Primordial non-gaussianity

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Overview

1. Defining the primordial curvature perturbation ζ.
2. Defining the non-gaussianity of ζ.
3. The magic formula for ζ.
4. The three main scenarios.

My main messages

To particle theorists I will show you how to calculate the primordial non-gaussianity predicted by your model for the origin of the curvature perturbation. (Cf. spectral tilt $n - 1 = 2\eta - 6\epsilon$, Liddle/DHL 1992).

To astronomers The trispectrum, even higher correlators, could be as important as the bispectrum. I will give you the dependence of each correlator on the wave-vectors.

Defining the curvature perturbation ζ

1. Smooth the Universe on the scale 10^{-2} Mpc.
2. Consider $t < 100$ s (before horizon entry $\equiv$ 'primordial')
3. Work on slicing of uniform energy density ρ
4. Define local scale factor: $g_{ij} = \tilde{a}^2(\mathbf{x}, t)\delta_{ij}$
5. Define $\zeta \equiv \ln \tilde{a}(\mathbf{x}, t) - \ln a(\mathbf{x}, t) \equiv \delta N$

- In words, ζ is the perturbation in the number of e-folds of expansion, starting from a *flat* slice.
- ζ is conserved iff pressure $P(\rho)$ is a unique function (adiabatic).
- Its constant value at $t \sim 10$ s determines (?) all observables.

The correlators

- Spectrum $\mathcal{P}$
- Bispectrum $f_{\rm NL}$ (Komatsu/Spergel 2000)
- Trispectrum $\tau_{\rm NL}$ (Boubekeur/DHL 2005)

$$\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \rangle = (2\pi)^3 \delta(\mathbf{k}+\mathbf{k}') K \mathcal{P} \quad (1)$$

$$-\frac{3}{5} \langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \zeta_{\mathbf{k}''} \rangle = (2\pi)^3 \delta(\mathbf{k}+\mathbf{k}'+\mathbf{k}'') K_2 \mathcal{P}^2 f_{\rm NL} \quad (2)$$

$$\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \zeta_{\mathbf{k}''} \zeta_{\mathbf{k}'''} \rangle_{\mathbf{c}} = (2\pi)^3 \delta(\mathbf{k}+\mathbf{k}'+\mathbf{k}''+\mathbf{k}''') K_3 \mathcal{P}^3 \tau_{\rm NL} \quad (3)$$

where the kinematic factors depend on the wave-vectors:

$$K \equiv k^3/2\pi^2 \quad (4)$$

$$K_2 \equiv K(k)K(k') + \text{ 5perms} \quad (5)$$

$$K_3 \equiv K(k)K(k'')K(|\mathbf{k}+\mathbf{k}'|) + 23\text{perms} \quad (6)$$

Theory and Observation

SLOW-ROLL INFLATION PREDICTION Spectrum $\mathcal{P}$, bispectrum $f_{\rm NL}$ and trispectrum $\tau_{\rm NL}$ are almost scale-independent.

OBSERVATION:

1. $\mathcal{P} = (5 \times 10^{-5})^2$ (WMAP+SDSS)
2. $|f_{\rm NL}| \lesssim 100 \ll \mathcal{P}^{-1/2}$ (WMAP)
3. $|\tau_{\rm NL}| \lesssim 10^8 \ll \mathcal{P}^{-1}$ (COBE (!!))

- From 2 and 3, ζ is almost gaussian.
- Observation eventually will give $|f_{\rm NL}| \lesssim 1$ (or detection) and $|\tau_{\rm NL}| \lesssim ???$ (or detection). A *strong discriminator* between models for the origin of ζ.

Magic formula 1 for ζ

Sasaki/Stewart 1995, DHL/Malik/Sasaki 2005)

$$\zeta(\mathbf{x}, t) = \delta N(\mathbf{x}, t). \tag{7}$$

Here $N(\phi_i(\mathbf{x}), \rho(t))$ is number of e-folds of expansion *starting* from a flat slice just after horizon exit (field values $\phi_i(\mathbf{x})$) and *ending* on a uniform-density slice at time t. We're interested in $t \sim 10\,$s.

