Signals of TeV Gravity at Neutrino Telescopes

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OUTLINE:

1. Motivation
2. TeV gravity
3. BH versus eikonal events
4. Signals at neutrino telescopes
5. Conclusions

Based on: *PRL* **93**, 151102 [hep-ph/0402279] and *PRD* **72**, 024003 [hep-ph/0504234]

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$$p + \gamma_{2.7K} \to \Delta^+ \to n + \pi^+ \; (p + \pi^0)$$

$$\hookrightarrow \nu, \;\; \gamma \;\; \ldots$$

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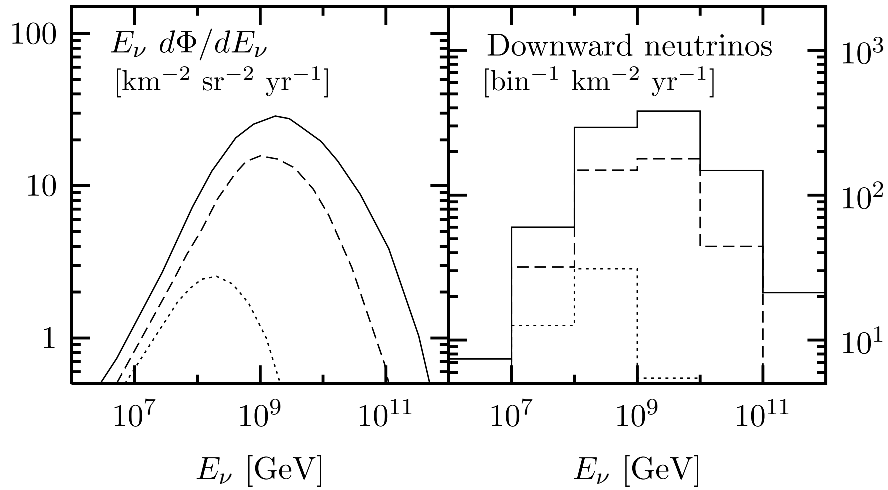

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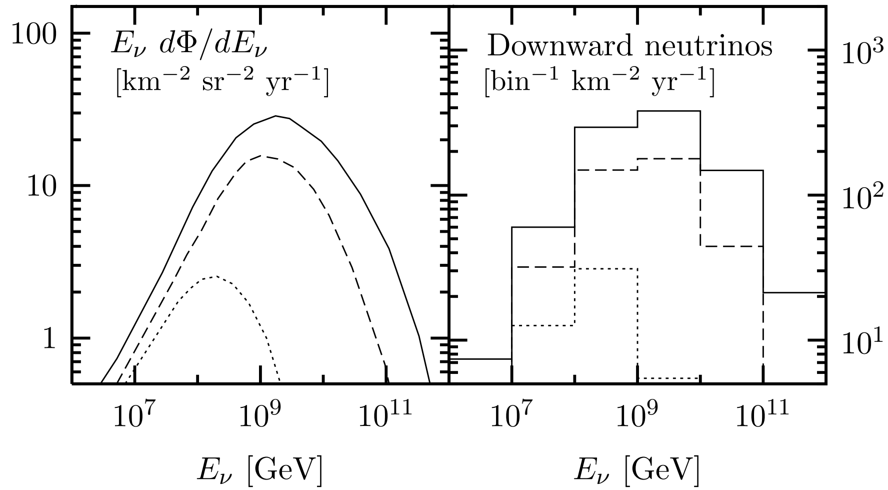

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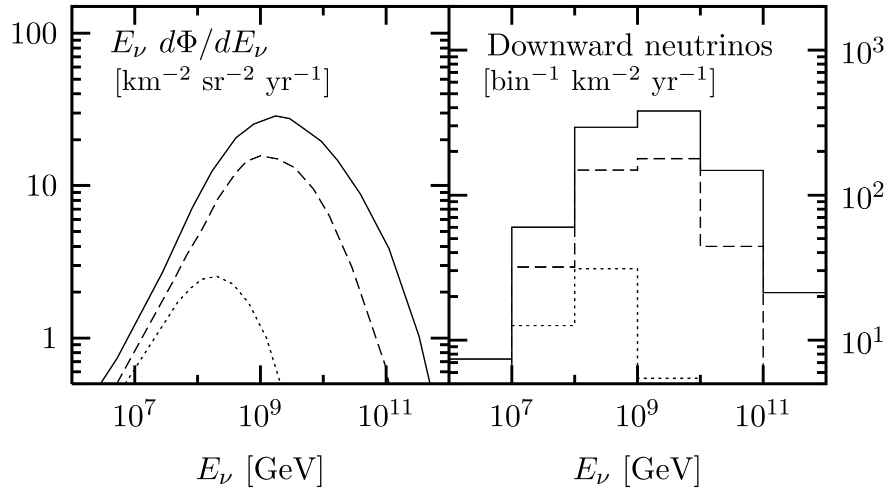

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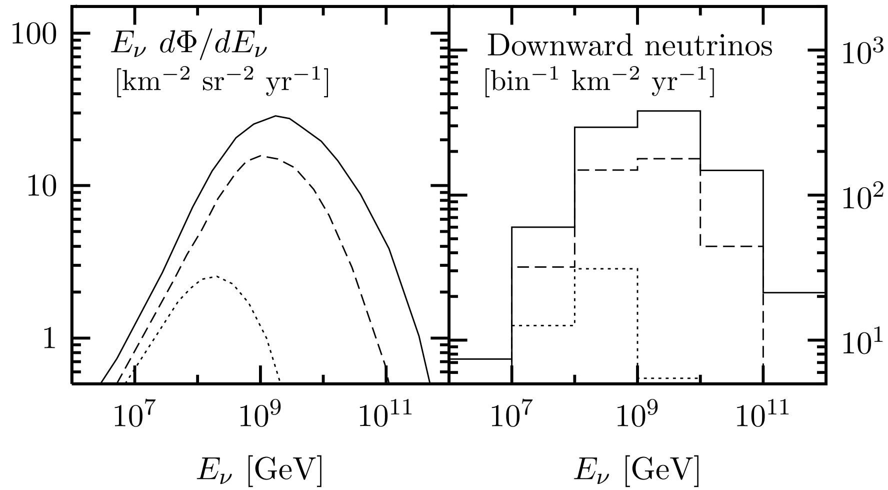

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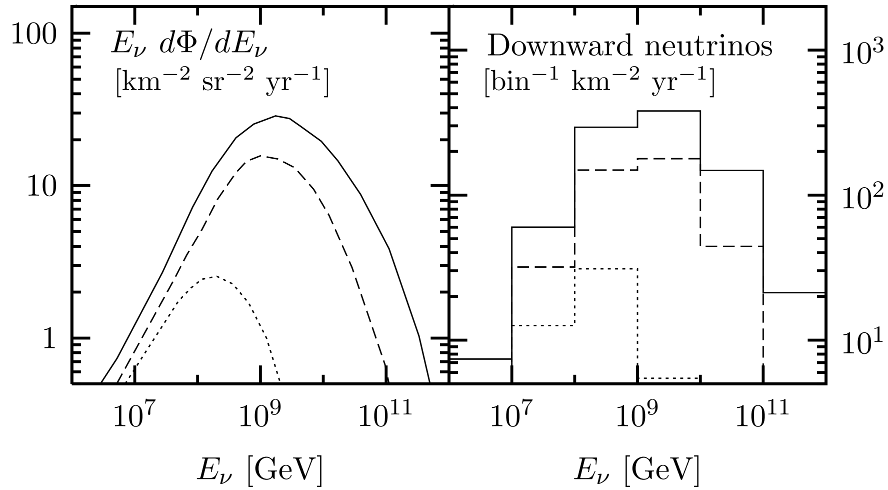

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- At larger distances ($b > R_S$) transplanckian gravitational interactions are elastic, cross sections are reliable (eikonal approximation) and independent of compactification details
- Neutrino telescopes are ideal experiments to detect these elastic processes

TeV gravity

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Two parameters: number $\boxed{n}$ of compact extra dimensions (only *seen* by gravity) and $\boxed{M_D}$:

(*e.g.* toroidal) $G_D \equiv (2\pi R)^n G_N = \frac{(2\pi)^n}{8\pi M_D^{n+2}}$, $G_N \equiv \frac{1}{M_P^2}$ $\Rightarrow M_P^2 = (8\pi R^n) M_D^{n+2}$

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Size R of extra dims if $M_D \sim 1$ TeV:
(isotropic toroidal compactification)

n	1	2	3	...	6
R	10^8 km	0.1 mm	1 nm	...	1 fm
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- Collider bounds (LEP, Tevatron): $M_D \gtrsim 1.4\ (0.6)$ TeV if $n = 2\ (6)$
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In transplanckian collisions: direct probe of M_D. Extra dimensions effectively 'infinite'

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  i.e. deflection angle $\theta^* \sim \frac{\sqrt{s}}{M_D^{n+2} b^{n+1}} \sim \left(\frac{R_S}{b}\right)^{n+1} \ll 1 \Rightarrow R_S \ll b$

