Constraints in codimension-2 brane cosmology

Antonios Papazoglou

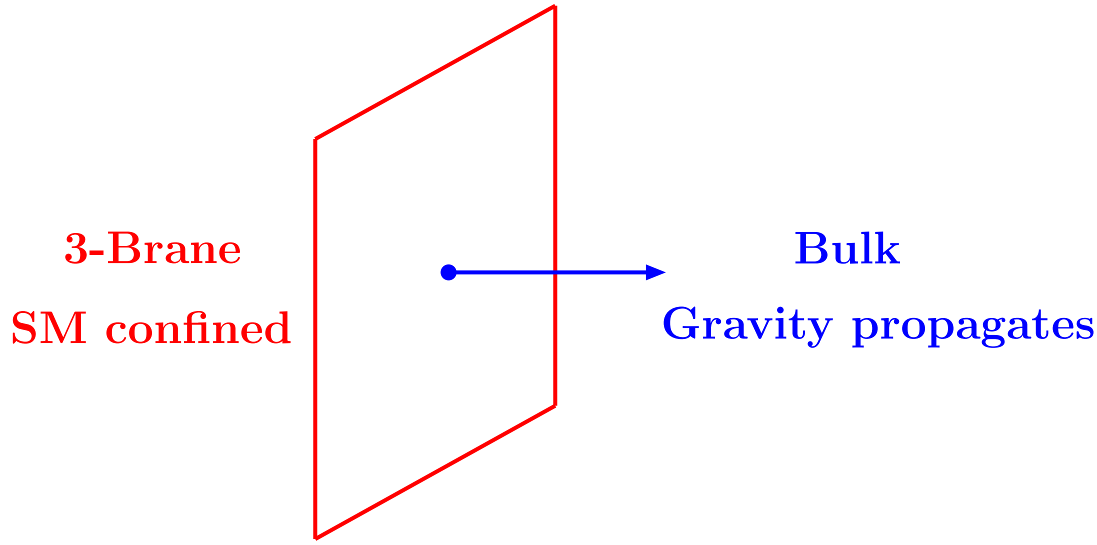
Lausanne, École Polytechnique

Outline

- Codimension-2 branes in 6D and cosmology
- Addition of induced gravity / Gauss-Bonnet term
- Bulk-brane constraints and relaxation of them

On hep-th/0501112, hep-th/0507xxx
by E. Papantonopoulos and A.P.

Brane worlds - 5D



- **Arena to explore long standing problems in physics under new perspective**
  - **Electroweak hierarchy problem**
  - **Cosmological constant problem**
- **Most thoroughly studied in 5D**

  One dim. $\perp$ to the brane $\equiv$ Codimension-1 brane
- **Einstein equation projected on the brane**

$$E^{(4)}_{\mu\nu} = \frac{1}{M^2_{Pl}} T^{(br)}_{\mu\nu} + \{T^2_{(br)}\}_{\mu\nu} + \{C\}_{\mu\nu} + \Lambda_4 g_{\mu\nu}$$

obtained by the junction condition $K_{\mu\nu} \equiv g'_{\mu\nu} \sim T^{(br)}_{\mu\nu}$

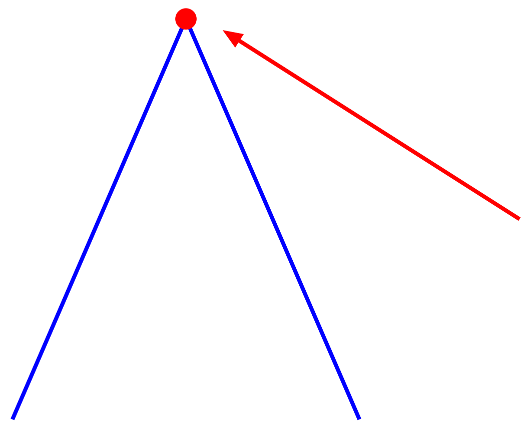
- **Example of cosmology on the brane**

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{1}{3M^2_{Pl}} \left[\rho + \frac{\rho^2}{2T} + \frac{C}{a^4} \right]$$

