

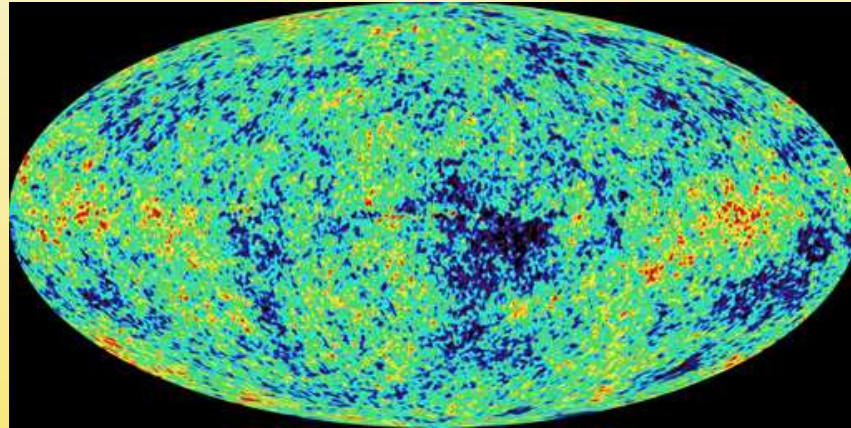
# NON-GAUSSIANITY FROM A SUBDOMINANT COMPONENT

Lotfi Boubekur  
Lancaster University.

Based on:

- LB and D.H. Lyth , [astro-ph/0504046](#)

## 2-POINT STATISTICS OF CMBR



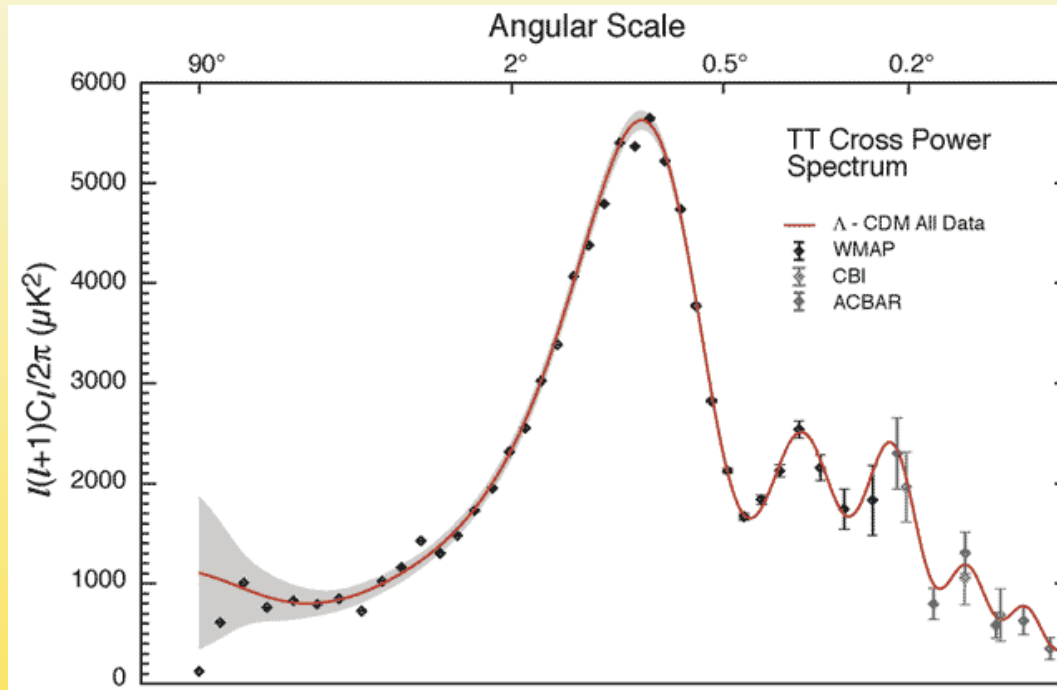
The observed temperature anisotropies are related to the primordial curvature perturbation  $\zeta$  through the Sachs-Wolfe effect

$$\frac{\delta T}{T} = -\frac{\zeta}{5}$$

The amplitude of the power spectrum of the curvature perturbation is defined as

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2) \left( \frac{2\pi^2}{k_1^3} \right) \mathcal{P}_\zeta(k_1)$$

with  $\mathcal{P}_\zeta(k) \propto (k/a_0 H_0)^{n_s - 1}$ .