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  $\Rightarrow$ Classical physics: small curvature, linearized gravity (negligible non-linear couplings)
- A new scale emerges if $n > 0$: $b_c \sim \frac{1}{M_D}\left(\frac{\sqrt{s}}{M_D}\right)^{\frac{2}{n}}$ $\Rightarrow$ two scattering regions:
  - $R_S \ll b \ll b_c$ (*i.e.* $q \gg b_c^{-1}$): classical trajectory
  - $b \gg b_c > R_S$ (*i.e.* $q \ll b_c^{-1}$): quantum mechanical effects (but not quantum gravity)

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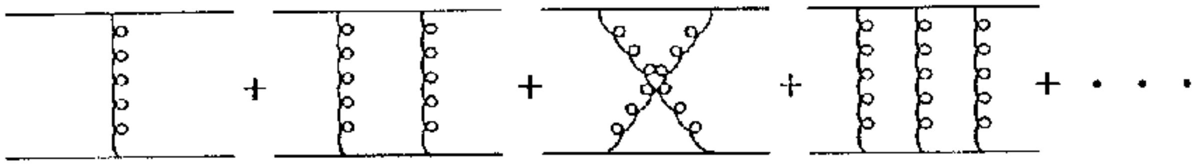
Elastic scatteing ν–(q, $\bar{q}$, g in N) exchanging D–dim gravitons ['t Hooft '87, Amati, Ciafaloni, Veneziano '87, Kabat, Ortiz '92]

$$\mathcal{A}_{\text{Born}} = -\frac{s^2}{M_D^{n+2}} \underbrace{\int \frac{d^n q_T}{t - q_T^2}}_{\text{KK tower}}$$



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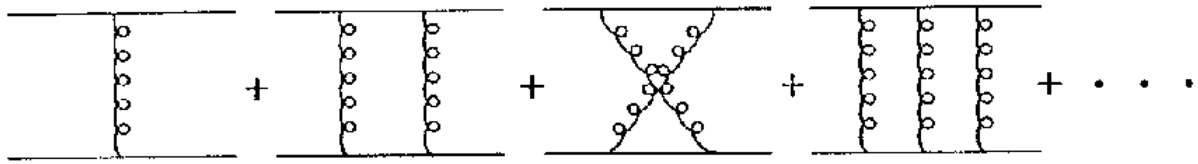

Resumming ladder and cross-ladder diagrams taking $-t/s \ll 1$:

$$\mathcal{A}_{\rm eik}(s,t) = \frac{2s}{i} \int d^2 b\ {\rm e}^{i q\cdot b} \left({\rm e}^{i\chi(s,b)} - 1\right), \quad \chi(s,b) = \frac{1}{2s} \int \frac{d^2 q}{(2\pi)^2} {\rm e}^{-iq\cdot b} \mathcal{A}_{\rm Born}(s,q^2)$$

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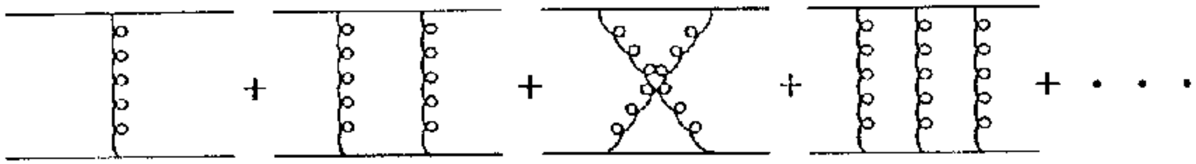

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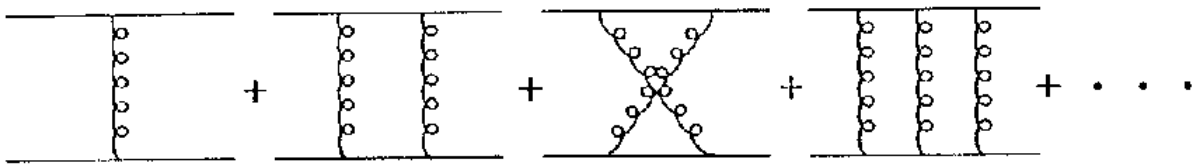

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UV cutoff dependence

Integrating q_T up to an UV cutoff Λ (*e.g.* M_D or $M_{\rm string}$), the eikonal phase gets:

$$\chi(s,b) = \left(\frac{b_c}{b}\right)^n \left[1 - \sqrt{\frac{\pi}{2\Lambda b}} {\rm e}^{-\Lambda b} A_n(\Lambda b)\right], \quad b_c^n \equiv \frac{(4\pi)^{\frac{n}{2}-1}}{2} \Gamma\left(\frac{n}{2}\right) \frac{s}{M_D^{n+2}}$$

with $A_n(\Lambda b \to \infty) \to 2^{2-n}/\Gamma^2(n/2)$

$\Rightarrow$ Cutoff dependence negligible as long as $b \gg \Lambda^{-1}$ $(b \gg R_S > M_D^{-1})$ $\surd$

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$$u\ll 1:\quad F_{n>2}(u)\to \frac{i}{2}\Gamma\left(1-\frac{2}{n}\right)\mathrm{e}^{-i\pi/n},\quad F_2(u)\to -\ln\frac{u}{1.4}+i\frac{\pi}{4}$$

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- Always non-perturbative: $\chi \gtrsim 1$

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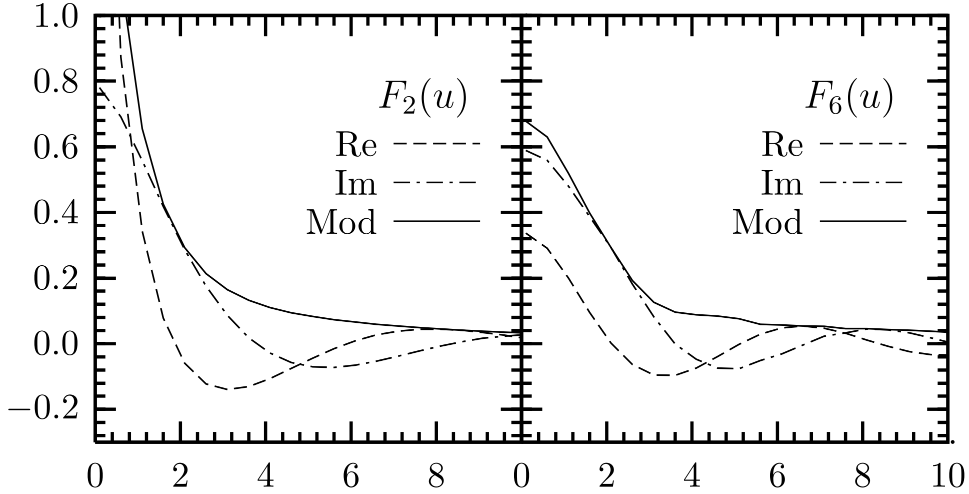

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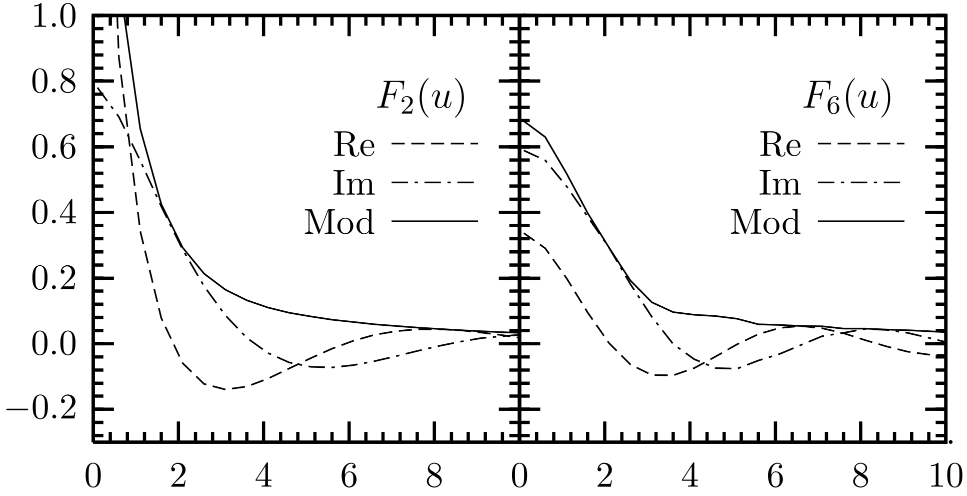

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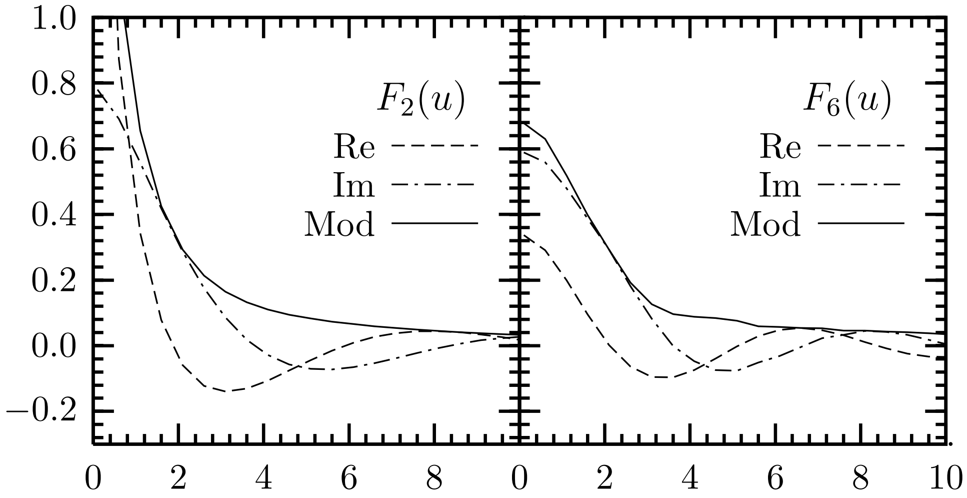