Early time cosmology (for $\rho \gg T \sim M^4_{Pl}$) is 5D

- **More exotic late time modifications also possible**

Brane worlds - 6D



- **Less understood are 6D brane worlds**

  Two dim. $\perp$ to the brane $\equiv$ Codimension-2 brane

- **Codimension-2 branes have an interesting property**
  - **General action of a brane model**

$$S = \int R^{(reg)} + R^{(sing)} + \mathcal{L}^{(bulk)} - T^{(br)}$$

  - **In general $R^{(sing)} \propto T^{(br)}$**
  - **Only for codimension-2 branes $R^{(sing)} = T^{(br)}$**
  - **Exact automatic cancellation $R^{(sing)}$ and $T^{(br)}$**

- **This happens because in 2 dimensions, sources do not curve the space, but only introduce a deficit angle**

- **Promising for realizing a self tuning scenario**

  i.e. **a situation where the 4D geometry is Minkowskian for any brane vacuum energy without any fine tuning of it with other parameters of the action**

[S.M.Carroll, M.M.Guica, hep-th/0302067]
[I.Navarro, hep-th/0302129]
[S.Radjbar-Daemi,V.Rubakov, hep-th/0407176]
[H.M.Lee,A.P., hep-th/0407208]

Problem with cosmology

[J.M.Cline,J.Descheneau,M.Giovannini,J.Vinet, hep-th/0304147]

- **6D brane models assume** $T_{\mu\nu} = -T g_{\mu\nu} \delta^{(2)}(\vec{r})$
- **For cosmology we need** $T_{\mu\nu} = \mathrm{diag}(-\rho, P, P, P)\delta^{(2)}(\vec{r})$
- **Metric ansatz**

$$ds^2 = -N^2(t,r)dt^2 + A^2(t,r)d\vec{x}^2 + dr^2 + L^2(t,r)d\theta^2$$

- **Equations of motion**

$$3\frac{A''}{A} + \frac{L''}{L} + \ldots = -\rho\ \delta^{(2)}(\vec{r}) \qquad (00)$$

$$2\frac{A''}{A} + \frac{N''}{N} + \frac{L''}{L} + \ldots = P\ \delta^{(2)}(\vec{r}) \qquad (ij)$$

- **For** $L \sim \beta r + \mathcal{O}(r^2)$ **we have**

$$R_{00} \sim \frac{N'}{r} + \ldots \quad , \quad R_{ij} \sim \frac{A'}{r}\delta_{ij} + \ldots$$

- **In general the geometry is singular at** $r = 0$
  - **Either we should regularize the brane**
  - **Or assume that that the brane is purely conical**
- **To have model independence, we choose to have conical branes**

  $\Rightarrow$ **No** $\frac{1}{r}$ **singularities force** $A' = N' = 0$ **at** $r = 0$

  $\Rightarrow$ **No singular part in** A'', N''

  $\Rightarrow$ **Singular part comes only from** L''

  $\Rightarrow$ **Only tension allowed** $\rho = -P$

Modifying the gravity action

[P.Bostock,R.Gregory,I.Navarro,J.Santiago, hep-th/0311074]

- **Need to modify the singularity structure of the equations of motion**

  $\Rightarrow$ **Add a bulk Gauss-Bonnet term**
  and a brane induced gravity term

$$\mathcal{S} = \frac{M_6^4}{2}\int d^6x\sqrt{G}\left[R^{(6)} + \alpha(R^{(6)\ 2} - 4R_{MN}^{(6)\ 2} + R_{MNK\Lambda}^{(6)\ 2})\right]$$
$$+ \frac{M_6^4}{2}r_c^2\int d^4x\sqrt{g}R^{(4)}\frac{\delta(r)}{2\pi L} + \int d^6x\mathcal{L}_{Bulk} + \int d^4x\mathcal{L}_{brane}\frac{\delta(r)}{2\pi L}$$