$$\frac{l(l+1)C_l^{TT}}{2\pi} = \int d \ln k \mathcal{P}_\zeta(k) [g_\ell^T(k)]^2$$

Latest measurements of WMAP gave

$$\mathcal{P}_\zeta^{1/2} \simeq 5 \times 10^{-5}$$

$$|n_s - 1| \lesssim 0.05$$

## INFLATIONARY PARADIGM

Inflation is a dominant paradigm for the early Universe.

- Solves the standard cosmological problems.

→ scalar field  $\phi$  (*inflaton*), with slowly varying potential

$$a(t) \sim e^{Ht}, \text{ with } H^2 \simeq \frac{V(\phi)}{3M_P^2}$$

- Provides the seed for structure formation.

Quantum fluctuations of the inflaton  $\delta\phi \simeq \frac{H}{2\pi}$



Classical curvature perturbation  $\zeta$ .

Single field inflation	Predictions	Experiments
$\frac{\delta\rho}{\rho}$	$\sim \frac{V^{3/2}}{M_P^3 V'}$	$\sim 10^{-5}$
GW	$r \equiv \frac{A_T}{A_S} \sim \frac{V}{(4 \times 10^{16} \text{GeV})^4}$	$\lesssim 0.46$
Scalar tilt	$ n_s - 1  \sim \frac{1}{N_e}$	$\lesssim 0.05$
Nature of $\frac{\delta\rho}{\rho}$	only adiabatic no isocurvature  Almost Gaussian	$\frac{A_{\text{iso}}}{A_S} \lesssim 0.1$  $\left(\frac{\delta\rho}{\rho}\right)_{\text{NG}} \lesssim 10^{-3}$

## ALTERNATIVE SCENARIOS FOR CURVATURE PERTURBATION

Despite this striking success, several alternatives for the generation of primordial curvature perturbation have been put forward.

- The curvaton scenario

Lyth & Wands  
Enqvist & Sloth  
Moroi & Takahashi

$$\frac{\delta\rho}{\rho} \sim \frac{\delta\sigma}{\sigma_*}$$

- Inhomogeneous reheating

Dvali, Gruzinov & Zaldarriaga

$$\frac{\delta\rho}{\rho} \sim \frac{\delta\Gamma_\phi}{\Gamma_\phi}$$

- Other scenarios (DBI, Ghost inflation, ...)

These scenarios are based on the hypothesis that there are other light field besides the inflaton during inflation.

They have distinctive features in the CMB: **Non-Gaussianity**, correlations, ... and typically no GW.

## 3-POINT STATISTICS OF CMBR

The curvature perturbation  $\zeta(\mathbf{x})$  can be expanded to 2<sup>nd</sup> order as

Komatsu & Spergel

$$\zeta(\mathbf{x}) = \zeta_g(\mathbf{x}) - \frac{3}{5} f_{\text{NL}} (\zeta_g^2(\mathbf{x}) - \langle \zeta_g \rangle)$$

where the coefficient  $f_{\text{NL}}$  measures the strength of the quadratic term.

The *bispectrum*  $B_\zeta(k_1, k_2, k_3)$  is defined as

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_\zeta(k_1, k_2, k_3)$$

The non-linearity parameter  $f_{\text{NL}}$  can thus be defined as

$$f_{\text{NL}} = -\frac{5}{6} \frac{B_\zeta(k_1, k_2, k_3)}{P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_1)P_\zeta(k_3) + P_\zeta(k_2)P_\zeta(k_3)}$$

which is generally scale-dependent.

- Prediction of  $f_{\text{NL}}$ .