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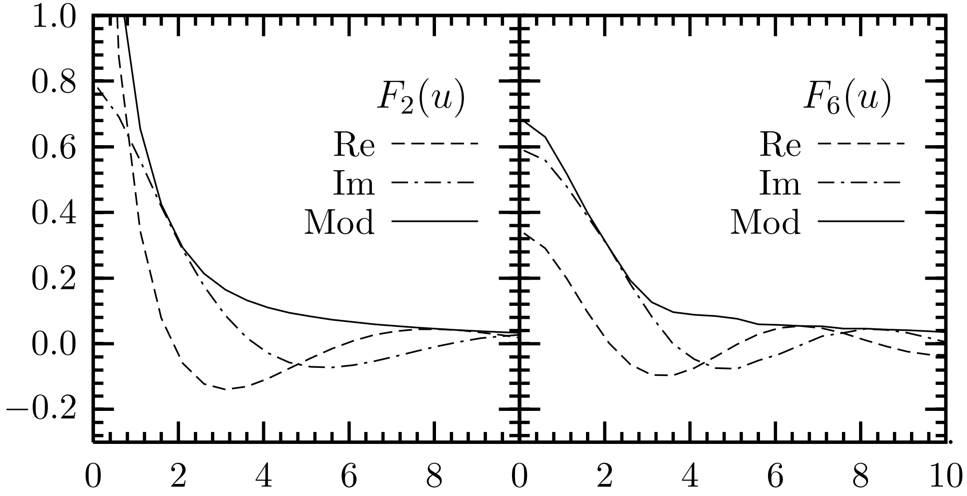

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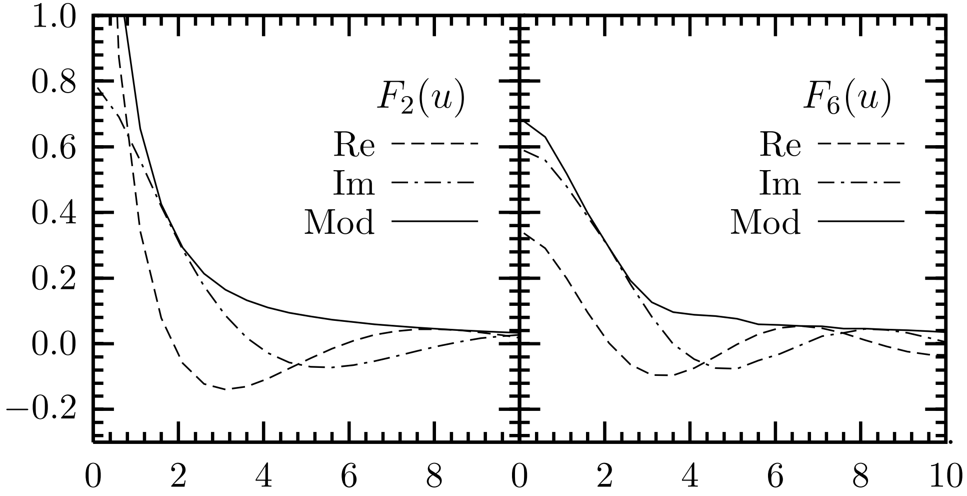

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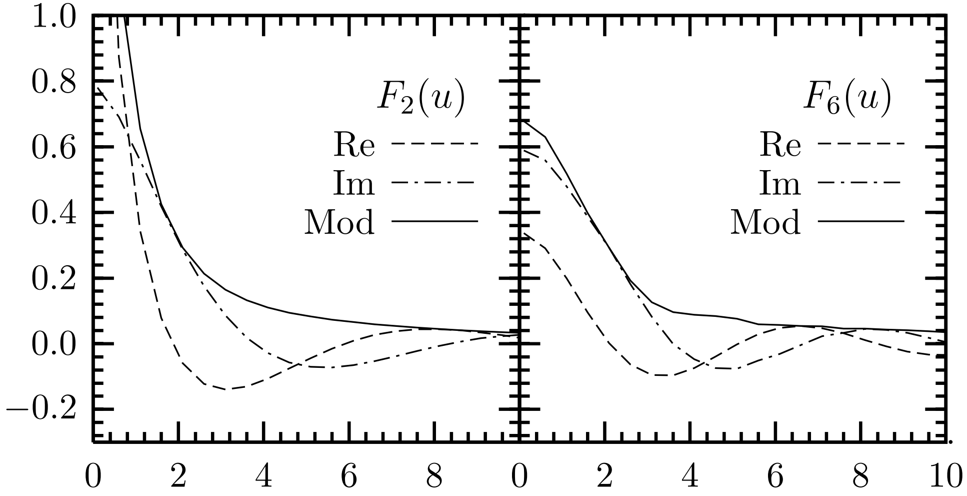

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- Black Hole: $\sigma_{\rm BH} = \pi R_S^2 \sim \hat{s}^{\frac{1}{n+1}} \lesssim \sigma_{\rm eik}$

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- (Soft) graviton emission $\leftrightarrow$ $\mathrm{Im}(\chi_H)$ negligible if $y = q^2/\hat{s} \ll 1$: [Giudice, Rattazzi, Wells '02]

[Amati, Ciafaloni, Veneziano '90]

$$b_r \equiv (b_c^n R_S^{2n+2})^{\frac{1}{3n+2}}, \quad N_{\text{soft}} = \mathrm{Im}(\chi_H) \approx \left(\frac{b_r}{b}\right)^{3n+2} \sim y^{\frac{3n+2}{2n+2}} \left(\frac{\hat{s}}{M_D^2}\right)^{\frac{n+2}{2n+2}}$$

with transverse momentum: $Q \sim \dfrac{N_{\text{soft}}}{b}, \quad b_s^{-1} \sim M_D \left(\dfrac{y M_D^2}{\hat{s}}\right)^{\frac{1}{n+2}}$

Radiated energy in lab frame (νN, N at rest): $E_{\text{rad}} = \gamma Q, \quad \gamma = \sqrt{\dfrac{E_\nu}{2 x m_N}}$

νN cross sections

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- Hard processes (BH formation): initial neutrino destroyed ($b \lesssim R_S$)

$$\sigma_{\rm BH}^{\nu N} = \int_{M_D^2/s}^{1} dx \; \pi R_S^2(\hat{s}) \sum_{i=q,\bar{q},g} f_i(x,\mu), \quad \hat{s} = xs, \quad s = 2 m_N E_\nu$$

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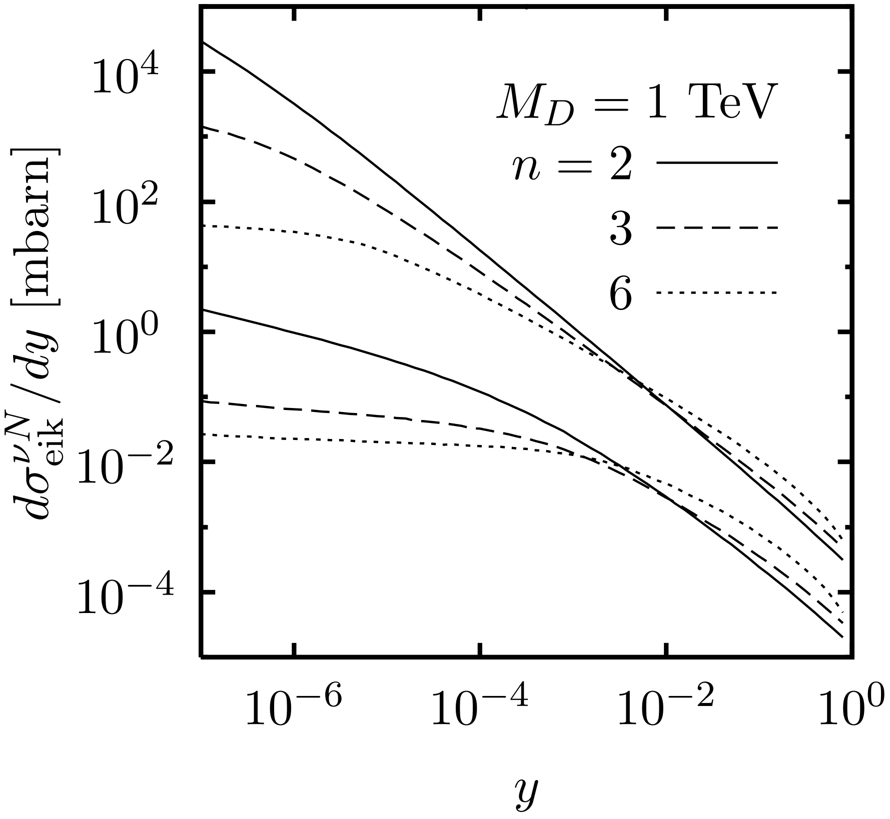
- Soft processes (eikonal): neutrino loses small fraction y of its energy and keeps going