- **Metric ansatz**

$$ds^2 = -g_{\mu\nu}(x,r)dx^\mu dx^\nu + dr^2 + L^2(x,r)d\theta^2$$

with $L = \beta r + \mathcal{O}(r^2)$

- **Singular terms**

$$\frac{L''}{L} = -(1-\beta)\frac{\delta(r)}{L} + \mathrm{non-singular}$$
$$\frac{K'_{\mu\nu}}{L} = K_{\mu\nu}\frac{\delta(r)}{L} + \mathrm{non-singular}$$

- **Since** $R_{\mu\nu}^{(6)} = -\frac{K_{\mu\nu}}{r} + \mathcal{O}(1)$**, we must have** $K_{\mu\nu} = 0$ **at** $r = 0$
- **The** (μr) **equation constrains** β

$$\frac{\partial_\mu L'}{L} = -\frac{K_{\mu\nu}}{r} + \mathcal{O}(1) \qquad \Rightarrow \qquad \beta = \mathrm{constant}$$

- **Then the** δ**-function part of the Einstein equations gives**

$$G_{\mu\nu}^{(4)} = \frac{1}{M_{Pl}^2}\left[T_{\mu\nu}^{(br)} - \Lambda_4 g_{\mu\nu}\right]$$

with, $M_{Pl}^2 = [8\pi(1-\beta)\alpha + r_c^2]M_6^4$ **and** $\Lambda_4 = -2\pi(1-\beta)M_6^4$

Bulk-brane matter relations

- **The 6D Einstein equations include more information in addition to the δ-function part**
- **In particular, the (rr) equation evaluated at $r = 0$ gives**

$$R^{(4)} + \alpha[R^{(4)\ 2} - 4R^{(4)\ 2}_{\mu\nu} + R^{(4)\ 2}_{\mu\nu\kappa\lambda}] = -\frac{2}{M_6^2} T_r^{(B)r}$$

- **We know $R^{(4)}$ and $R^{(4)}_{\mu\nu}$ from the brane Einstein equation, but $R^{(4)}_{\mu\nu\kappa\lambda}$ is in general arbitrary**
- **For $\alpha = 0$, *i.e.* only brane induced gravity**

$$T_r^{(B)r} = -\frac{M_6^4}{2} R^{(4)} = \frac{1}{2r_c^2}[T_\mu^{(br)\mu} + 8\pi M_6^4(1-\beta)]$$

Brane matter is tuned to bulk matter

- **For $\alpha \neq 0$, it depends on the symmetry of the space**
- **If we assume isotropic metric, *e.g.***

$$ds^2 = -N^2(t,r)dt^2 + A^2(t,r)d\vec{x}^2 + dr^2 + L^2(t,r)^2 d\theta^2$$

then $R^{(4)}_{\mu\nu\kappa\lambda}$ is related to $R^{(4)}$ and $R^{(4)}_{\mu\nu}$

- **For an isotropic metric we have always a tuning**

$$T_r^{(B)r} = f(T_\nu^{(br)\mu})$$

- **Different from the $5D$ brane cosmology**

In $5D$ $K_{\mu\nu} \neq 0$ on the brane

$\Rightarrow$ **Independence of brane matter from bulk matter**

The constrained isotropic case

- **Let us assume the following**
  - **the brane vacuum energy cancels Λ_4**

$$\rho = -\Lambda_4 + \rho_m \quad , \quad P = \Lambda_4 + w\rho_m$$

  - **the bulk has only cosmological constant**

$$T^{(B)}_{MN} = -\Lambda_B G_{MN}$$

- **The brane Einstein equations give**

$$\frac{\dot{a}^2}{a^2} = \frac{\rho_m}{3M_{Pl}^2} \quad , \quad \frac{\ddot{a}}{a} = -(3w+1)\frac{\rho_m}{6M_{Pl}^2}$$