Scenario	$f_{\text{NL}}$ ( $k$ -independent part)
Single-field inflation	$-\frac{5}{12}(n_s - 1)$
Curvaton scenario	$\sim \frac{1}{r_\sigma}; \quad r_\sigma \equiv \left(\frac{\rho_\sigma}{\rho_{\text{tot}}}\right)_{\text{decay}}$
Inhomogeneous decay scenario	$O(1)$ simplest case
Other scenarios	$\gtrsim O(50)$

- Experimental constraints on  $f_{\text{NL}}$

$$-58 < f_{\text{NL}} < 134 \quad 95\% \text{ C.L.} \quad \text{WMAP}$$



- Prediction of  $f_{\text{NL}}$ .

Scenario	$f_{\text{NL}}$ ( $k$ -independent part)
Single-field inflation	$-\frac{5}{12}(n_s - 1)$
Curvaton scenario	$\sim \frac{1}{r_\sigma}; \quad r_\sigma \equiv \left(\frac{\rho_\sigma}{\rho_{\text{tot}}}\right)_{\text{decay}}$
Inhomogeneous decay scenario	$O(1)$ simplest case
Other scenarios	$\gtrsim O(50)$

- Experimental constraints on  $f_{\text{NL}}$

$$-58 < f_{\text{NL}} < 134 \quad 95\% \text{ C.L.} \quad \text{WMAP}$$

$$|f_{\text{NL}}| \sim O(5) \quad \text{PLANCK} \sim 2008$$

## A SMALL NON-GAUSSIAN COMPONENT

In the simplest case there is only one light field during inflation (inflaton, or curvaton, ...)  $\rightarrow$  one can hope to identify it through its non-Gaussianity.

But in general there could be other light scalar fields during inflation besides the inflaton with subdominant spectrum.

$$\zeta = \zeta_\phi + \zeta_X, \quad \mathcal{P}_X \ll \mathcal{P}_\phi$$

## A SMALL NON-GAUSSIAN COMPONENT

In the simplest case there is only one light field during inflation (inflaton, or curvaton, ...)  $\rightarrow$  one can hope to identify it through its non-Gaussianity.

But in general there could be other light scalar fields during inflation besides the inflaton with subdominant spectrum.

$$\zeta = \zeta_\phi + \zeta_X, \quad \mathcal{P}_X \ll \mathcal{P}_\phi$$

- Cosmic strings.

## A SMALL NON-GAUSSIAN COMPONENT

In the simplest case there is only one light field during inflation (inflaton, or curvaton, ...)  $\rightarrow$  one can hope to identify it through its non-Gaussianity.

But in general there could be other light scalar fields during inflation besides the inflaton with subdominant spectrum.

$$\zeta = \zeta_\phi + \zeta_X, \quad \mathcal{P}_X \ll \mathcal{P}_\phi$$

- Cosmic strings.
- Magnetic fields.

## A SMALL NON-GAUSSIAN COMPONENT

In the simplest case there is only one light field during inflation (inflaton, or curvaton, ...)  $\rightarrow$  one can hope to identify it through its non-Gaussianity.

But in general there could be other light scalar fields during inflation besides the inflaton with subdominant spectrum.

$$\zeta = \zeta_\phi + \zeta_X, \quad \mathcal{P}_X \ll \mathcal{P}_\phi$$

- Cosmic strings.
- Magnetic fields.
- Axion CDM.

## A SMALL NON-GAUSSIAN COMPONENT

In the simplest case there is only one light field during inflation (inflaton, or curvaton, ...)  $\rightarrow$  one can hope to identify it through its non-Gaussianity.

But in general there could be other light scalar fields during inflation besides the inflaton with subdominant spectrum.

$$\zeta = \zeta_\phi + \zeta_X, \quad \mathcal{P}_X \ll \mathcal{P}_\phi$$

- Cosmic strings.
- Magnetic fields.
- Axion CDM.
- Any other light scalar field (curvaton, fluctuating decay rate...).