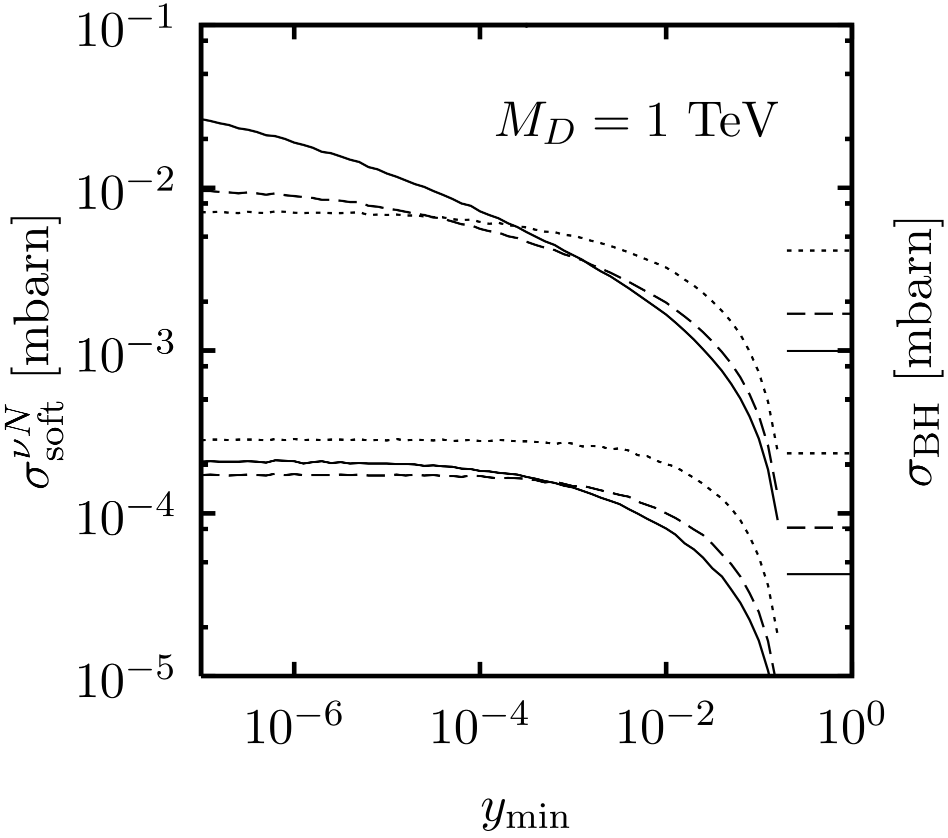
$$\frac{d\sigma_{\rm eik}^{\nu N}}{dy} = \int_{M_D^2/s}^{1} dx \; \hat{s} \; \pi b_c^4 |F_n(b_c q)|^2 \sum_{i=q,\bar{q},g} f_i(x,\mu), \quad y = \frac{E_\nu - E'_\nu}{E_\nu}$$

$$q = xys, \quad \mu = \langle b \rangle^{-1}, \quad \langle b \rangle \equiv \begin{cases} b_s & , \quad q > b_c^{-1} \\ b_c & , \quad q < b_c^{-1} \end{cases}$$

$$\sigma_{\rm soft}^{\nu N} = \int_{y_{\rm min}}^{y_{\rm max}} dy \; \frac{d\sigma_{\rm eik}^{\nu N}}{dy}, \quad y_{\rm max} \equiv 0.2 \quad (\text{eik: } b \gtrsim R_S), \quad y_{\rm min} = E_{\rm thres}/E_\nu$$

BH versus eikonal events

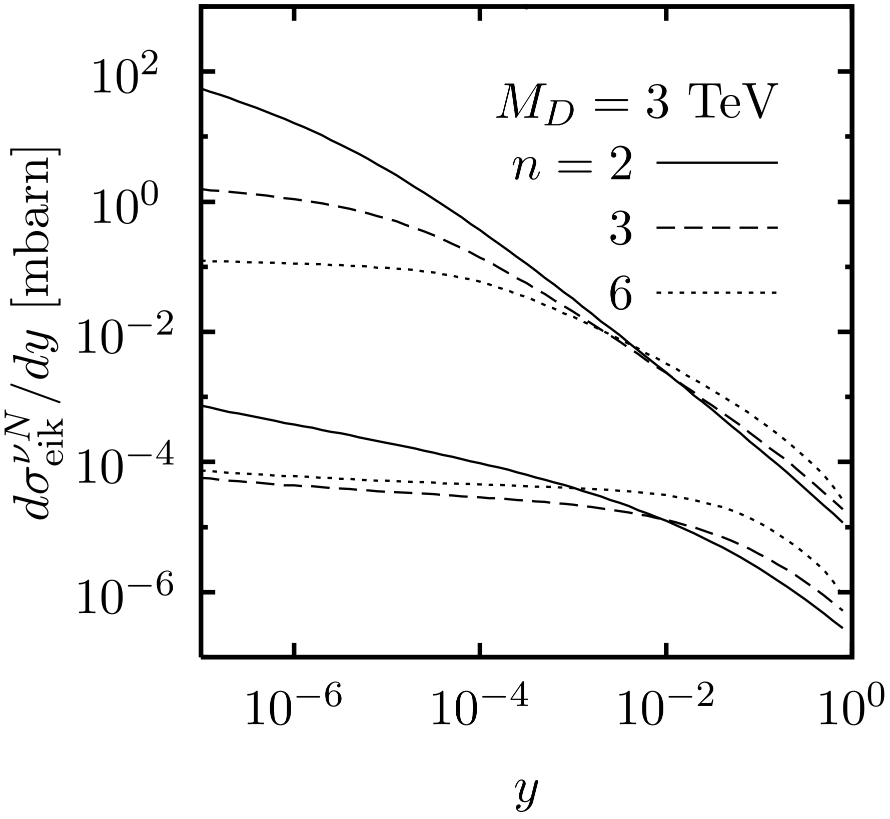


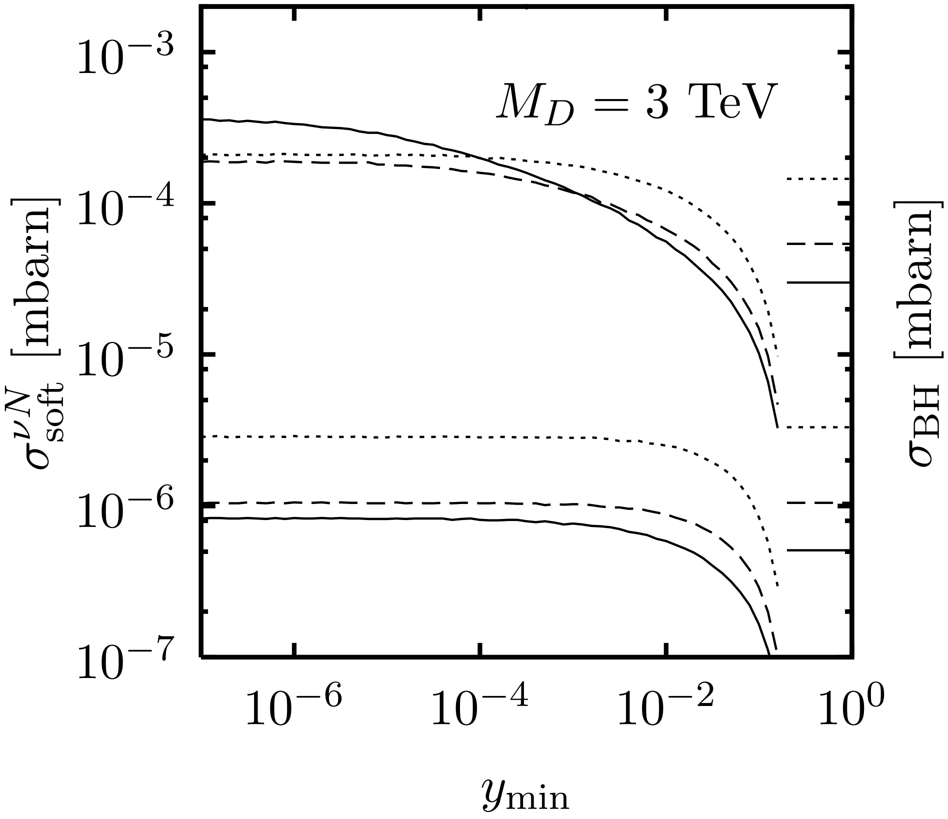


Upper lines: $E_\nu = 10^{10}$ GeV

Lower lines: $E_\nu = 10^8$ GeV

BH versus eikonal events





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Example

One UHE neutrino of $E_\nu = 10^{10}$ GeV with $E_{\rm thres} = 100$ TeV and $M_D = 1$ TeV

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	$n=2$		$n=6$	
$\sigma_{\rm BH}$	9.82×10^{-4}	mbarn	4.07×10^{-3}	mbarn
$\sigma_{\rm soft}$	1.94×10^{-2}	mbarn	6.88×10^{-3}	mbarn
$L_{\rm BH}/L_{\rm eik}$	19.8		1.69	
$L_{\rm BH}$ in ice	17	km	4	km
$E^{\rm loss}_{\rm eik}$ in $L_{\rm BH}$	1.0×10^9	GeV	5.0×10^8	GeV
$E^{\rm loss}_{\rm rad}$ in $L_{\rm BH}$	1.6×10^9	GeV	4.9×10^8	GeV
$E^{\rm loss}_{\rm eik}$ in 1 km	5.9×10^7	GeV	1.2×10^8	GeV
$E^{\rm loss}_{\rm rad}$ in 1 km	9.2×10^7	GeV	1.2×10^8	GeV