- **The brane matter tuning is**

$$-\frac{\Lambda_B}{M_6^4} = \frac{\rho_m}{M_{Pl}^2}\left[\frac{1}{2}(3w-1) + \frac{2}{3}(3w+1)\alpha\frac{\rho_m}{M_{Pl}^2}\right]$$

- **Note that w cannot be constant but evolves as**

$$\dot{w} + 3(1+w)\rho_m \frac{\partial w}{\partial \rho_m}\frac{\dot{a}}{a} = 0$$

- **The evolution of this constrained system depends on Λ_B**
  - **For $\Lambda_B = 0$, there is an attractor with $(\rho_m, w) = (0, 1/3)$**
  - **For $\Lambda_B > 0$, there is an attractor with $(\rho_m, w) = (\rho_f, -1)$ with $\frac{\alpha\rho_f}{M_{Pl}^2} = -\frac{3}{4} + \frac{3}{4}\sqrt{1 + \frac{4}{3}\frac{\Lambda_B}{M_6^4}} > 0$**
  - **For $\Lambda_B < 0$, the system runs away to $w \to \infty$**
- **Potentially interesting evolution for $0 < \Lambda_B/M_6^4 \ll 1$ with**

$$w_{ini} \sim 1/3 \quad \Rightarrow \quad w \sim 0 \quad \Rightarrow \quad w_{fin} \sim -1$$

The unconstrained anisotropic case

- **Let us see if we can avoid the above tuning of ρ and w by adding anisotropy to the geometry**

- **Consider the simple case where**

$$ds^2 = -N^2(t,r)dt^2 + \sum_i A_i^2(t,r)(dx^i)^2 + dr^2 + L^2(t,r)d\theta^2$$

and with a particular anisotropy

$$\begin{aligned} A_1(t,r) &= a(t)b(t) + \xi_1(t)r^2 + \ldots \\ A_2(t,r) &= \frac{a(t)}{b(t)} + \xi_2(t)r^2 + \ldots \\ A_3(t,r) &= a(t) + \xi_3(t)r^2 + \ldots \end{aligned}$$

- **We keep the brane fluid isotropic**

- **The brane Einstein equations give**

$$\frac{\dot{a}^2}{a^2} - \frac{1}{3}\cdot\frac{\dot{b}^2}{b^2} = \frac{\rho_m}{3M_{Pl}^2} \quad , \quad \frac{\ddot{a}}{a} + \frac{2}{3}\cdot\frac{\dot{b}^2}{b^2} = -(3w+1)\frac{\rho_m}{6M_{Pl}^2}$$
$$\frac{\ddot{b}}{b} - \frac{\dot{b}^2}{b^2} \; + \; 3\frac{\dot{a}\dot{b}}{ab} = 0$$

- **The (rr) equation gives a "Hubble" equation for b**

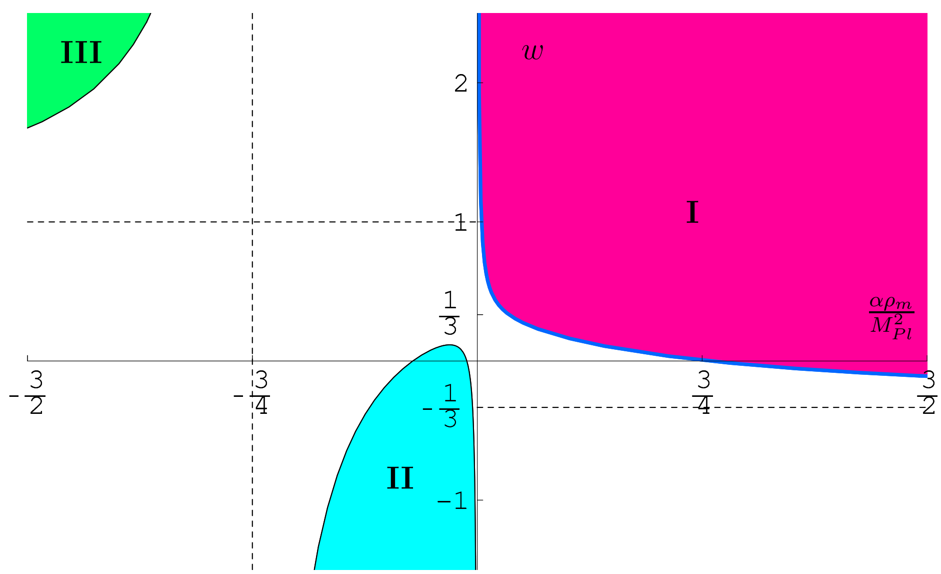
$$\frac{\dot{b}^2}{b^2} = -\frac{\rho_m}{4M_{Pl}^2} + \sqrt{\frac{3}{32\alpha}}\sqrt{2\frac{\Lambda_B}{M_6^4} + \frac{\rho_m}{M_{Pl}^2}\left[(3w-1) + 2(2w+1)\alpha\frac{\rho_m}{M_{Pl}^2}\right]} \equiv f(\rho_m, w)$$