## FRAMEWORK

- Assume the curvature perturbation is sourced by two sources: the first one is Gaussian  $\zeta_g$  (say the inflaton) and the second one  $\chi^2$ -distributed:

$$\zeta(\mathbf{x}) = \zeta_g(\mathbf{x}) + \sigma^2(\mathbf{x}) - \langle \sigma^2 \rangle$$

- The two sources are assumed to be completely uncorrelated i.e.  $\langle \zeta_g \sigma \rangle = 0$ , and  $\sigma$  is Gaussian i.e.  $\langle \sigma \rangle = 0$ .

- The inflaton  $\zeta_g$  is assumed to dominate the power spectrum

$$\mathcal{P}_\zeta = \mathcal{P}_{\zeta_g} + \mathcal{P}_{\sigma^2} \simeq \mathcal{P}_{\zeta_g} \gg \mathcal{P}_{\sigma^2}$$

Furthermore since the 2 sources are uncorrelated the bispectrum is

$$B_\zeta(k_1, k_2, k_3) = B_{\sigma^2}(k_1, k_2, k_3)$$

## FRAMEWORK

- Assume the curvature perturbation is sourced by two sources: the first one is Gaussian  $\zeta_g$  (say the inflaton) and the second one  $\chi^2$ -distributed:

$$\zeta(\mathbf{x}) = \zeta_g(\mathbf{x}) + \sigma^2(\mathbf{x}) - \langle \sigma^2 \rangle$$

- The two sources are assumed to be completely uncorrelated i.e.  $\langle \zeta_g \sigma \rangle = 0$ , and  $\sigma$  is Gaussian i.e.  $\langle \sigma \rangle = 0$ .

- The inflaton  $\zeta_g$  is assumed to dominate the power spectrum

$$\mathcal{P}_\zeta = \mathcal{P}_{\zeta_g} + \mathcal{P}_{\sigma^2} \simeq \mathcal{P}_{\zeta_g} \gg \mathcal{P}_{\sigma^2}$$

Furthermore since the 2 sources are uncorrelated the bispectrum is

$$B_\zeta(k_1, k_2, k_3) = B_{\sigma^2}(k_1, k_2, k_3)$$

What is the level of non Gaussianity in such a scenario?



## THE POWER SPECTRUM $\mathcal{P}_{\sigma^2}$

The Fourier components of  $\sigma^2$  are given by

$$(\sigma^2)_{\mathbf{k}} = \frac{1}{(2\pi)^3} \int d^3q \sigma_{\mathbf{q}} \sigma_{\mathbf{k}-\mathbf{q}}.$$

The 2-point correlation function for  $\mathbf{k}, \mathbf{k}' \neq 0$  reads

$$\langle (\sigma^2)_{\mathbf{k}} (\sigma^2)_{\mathbf{k}'} \rangle = 2\delta^{(3)}(\mathbf{k} + \mathbf{k}') \int d^3p P_{\sigma}(p) P_{\sigma}(|\mathbf{k} - \mathbf{p}|)$$

This for flat  $\mathcal{P}_{\sigma}$  gives

$$\mathcal{P}_{\sigma^2}(k) = \frac{k^3}{2\pi} \mathcal{P}_{\sigma}^2 \int d^3p \frac{1}{p^3 |\mathbf{p} - \mathbf{k}|^3},$$

which diverges for  $\mathbf{p} = 0$  and  $\mathbf{p} = \mathbf{k}$ . To regularize it, we work in a finite sphere of radius  $L$ , giving

$$\mathcal{P}_{\sigma^2}(k) = 4\mathcal{P}_{\sigma}^2 \ln(kL),$$

which gives the correct variance normalization

$$\langle (\sigma^2 - \langle \sigma^2 \rangle) \rangle = 2\langle \sigma^2 \rangle^2$$

## THE BISPECTRUM

Likewise we can compute the bispectrum of  $\sigma^2$ , giving

$$B_{\sigma^2}(k_1, k_2, k_3) = (2\pi)^3 \mathcal{P}_\sigma^3 \int d^3p \frac{1}{p^3 |\mathbf{p} - \mathbf{k}_1|^3 |\mathbf{p} + \mathbf{k}_2|^3}$$

with  $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0$ .