Signals at neutrino telescopes

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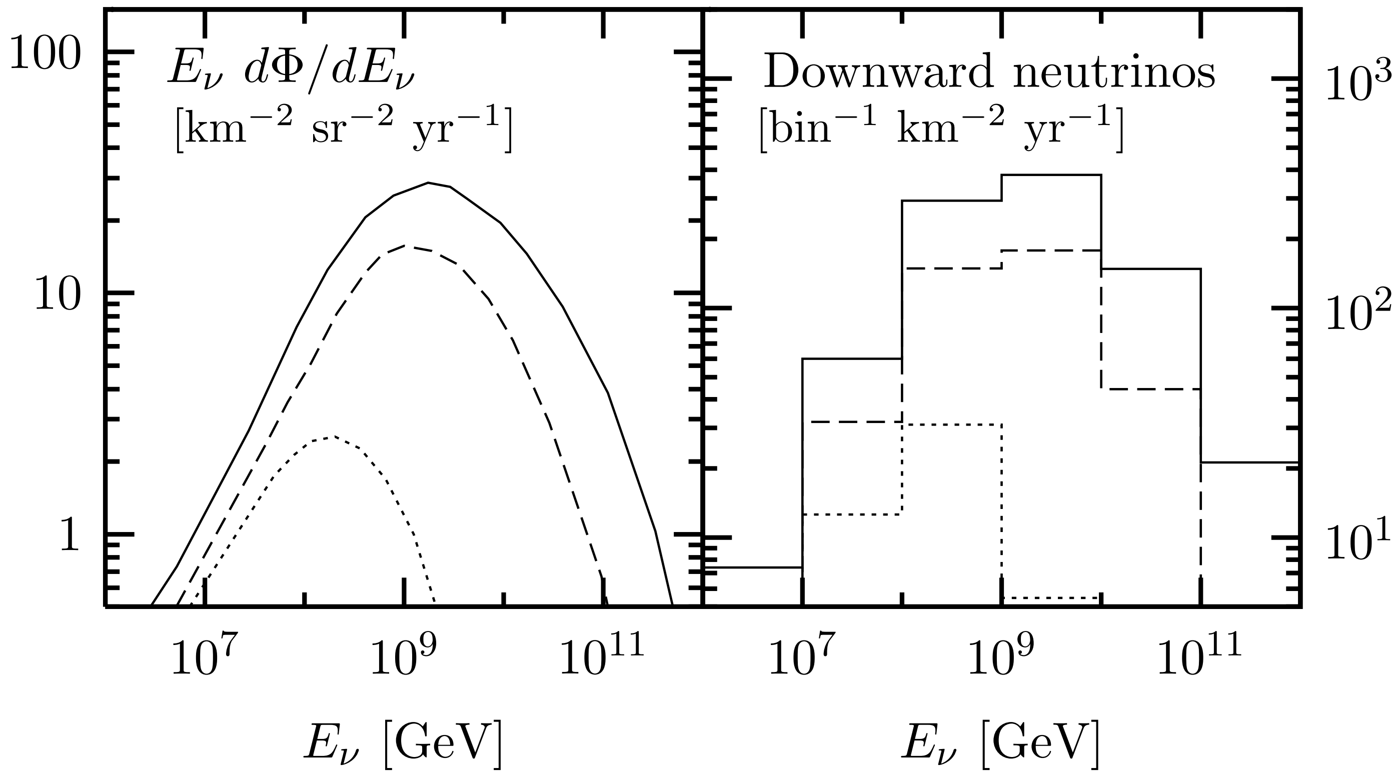
- Cosmogenic neutrinos per flavour (consistent with p and γ at AGASA/HiRes and EGRET)



Higher :	100 % EGRET	820 km^{-2} yr^{-1} in $[10^8, 10^{11}]$ GeV		
Lower :	20 % EGRET	370 km^{-2} yr^{-1}	"	[Semikoz, Sigl '04]
Minimal :	No protons above E_{GZK}	35 km^{-2} yr^{-1}	"	[Fodor, Katz, Ringwald '03]

- Survival probability: $P_{\rm surv}(E_\nu,\theta_z) = {\rm e}^{-x(\theta_z) N_A (\sigma_{\rm SM}+\sigma_{\rm BH})}$

  Column density: $x(\theta_z) = \int_{\theta_z} dl\ \rho(l,\theta_z) \approx \rho_{\rm ice} d(\theta_z, d_v, R_\oplus)$ (atmosphere negligible)

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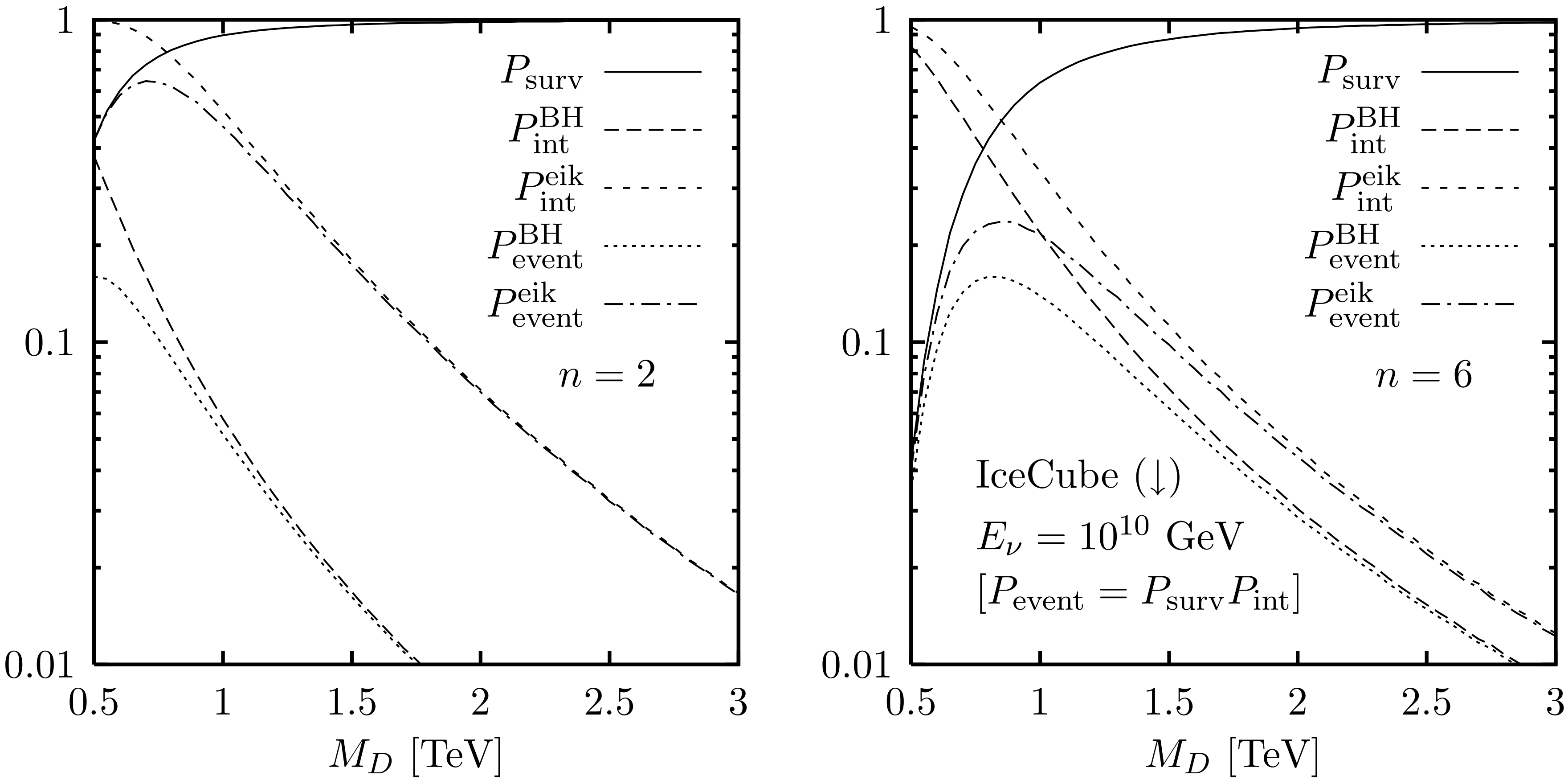