- **Again w cannot be constant but evolves as**

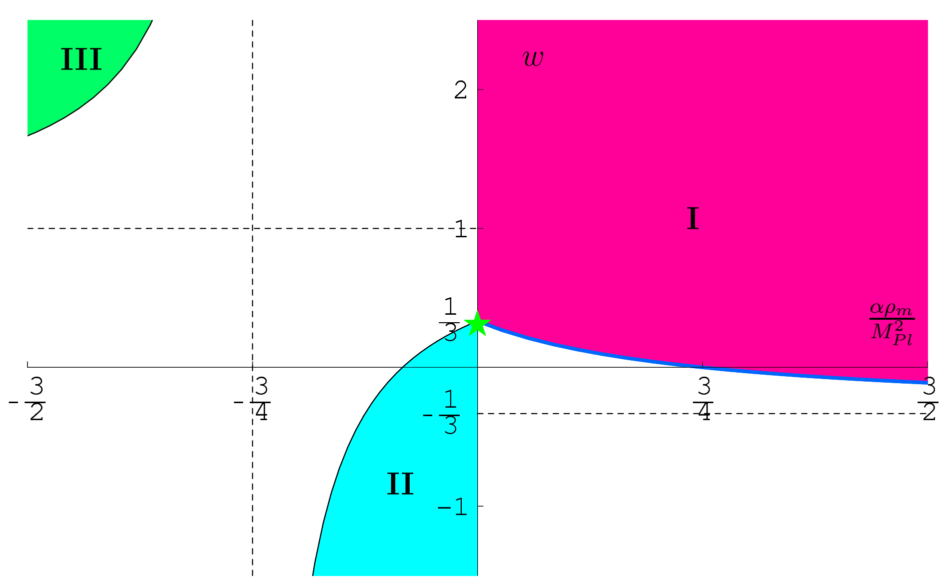
$$\frac{\partial f}{\partial w}\dot{w} + 3\left[2f - (1+w)\rho_m\frac{\partial f}{\partial \rho_m}\right]\frac{\dot{a}}{a} = 0$$