As for the power spectrum, we must regularize the divergences at  $\mathbf{p} = 0$ ,  $\mathbf{p} = \mathbf{k}_1$  and  $\mathbf{p} = \mathbf{k}_2$  by working in a finite volume. We get

$$B_{\sigma^2}(k_1, k_2, k_3) \simeq \frac{6(2\pi)^4}{k^6} \mathcal{P}_\sigma^3 \ln(kL),$$

from which we can read off  $f_{\text{NL}}$

$$\begin{aligned} f_{\text{NL}} &\simeq -\frac{20}{3} \frac{\mathcal{P}_\sigma^3}{\mathcal{P}_\zeta^2} \ln(kL) \\ &= -\frac{1}{\sqrt{\ln(kL)}} \left( 25.5 \frac{\mathcal{P}_{\sigma^2}^{1/2}}{\mathcal{P}_\zeta^{1/2}} \right)^3 \end{aligned}$$

## OBSERVATIONAL CONSEQUENCES

The current WMAP bound  $|f_{\text{NL}}| \lesssim 100$  gives

$$\left(\frac{\mathcal{P}_{\sigma^2}}{\mathcal{P}_{\zeta}}\right)^{1/2} \lesssim 0.18$$

PLANCK will be able to go down to

$$\left(\frac{\mathcal{P}_{\sigma^2}}{\mathcal{P}_{\zeta}}\right)^{1/2} \lesssim 0.04$$

## A GAUSSIAN SQUARED ISOCURVATURE PERTURBATION

- The axion solves the strong CP problem.
- It also provides an attractive and economical candidate for CDM.
- It can have isocurvature perturbations

$$S_a = \frac{\delta n_a}{n_a} - \frac{\delta n_\gamma}{n_\gamma},$$

that can feed into the CMB temperature anisotropies through the Sachs-Wolfe formula

$$\left(\frac{\Delta T}{T}\right)_{\text{SW}} = \left(\frac{\Delta T}{T}\right)_{\text{A}} + \left(\frac{\Delta T}{T}\right)_{\text{S}} = -\frac{\zeta_{\text{g}}}{5} - \frac{2}{5}S_a$$

There are two possible regimes

Lyth '91

$$n_a \sim m_a^2 a^2 \Rightarrow \frac{\delta n_a}{n_a} = \frac{\delta a}{a} + \frac{1}{2} \left( \frac{\delta a}{a} \right)^2$$

- $\delta a \ll a \Rightarrow S_a \sim \frac{\delta a}{a}$  (Gaussian).

- $\delta a \gg a \Rightarrow S_a \sim \left( \frac{\delta a}{a} \right)^2$  (Highly non-Gaussian).

Applying the previous bounds, one gets

$$S_a \lesssim 0.18 \quad \text{WMAP}$$

$$S_a \lesssim 0.04 \quad \text{PLANCK}$$

## EXPERIMENTAL SMOKING GUN

The scenario can be tested by combining data on gravitational waves and non-Gaussianity.

- If the B-modes (tensors) are detected at a level expected from inflation, then primordial curvature perturbation is likely to be dominated by inflaton fluctuations.
- If the amount of measured NG is significant i.e.  $f_{\text{NL}} \gg |n_s - 1|$ , then this will be attributed to a subdominant non-Gaussian source, not to the inflaton.

The scenario can be confirmed or ruled out as accuracy of CMB experiments increases.

## CONCLUSIONS

- We studied a new possible source of non-Gaussianity, sourced by a subdominant uncorrelated component of  $\zeta$

$$\zeta = \zeta_g(\mathbf{x}) + \sigma^2(\mathbf{x}) - \langle \sigma^2 \rangle$$

$$\sigma \neq \zeta_g \quad \mathcal{P}_{\zeta_g} \gg \mathcal{P}_{\sigma^2}$$

- We computed the non-linearity parameter

$$f_{\text{NL}} \simeq \frac{\mathcal{P}_{\sigma^2}^{3/2}}{\mathcal{P}_{\zeta}^2}$$

- Can be applied to (uncorrelated) axion CDM.
- Future observations can test such a scenario.