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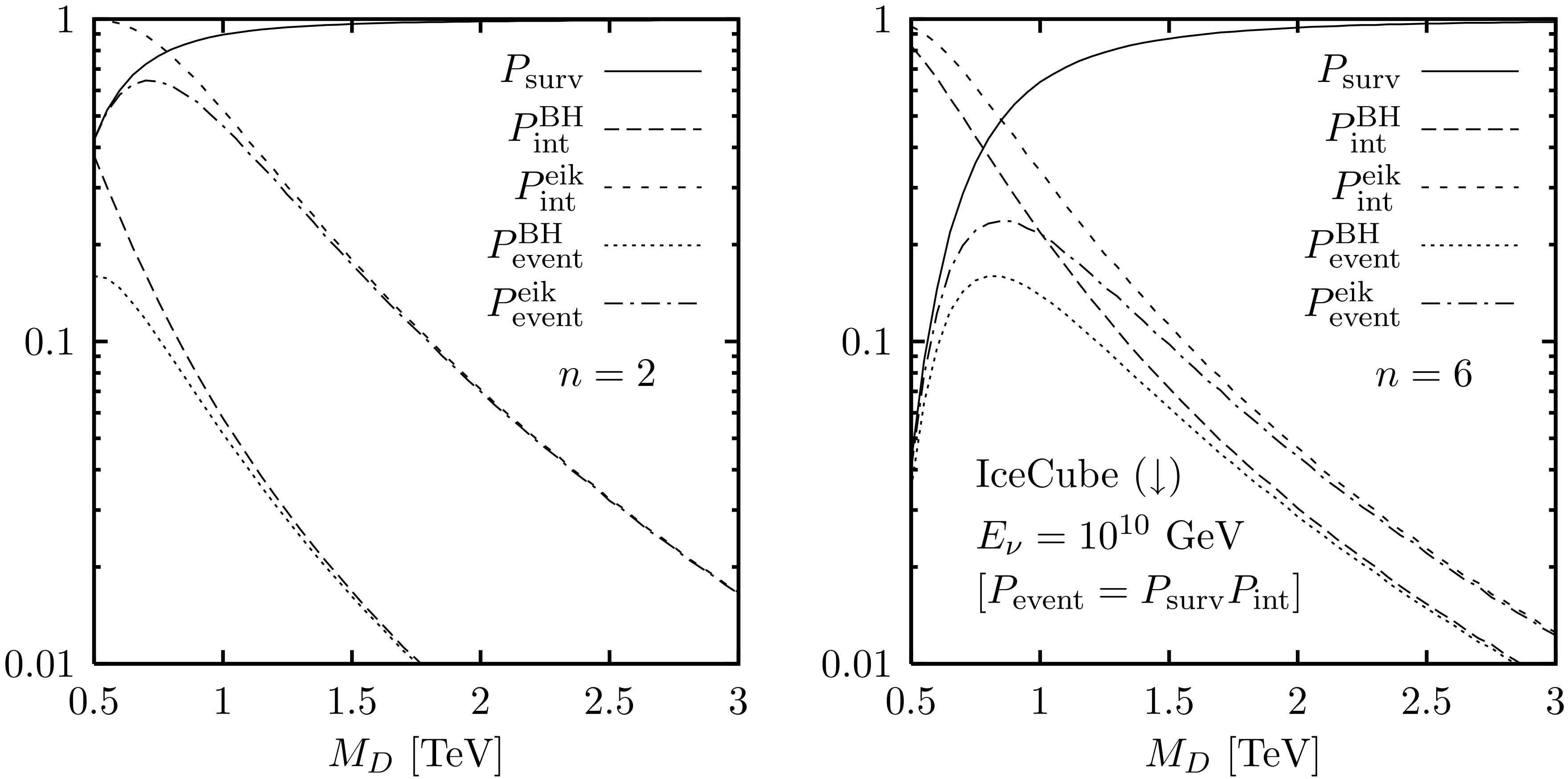
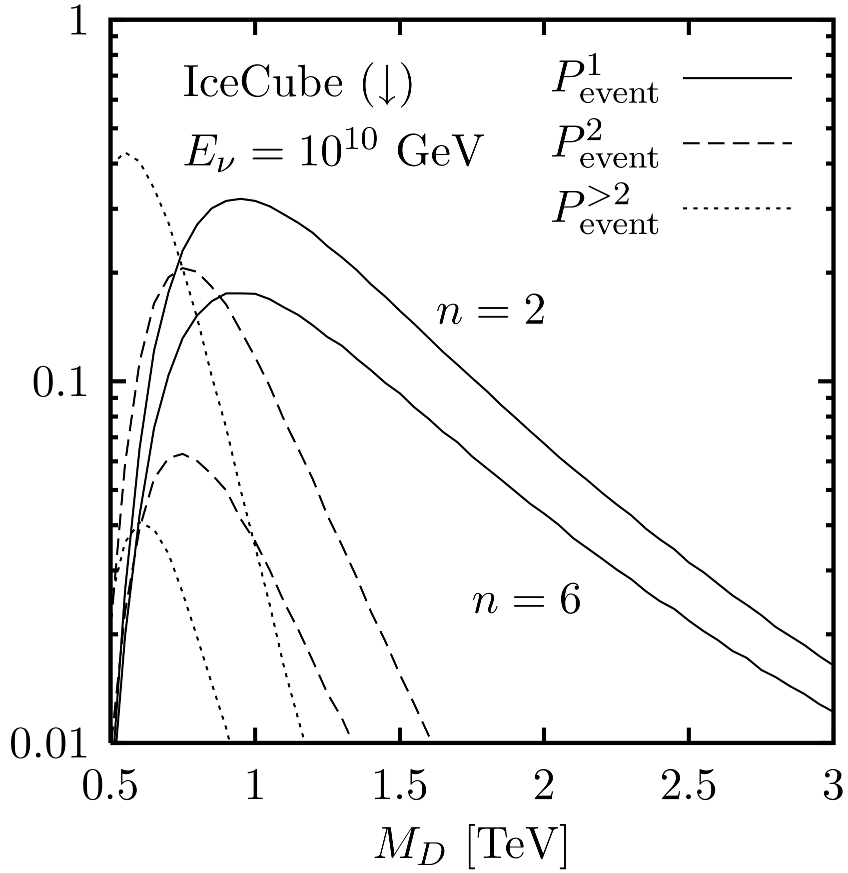


- $N_{\rm events} = 2\pi A T \int dE_\nu \sum_{\nu_i,\bar\nu_i} \frac{d\phi_{\nu_i}}{dE_\nu} \int d\cos\theta_z P_{\rm surv} P_{\rm int}$ in time T for detector area A

- Multiple-bang events

If detector L larger than interaction length $L_0 = (\rho N_A \sigma_{\rm eik})^{-1}$, prob of $N > 1$ bangs:

$[P^N_{\rm event} = P_{\rm surv} P_N]$



$$P_N(L) = {\rm e}^{-L/L_0} \frac{(L/L_0)^N}{N!}$$

Probability of at least one interaction:

$$P_{\rm int} = \sum_{N=1}^{\infty} P_N = 1 - {\rm e}^{-L/L_0}$$

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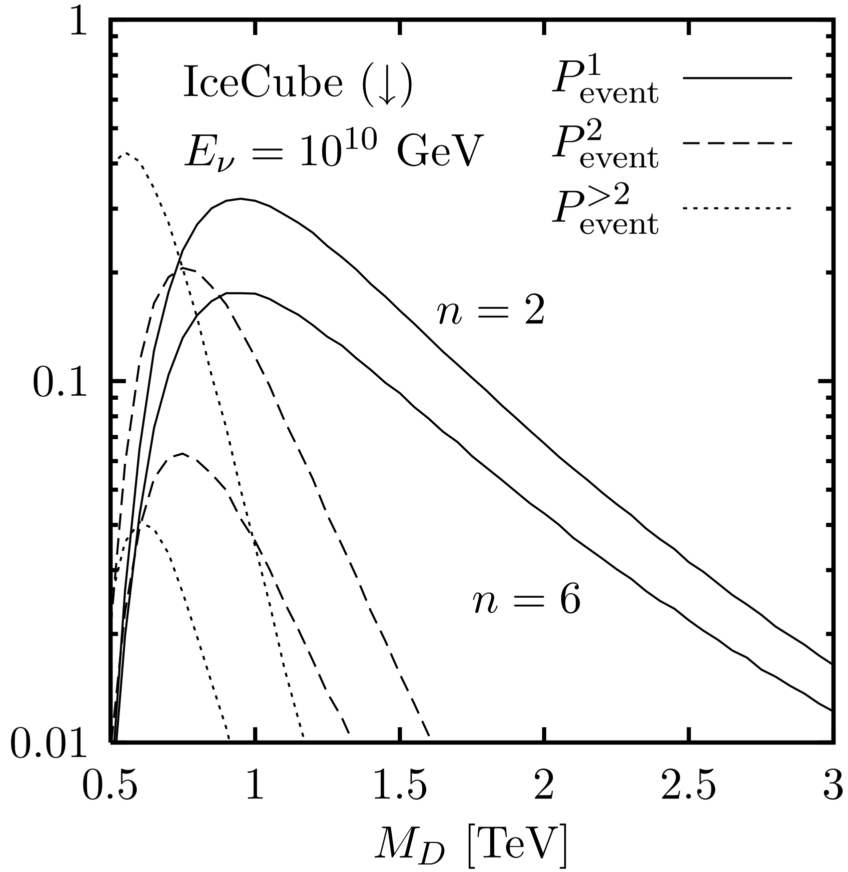

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– In a SM CC interaction (or in BH evaporation) a double-bang ν_τ event may occur only if $2.5 \times 10^6 < E_\tau/\text{GeV} < 10^7$ in IceCube $[125 \text{ m} < c\tau < 1 \text{ km}]$. Prob is just 6.8×10^{-5}