Parameter spaces

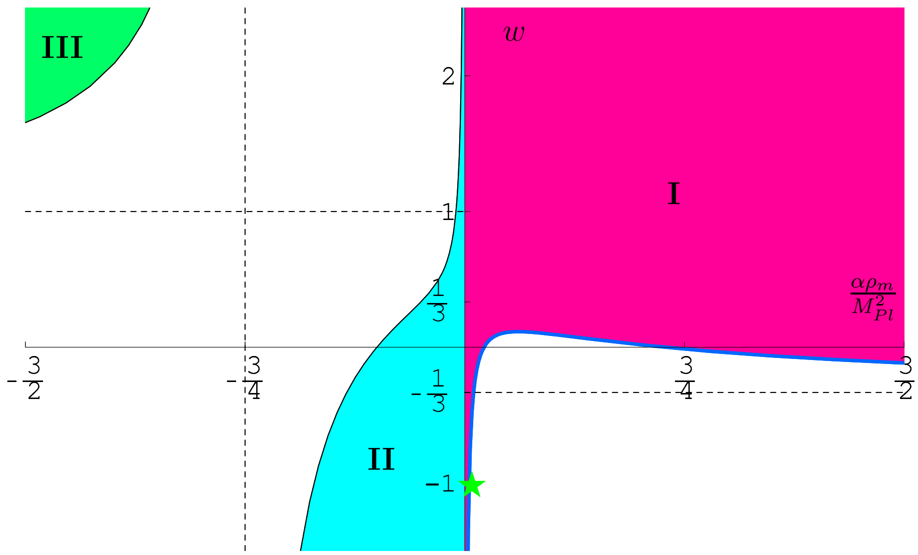
- **In order that the Hubbles H_a, H_b are not imaginary, there are only certain allowed regions in the (ρ_m, w) plane**
- **To be compared with the line of isotropic tuning (—)**
- **The fixed points are in the allowed region (★)**
- **For $\Lambda_B < 0$**



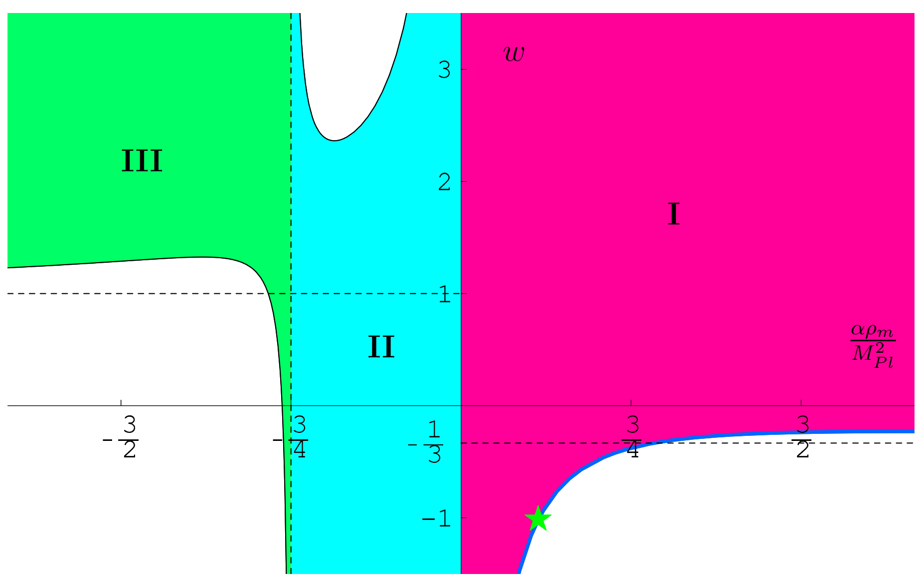
- **For $\Lambda_B = 0$**



- **For** $0 < \frac{\Lambda_B}{M_6^4} < \frac{3}{4}$



- **For** $\frac{\Lambda_B}{M_6^4} > \frac{3}{4}$



- **Although Regions I, II, III may be connected, the evolutions never cross the** $r_m = 0$ **and** $\frac{\alpha\rho_m}{M_{Pl}^2} = -\frac{3}{4}$ **lines**

Cosmological evolutions

- **There are no new fixed points in the (w, ρ_m) plane apart from the ones on the lines of isotropic tuning**

- **In Region I, the solutions for $\Lambda_B \geq 0$ evolve towards the fixed points**

