mu_0 \sim M$ there are heavy particles, which generate the magnetic moments and light SM particles:

$$\mathcal{L}_{\rm full} = \mathcal{L}_H(h, l) + \mathcal{L}(l).$$

At a low scale $M_W \sim \mu < \mu_0$ we obtain an effective Lagrangian (the Standard Model plus the $D > 4$ Operators):

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{SM}(l) + \delta\mathcal{L}(l)$$

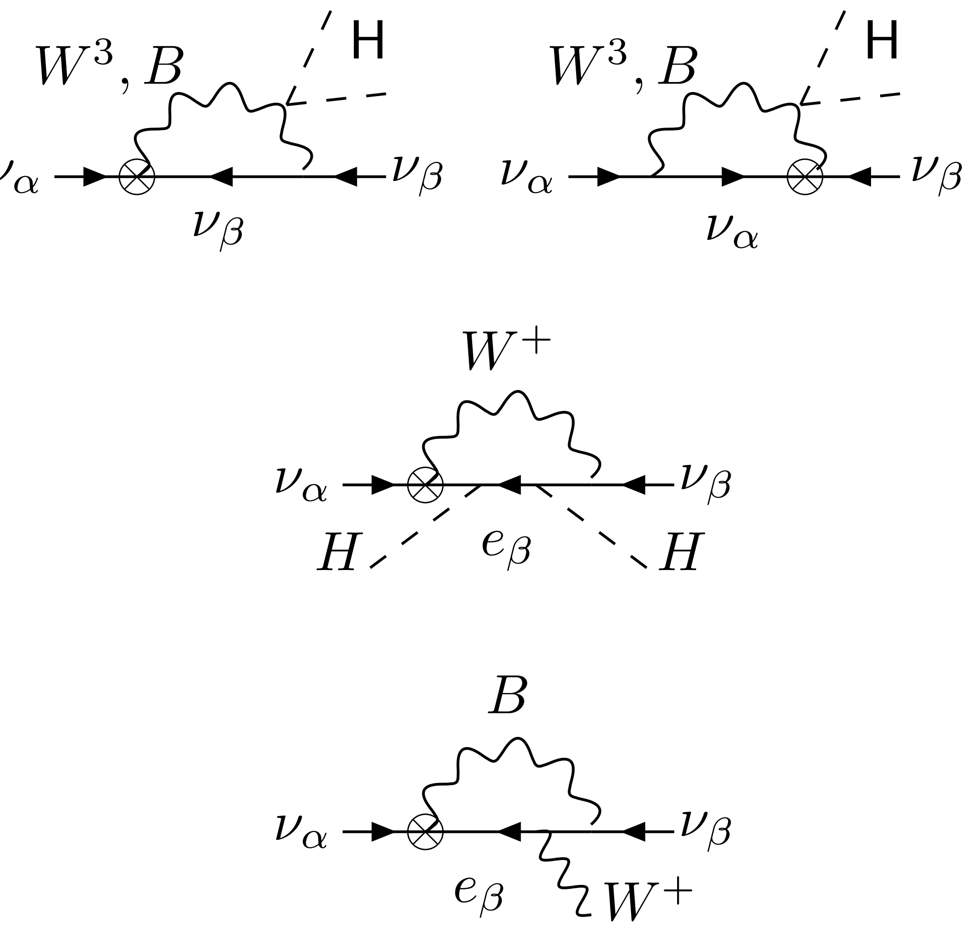
The calculation takes three steps

- Matching of $\mathcal{L}_{\rm full}$ and $\mathcal{L}_{\rm eff}$ at μ_0 gives $\delta\mathcal{L}(L)$; at LO this is model independent $\rightarrow$ constrain arbitrary model which generates the magnetic moments at the high scale
- With the help of the Renormalisation Group Equation (RGE) we can relate the effective Lagrangian at the high scale to the low scale one

$$\mathcal{L}_{\rm eff} \quad \text{at} \quad \mu_0 \rightarrow \mathcal{L}_{\rm eff} \quad \text{at} \quad \mu$$

- Calculation of the matrix elements; at LO this is tree level

One loop Mixing



- Mixing of γ and Z magnetic moments vanish at one loop order,due to $[\mu]_{\alpha\beta}$ being antisymmetric
- For the W^+ magnetic moments the antisymmetry argument is not anymore valid, since the external Higgses are sensitive to e_β
- The B magnetic moment does mix into the W one. This together with the mixing of the W operator will give the leading contribution