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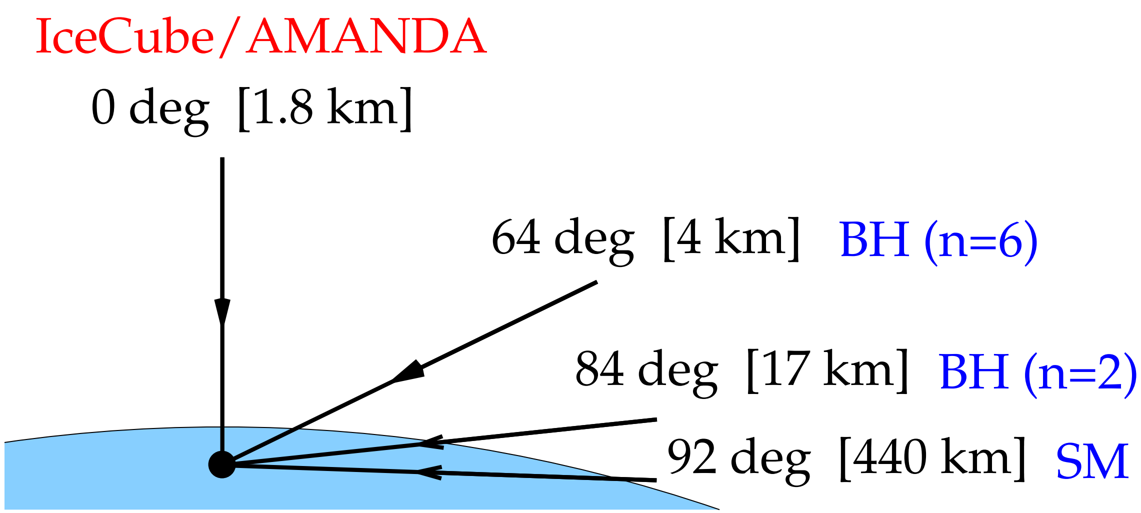

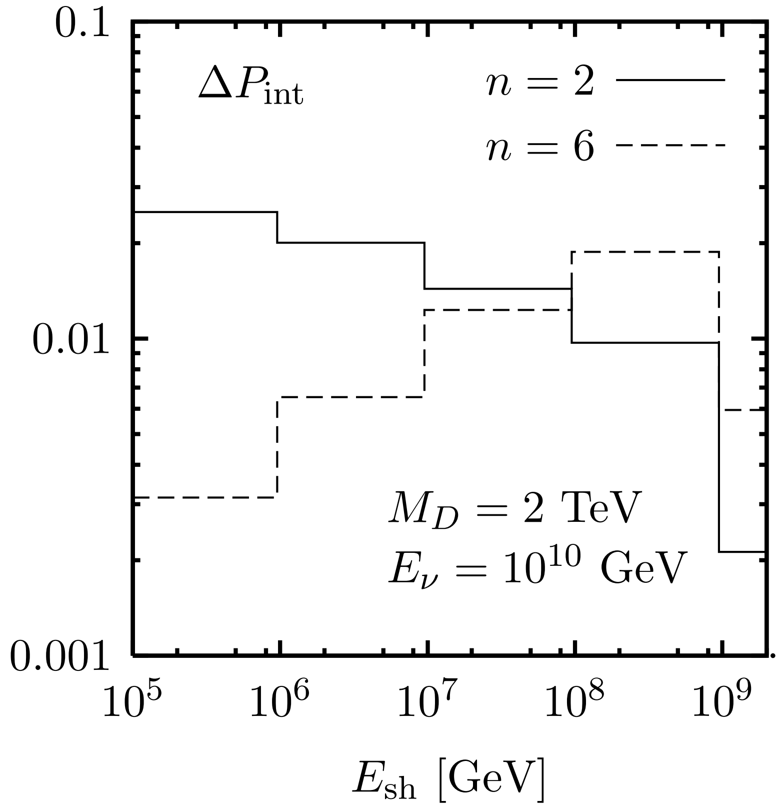

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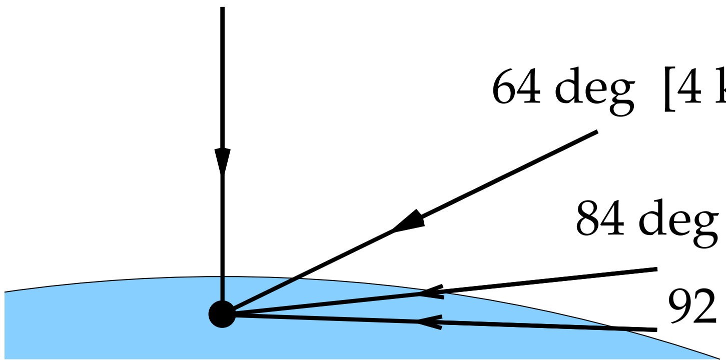

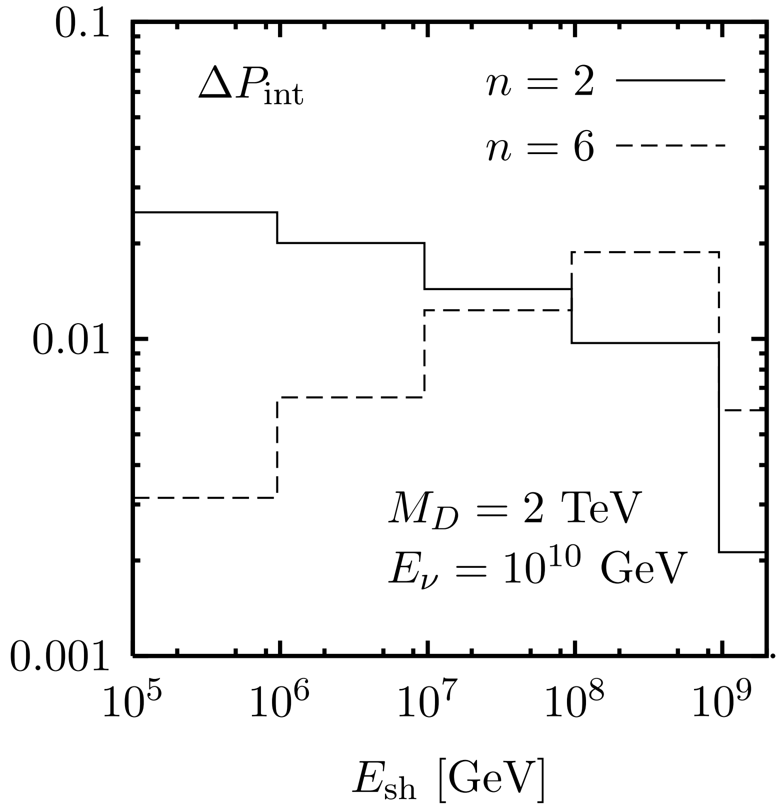

$L = 1$ km	$P_{\rm int}^{\rm SM} = 0.0022$
$M_D = 1$ TeV $n = 2\ (6)$	$P_{\rm int}^{\rm BH} = 0.06\ (0.22)$ $P_1^{\rm eik} = 0.36\ (0.27)$ $P_2^{\rm eik} = 0.15\ (0.06)$ $P_{>2}^{\rm eik} = 0.05\ (0.008)$

Bounds from air shower experiments

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- Hadronic shower energy (in air or ice):
  - BH evaporation: $E_{\rm sh} \approx 0.8 E_\nu$ (around 80% to hadrons)
  - Eikonal events: $E_{\rm sh} = y E_\nu$ (typically $y \ll 1$)