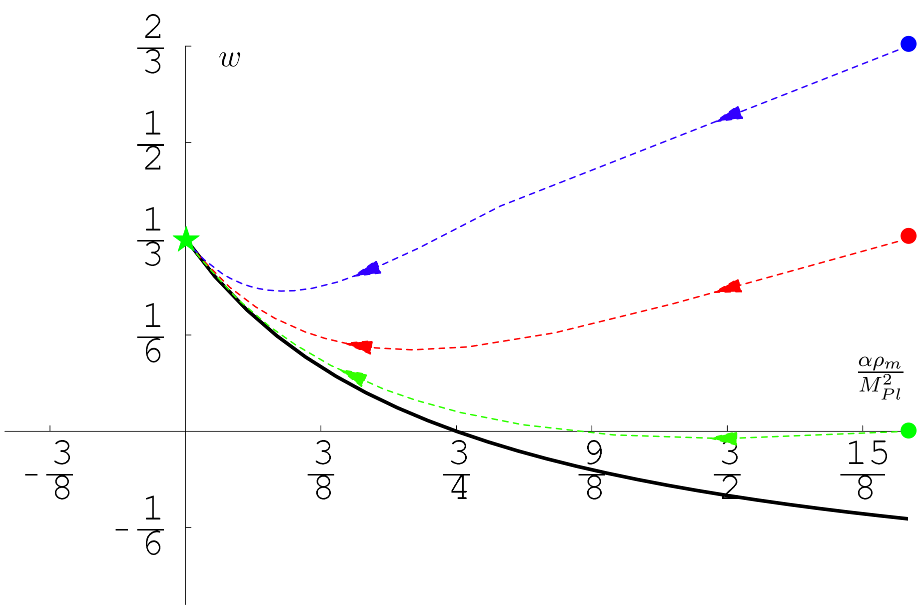
  $\Rightarrow$ $(\rho_m, w) = (0, 1/3)$ **if** $\Lambda_B = 0$

  $\Rightarrow$ $(\rho_m, w) = (\rho_f, -1)$ **if** $\Lambda_B > 0$

$$\left[\frac{\alpha\rho_f}{M_{Pl}^2} = -\frac{3}{4} + \frac{3}{4}\sqrt{1 + \frac{4}{3}\frac{\Lambda_B}{M_6^4}} > 0\right]$$

- **The lines of isotropic tuning are not attractors, only the fixed points are attractors**

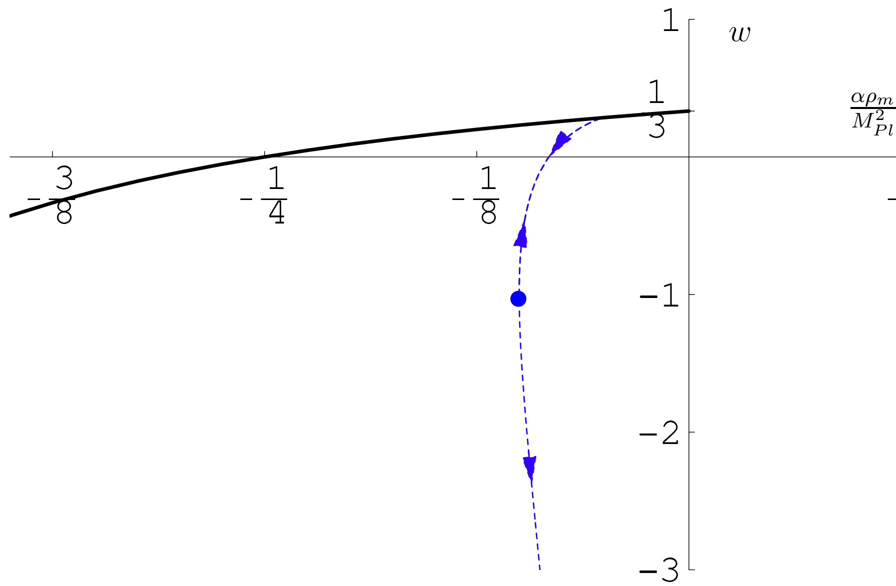
- **Example for $\Lambda_B = 0$**



- **In Region I, the solutions for $\Lambda_B < 0$ have a runaway behaviour with $w \to +\infty$ and $\rho_m \to 0^+$**