Leading Contribution to the Mass Matrix

- Contribution from O_W depends on the charged lepton mass m_α^e

$$\frac{1}{2}[\delta m]_{\alpha\beta} \simeq \left| \frac{m_\alpha^{e2} - m_\beta^{e2}}{m_\tau^2} \frac{[\mu]_{\alpha\beta}}{10^{-12}\mu_B} \right| \log\left(\frac{\Lambda_{NP}^2}{m_W^2}\right) \times .014 \text{ eV}$$

- Contribution from O_B

$$.014 \text{ eV} \to \log\left(\frac{\Lambda_{NP}^2}{m_W^2}\right) \times 1.7 \times 10^{-5} \text{ eV}$$

O_W can be relevant for $[m_\nu]_{\alpha\beta}$

O_B is negligible for $[m_\nu]_{\alpha\beta}$

Results for the Masses

- ν Mass Matrix $[m_\nu]_{\alpha\beta} = U_{\alpha k} U_{\beta k} m_k$ in hierarchical interpretation

$$[m_\nu] \simeq \begin{bmatrix} .30k_2 & .32k_2 + .35s & -.32k_2 + .35s \\ .32k_2 + .35s & .35k_2 + .25 & -.35k_2 + .25 \\ -.32k_2 + .35s & -.35k_2 + .25 & .35k_2 + .25 \end{bmatrix} \times .1\text{eV}$$

$k_2 = e^{2i(\alpha-\beta)} m_2/m_3$, $s = \sin\theta_{13} e^{-i\delta}$, α and β Majorana phases

- Contribution from the O_W magnetic moment $(\tilde{\mu} = \mu/10^{-12}\mu_B)$

$$[\delta m_\nu] \simeq \begin{bmatrix} 0 & 0.004\tilde{\mu}_{e\mu} & \tilde{\mu}_{e\tau} \\ 0.004\tilde{\mu}_{e\mu} & 0 & \tilde{\mu}_{\mu\tau} \\ \tilde{\mu}_{e\tau} & \tilde{\mu}_{\mu\tau} & 0 \end{bmatrix} \times .1\text{eV}$$

Bounds from the Mixing

- Bounds on $[\mu]_{\mu\tau}$ from O_W for hierarchical and inverted scenario:

$$[\mu]_{\mu\tau} \leq 3 \times 10^{-13}$$

- Bounds on $[\mu]_{e\tau}$ from O_W
  - hierarchical

$$[\mu]_{e\tau} \leq 10^{-13}$$

  - inverted

$$[\mu]_{e\tau} \leq 3 \times 10^{-13}$$

- Bounds from O_B on $[\mu]_{e\tau} < 10^{-10}$ less than the experimental bound but exceeds the astrophysical bound

Dirac Magnetic Moments

- No antisymmetry $\rightarrow$ Bounds on Dirac magnetic moments are stronger
- Follow [Bell et. al. hep/ph0504134] there are three operators

$$\begin{aligned} \mathcal{O}_1 &= \bar{L}\tilde{\phi}\sigma^{\mu\nu}\nu_R B_{\mu\nu} \\ \mathcal{O}_2 &= \bar{L}\tau^a\tilde{\phi}\sigma^{\mu\nu}\nu_R W^a_{\mu\nu} \\ \mathcal{O}_3 &= \bar{L}\tilde{\phi}\nu_R\left(\phi^\dagger\phi\right) \end{aligned}$$

- For Dirac magnetic moments they find the following bound

$$\boxed{\mu \leq 10^{-14}\mu_B}$$

Conclusions

- Current bounds for magnetic moment $10^{-10}\mu_B$ (lab) $3 \times 10^{-12}\mu_B$ (astro)
- Two types of $\mathrm{SU}(2) \times \mathrm{U}(1)$ magnetic moments O_W and O_B
- For O_W and $\delta m_\nu \leq m_\nu$
  - non-degenerate: $\mu_{\alpha\tau} \leq 3 \times 10^{-13}$
  - hierarchical: $\mu_{e\tau} \leq 10^{-13}$
- For O_B best limit comes from astro physics
- Stronger bounds for Dirac magnetic moments $\mu \leq 10^{-14}$