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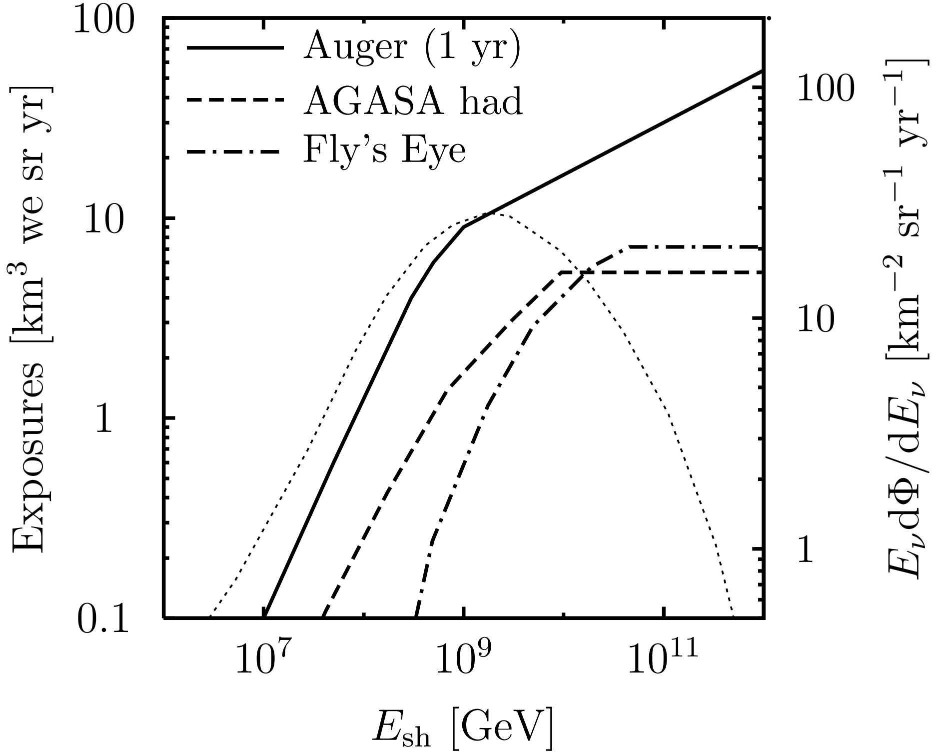
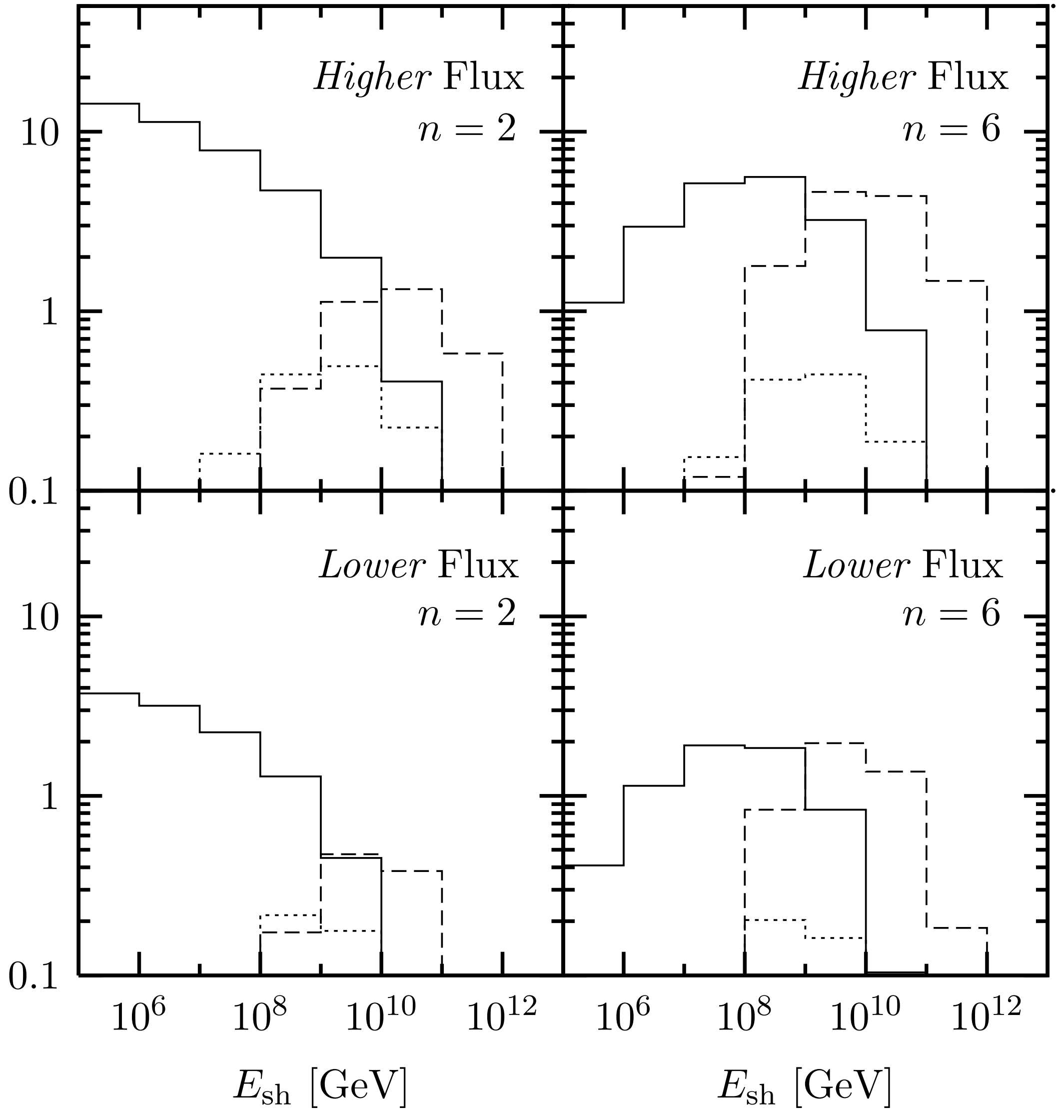
  - BH evaporation: $E_{\rm sh} \approx 0.8 E_\nu$ (around 80% to hadrons)
  - Eikonal events: $E_{\rm sh} = y E_\nu$ (typically $y \ll 1$)
- Air shower experiments sensitive to $E_{\rm sh} \gtrsim 10^9$ GeV



$\Rightarrow$ Limits on M_D from eikonal events similar to those from BH production: $M_D \gtrsim 1\ (1.5)$ TeV for $n = 2\ (6)$

Shower energy distribution at IceCube

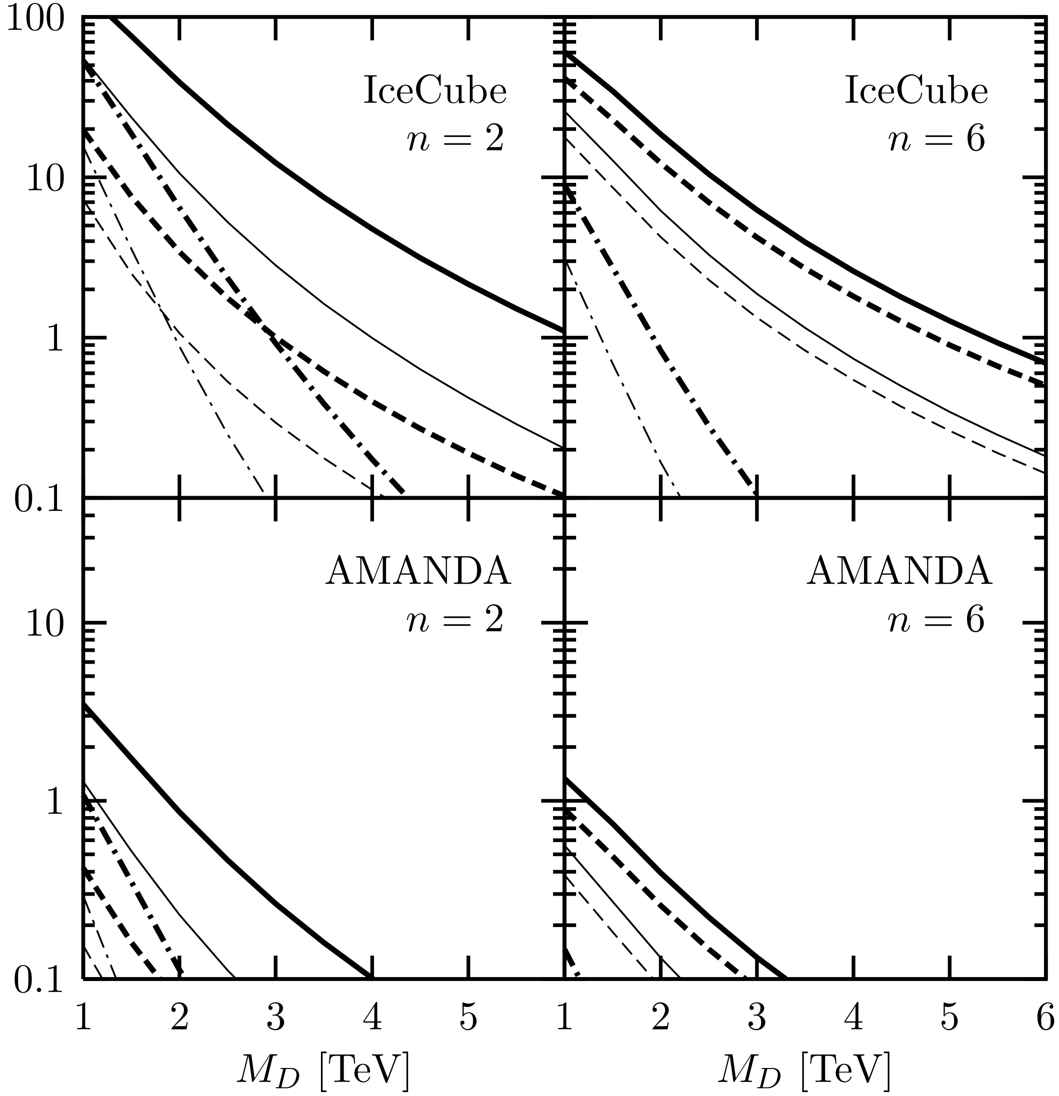


Number	↓	SM (dotted) CC	NC	SM	2-bang
HF	910	0.94	0.38	1.32	0.008
LF	410	0.36	0.14	0.50	0.003

Events per bin and year for $M_D = 2$ TeV for $n = 2\ (6)$:

Number	BH (dashed)	Eikonal (solid)
HF	3.43 (12.2)	39.1 (18.5)
LF	1.07 (4.25)	10.6 (6.20)

Contained events at IceCube and AMANDA



IceCube:	$L = 1$ km	$A = 1$ km^2
AMANDA:	$L = 0.7$ km	$A = 0.03$ km^2

Contained events per year:

Higher Flux (thick)

Lower Flux (thin)

Eikonal (solid)

Multi-bang (dashed-dotted)

BH (dashed)

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