- The magic formula gives ζ knowing evolution of a family of unperturbed local universes (separate universe picture, DHL/Malik/Liddle/Wands 1999).
- It's not cosmological perturbation theory!

Magic formula 2 for ζ

(Sasaki/Stewart 1995, DHL/Rodriguez 2005)

$$\zeta(\mathbf{x},t) = \sum_i \frac{\partial N}{\partial \phi_i}\delta\phi_i(\mathbf{x}) + \frac{1}{2}\sum_{ij}\frac{\partial^2 N}{\partial\phi_i\partial\phi_j}\delta\phi_i(\mathbf{x})\delta\phi_j(\mathbf{x}) \qquad (8)$$

where $\delta\phi_i(\mathbf{x}) \equiv \phi_i(\mathbf{x}) - \delta\phi_i$.

- The $\delta\phi_i$ have spectrum $(H/2\pi)^2$ and are practically gaussian (Seery/Lidsey 2005, DHL/Zaballa 2005).
- Magic formula gives $\mathcal{P} = (H/2\pi)^2 \sum N_i^2$, and determines non-gaussianity if it's significant.

NOTE: We discount exotic inflation models (D-celeration, ghost inflation) giving non-gaussian inflaton perturbation $\delta\phi$.

Scenario 1; The inflaton ϕ generates ζ

$$\zeta \simeq N_\phi \delta\phi. \tag{9}$$

- Spectrum is $\mathcal{P} \simeq (H^2/2\pi\dot{\phi})^2$ (Starobinsky 1982).
- Negligible non-gaussianity (Seery/Lidsey 2005, see also Maldacena 2003).

Scenario 2: Some non-inflaton σ generates ζ

This scenario is *not exotic* (discovered unknowingly by Hamaguchi, Murayama & Yanagida, 2001).

$$\zeta = N_\sigma \delta\sigma + \frac{1}{2} N_{\sigma\sigma} (\delta\sigma)^2 \equiv \zeta_{\rm g} - \frac{5}{3} f_{\rm NL} \zeta_{\rm g}^2 . \qquad (10)$$

Trispectrum $\tau_{\rm NL} = 4(3 f_{\rm NL}/5)^2$ (Boubekeur/DHL 2005). Could be competitive with $f_{\rm NL}$ if both bounded by same data.

APPLICATION to the curvaton scenario (DHL/Rodriguez 2005):

$$-(3/5) f_{\rm NL} = -1 - \frac{1}{2} r + \frac{3}{4} \frac{1}{r} \left(1 + \frac{g g''}{g'} \right) \qquad (11)$$

This magically reproduces previous findings of first-order *and second-order* cosmological perturbation theory.

Scenario 3: mixture of 1 and 2

Mainly inflaton with some non-inflaton. *Not exotic* eg. GUT inflation with σ a string axion.

$$\zeta = N_\phi \delta\phi + \frac{1}{2} N_{\sigma\sigma} (\delta\sigma)^2 \equiv A + B^2 \qquad (12)$$

PREDICTIONS (DHL/Boubekeur 2005): $f_{\rm NL} \sim 10^4 \langle B^2/A \rangle^3$ and $\tau_{\rm NL} \sim 10^{10} \langle B^2/A \rangle^4$. *If $\tau_{\rm NL}$ and $f_{\rm NL}$ were constrained by same data, $\tau_{\rm NL}$ would give a stronger constraint on $\langle B^2/A \rangle$.*

APPLICATION: two-component inflation

$$V = V_0 + \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{2} m_\sigma^2 \sigma^2, \qquad (\text{unperturbed } \sigma = 0) \qquad (13)$$

Magic formula for $\langle B^2/A \rangle$ (DHL/Rodriguez 2005) *agrees* with 2nd-order cos. pert. theory (DHL/Malik in preparation). Negligible non-gaussianity.

Final thoughts

1. Non-gaussianity may discriminate between models more strongly than spectral tilt, tensor, correlated or un-correlated isocurvature, cosmic string signals etc.

2. Most scenarios predict that at least one of the above will be observed. So we will learn a lot even if none of them are observed!