Gravity on the Thick Brane

José Santiago
FNAL (USA)

SUSY05, Durham, July 18-23, 2005

I. Navarro, J. S., JHEP 0502 ('05) 007 [hep-th/0411250]

I. Navarro, J. S., [hep-th/0505156]

Outline

- Motivation and Philosophy
- Einstein Equations
  - Matching Conditions: Encode effects on the surroundings
  - Constraints on Induced Gravity: $\hat{T}_{\mu\nu}$ conservation, 4D Einstein Equations
- Some Topics on Braneworld Gravity:
  - Codimension 1 Brane: Dust sourced acceleration
  - Codimension 2 Brane Towards realistic self-tuning
  - Higher Codimension Branes: matching to 4D gravity?
- Conclusions and Open Questions

Motivation and Philosophy

Braneworld gravity could help explain some of the most dramatic conundrums in theoretical physics: cosmological constant, gauge hierarchy, quantum gravity, ...

Thin branes badly singular for codimension higher than 1 $\Rightarrow$ *Go Thick !*

- Our set-up:
  - "Spherical" symmetry

$$\mathrm{d}s^2 = g_{\mu\nu}(x,r)\mathrm{d}x^\mu \mathrm{d}x^\nu - e^{2\phi(x,r)}\mathrm{d}r^2 - L^2(x,r)\mathrm{d}\Omega_d^2$$

  - Thick brane with EMT T^{br} localized at $r < \epsilon$
  - $5+d-$dimensional Einstein Equations

$$M_*^{d+3}\left(R_N^M - \frac{1}{2}\delta_N^M R\right) = T_N^M \Leftrightarrow M_*^{d+3} R_N^M = T_N^M - \frac{1}{d+3}\delta_N^M T$$

- Our goal:
  - Describe the leading effects of the brane (independent of its detailed inner structure) on its surroundings and on brane observers

Einstein Equations

- Total derivatives (plus small terms)

$$e^{\phi}\sqrt{g}L^d R^{\mu}_{\nu} = \frac{1}{2}\left[e^{-\phi}\sqrt{g}L^d K^{\mu}_{\nu}\right]' + e^{\phi}\sqrt{d}L^d\left(R^{\mu}_{\nu}(g) - d\frac{\nabla^{\mu}\nabla_{\nu}L}{L} - \nabla^{\mu}\nabla_{\nu}\phi\right)$$

$$e^{\phi}\sqrt{g}L^d R^{i}_{j} = \left\{\left[e^{-\phi}\sqrt{g}L^{d-1}L'\right]' + e^{\phi}\sqrt{g}L^d\left(\frac{\kappa_d}{dL^2} - \frac{\nabla^{\rho}\nabla_{\rho}L}{L} - (d-1)\frac{\nabla^{\rho}L\nabla_{\rho}L}{L^2} - \frac{\nabla^{\rho}\phi\nabla_{\rho}L}{L}\right)\right\}\delta^i_j$$

- No second derivatives

$$G^r_r = \boxed{-\frac{1}{2}R(g)} - \frac{d}{2}e^{-2\phi}K\frac{L'}{L} + \frac{1}{8}e^{-2\phi}\left[K^{\rho}_{\sigma}K^{\sigma}_{\rho} - K^2\right] - \frac{1}{2}\left[d(d-1)e^{-2\phi}\left(\frac{L'}{L}\right)^2 - \frac{\kappa_d}{L^2}\right] + d\frac{\nabla^{\rho}\nabla_{\rho}L}{L} + \frac{d(d-1)}{2}\frac{\nabla^{\rho}L\nabla_{\rho}L}{L^2}$$

$$e^{-\phi}G_{\mu r} = \frac{1}{2}\nabla^{\nu}\left[e^{-\phi}(K_{\mu\nu} - g_{\mu\nu}K)\right] - d\frac{\partial_{\mu}(e^{-\phi}L')}{L} + \frac{d}{2}e^{-\phi}K^{\nu}_{\mu}\frac{\partial_{\nu}L}{L}$$

where $K_{\mu\nu} \equiv \partial_r g_{\mu\nu}$.

Matching Conditions

Integrate along the brane appropriate combinations of Einstein Equations

$$e^{-\phi}\frac{V_d}{2}\frac{K^\mu_\nu|_\epsilon}{M_b^d} = \frac{\hat{T}^\mu_\nu - \frac{1}{d+3}\delta^\mu_\nu(\hat{T}^\rho_\rho + \hat{T}^r_r + d\hat{T}_i)}{M_*^{d+3}}$$

$$e^{-\phi}V_d\left(\beta - \frac{\sqrt{g_0}}{\sqrt{g_\epsilon}}\right) = -\frac{M_b^{d-1}}{(d+3)M_*^{d+3}}[\hat{T}^\rho_\rho + \hat{T}^r_r - 3\hat{T}_i] - V_d\frac{\kappa_d}{d}\left(\frac{M_b}{M_\kappa}\right)^{d-1}$$

where $\beta \equiv L'(\epsilon)$, $M_b \equiv L^{-1}(\epsilon)$, the effective four-dimensional EMT reads

$$\hat{T}^M_N \equiv \frac{1}{\sqrt{g}|_\epsilon}\int dy^d\sqrt{k}\int_0^\epsilon dr\, e^\phi\sqrt{g}L^d T^M_N\,,$$

and the mass M_κ is given by

$$M_\kappa^{-(d-1)} \equiv \frac{1}{\sqrt{g}|_\epsilon}\int_0^\epsilon dr\, e^\phi\sqrt{g}L^{d-2},$$

Encode the effect of the brane on its surroundings

Induced Gravity

Evaluate the rest of Einstein Eqs. at the brane boundary

- G_{rr}: (Trace of) Einstein eqs. for induced metric

$$\boxed{\frac{1}{2}R(g)} = -\frac{T^r_r|_\epsilon}{M_*^{n+2}} - \frac{d}{2}e^{-2\phi}K\frac{L'}{L} + \frac{1}{8}e^{-2\phi}[K^\rho_\sigma K^\sigma_\rho - K^2]$$
$$-\frac{1}{2}\left[d(d-1)e^{-2\phi}\left(\frac{L'}{L}\right)^2 - \frac{\kappa_d}{L^2}\right] + d\frac{\nabla^\rho\nabla_\rho L}{L} + \frac{d(d-1)}{2}\frac{\nabla^\rho L\nabla_\rho L}{L^2}$$

- $G_{\mu r}$: EMT conservation

$$\nabla^\nu \hat{T}_{\mu\nu} = -\nabla_\mu \hat{T}^r_r - V_d M_*^{d+3}\kappa_d \nabla_\mu M_\kappa^{-(d-1)}$$
$$+\left[d(\hat{T}^r_r + \hat{T}_i) + (d-1)V_d\kappa_d\frac{M_*^{d+3}}{M_\kappa^{d-1}}\right] M_b \partial_\mu M_b^{-1}$$

Cosmology of Codimension 1 Thick Branes

$$ds^2 = n^2(r,t)dt^2 - a^2(r,t)d\mathbf{x}^2 - e^{2\phi(r,t)}dr^2$$

Matching Conditions

$$e^{-\phi}\frac{2n'}{n}|_\epsilon = \frac{1}{M_*^3}\left[\frac{2}{3}\rho + p + \frac{1}{3}p_r\right]$$
$$e^{-\phi}\frac{2a'}{a}|_\epsilon = \frac{1}{M_*^3}\left[\frac{1}{3}(p_r - \rho)\right]$$

We expand around a large constant tension ($T, T_r \gg \rho_m$) to leading order

$$\rho = T + \rho_m, \quad p = -T + \omega\rho_m, \quad p_r = -T_r + \omega_r\rho_m.$$

The energy-momentum conservation equation

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}(1+\omega)\rho_m - \omega_r\dot{\rho}_m = 0$$

can be integrated to give the scale factor

$$\rho_m = \rho_{m0}a^{-3\frac{1+\omega}{1-\omega_r}}$$

Cosmology of Thick Branes (cont'd)

We can plug it into Friedmann equation

$$3\left(\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}\right)=\underbrace{\frac{T+T_r}{12M_*^6}}_{1/2M_P^2}(1-3w-4w_r)\rho_m+\underbrace{\left[\frac{(T+T_r)^2}{6M_*^6}+\frac{\Lambda}{M_*^3}\right]}_{\Lambda_{\text{eff}}}$$

and solve for $\Lambda_{\text{eff}} \approx 0$ to obtain

$$H^2=\mathcal{C}a^{-4}+\frac{T+T_r}{18M_*^6}\left(1-\omega_r\right)\rho_{m0}a^{-3\frac{1+\omega}{1-\omega_r}}.$$

It is equivalent to a fluid with an effective equation of state parameter

$$\boxed{\omega_{\text{eff}}=\frac{\omega+\omega_r}{1-\omega_r}}$$

A universe filled with dust ($\omega = 0$) will be accelerating if $w_r < -\frac{1}{2}$.

Matching to Global Solutions

We can also match to a RS model

$$ds^2 = e^{-2kr}\eta_{\mu\nu}dx^\mu dx^\nu - dr^2$$

giving

$$\frac{n'}{n} = \frac{a'}{a} = -k = -\frac{1}{6M_*^3}(T + T_r)$$

we therefore have

$$M_P^2 = \underbrace{\frac{6M_*^6}{T+T_r} = \frac{M_*^3}{k} = \overbrace{4M_*^3\mathcal{V}_{AdS}}^{\text{global}}}_{\text{local}}$$

We get a nice agreement between the local analysis (matching conditions plus modified Friedmann equation) and the global one (KK reduction).

Similar behaviour for codimension 2 branes.

Cosmology of Higher Codimension Branes

We match to an $AdS_5 \times S^d$ geometry (Gherguetta, Roessl, Shaposhnikov '00)

$$ds^2 = e^{-kr}\eta_{\mu\nu}dx^\mu dx^\nu - dr^2 - R_0^2 d\Omega_d^2$$

Imposing

$$L(x, r > \epsilon) = R_0 \Rightarrow L'(x, \epsilon) = 0$$

The cosmology is then identical to that of an infinitesimal codimension 1 brane with EMT ($d = 0$ for a thick codimension 1 brane)

$$T^{\text{eff}}_{\mu\nu} = \hat{T}_{\mu\nu} - \frac{1}{d+3} g_{\mu\nu} \left[d\hat{T}^\sigma_\sigma - 3(\hat{T}^r_r + d\hat{T}^\theta_\theta) \right]$$

Solving the matching conditions and Friedmann equations (local analysis) we find that 4D cosmology is not reproduced, contrary to what happens in codimension 1 and 2 branes

Conclusions and Open Questions

- Higher codimension branes are singular in the thin brane limit $\Rightarrow$ finite width regularisation is required
- It is possible to compute matching conditions and the induced gravity for arbitrary codimension independently of the detailed structure of the brane
- Interesting *new effects* on thick branes: dust sourced acceleration
- Matching of cod 1 and 2 branes with global sols: nice agreement between global and local calculations, 4D gravity consistent with KK reduction
- Intriguing behaviour for higher codimensions: matching to a 4D theory?
- Future avenues:
  - discussion of localized modes with $p_r, p_i \sim \rho_m$
  - 4D description of dust acceleration: effective theory of radion
  - study of explicit models of higher cod'n to understand the matching
  - phenomenological implications for cosmology