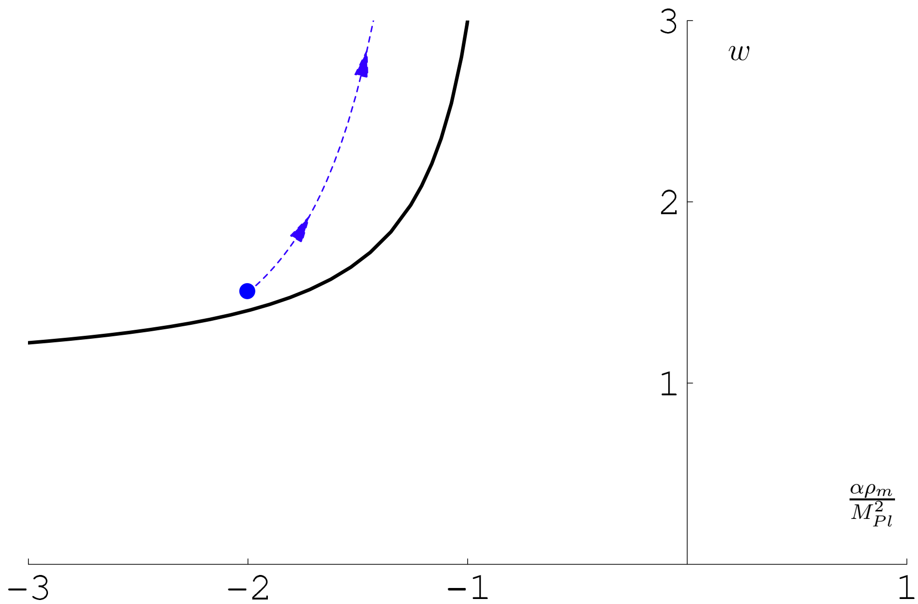
- **In Region II the solutions for any Λ_B have a runaway with $w \to \pm\infty$ and $\rho_m \to 0^-$**

- **Example for $\Lambda_B = 0$**



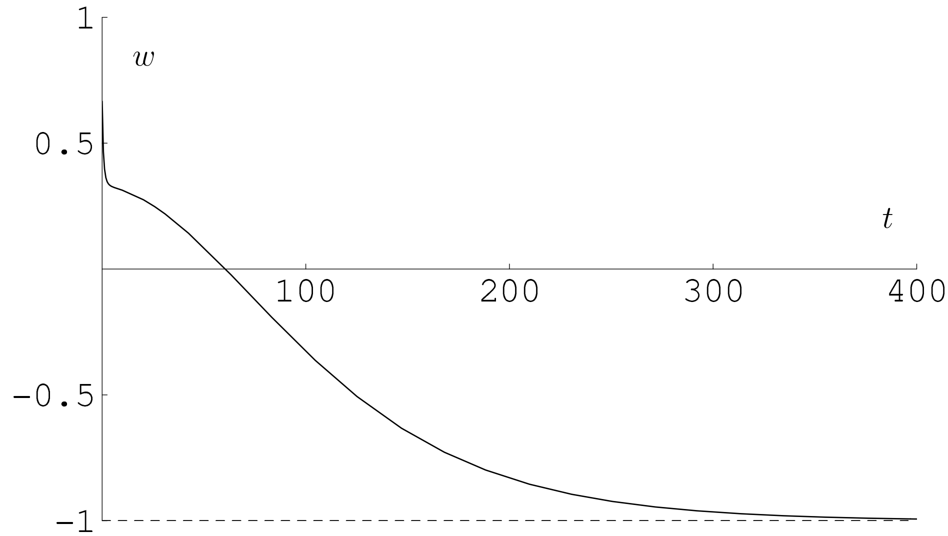
- **In Region III the solutions for any Λ_B have a runaway with $w \to +\infty$ and $\frac{\alpha\rho_m}{M_{Pl}^4} \to -\frac{3}{4}$**

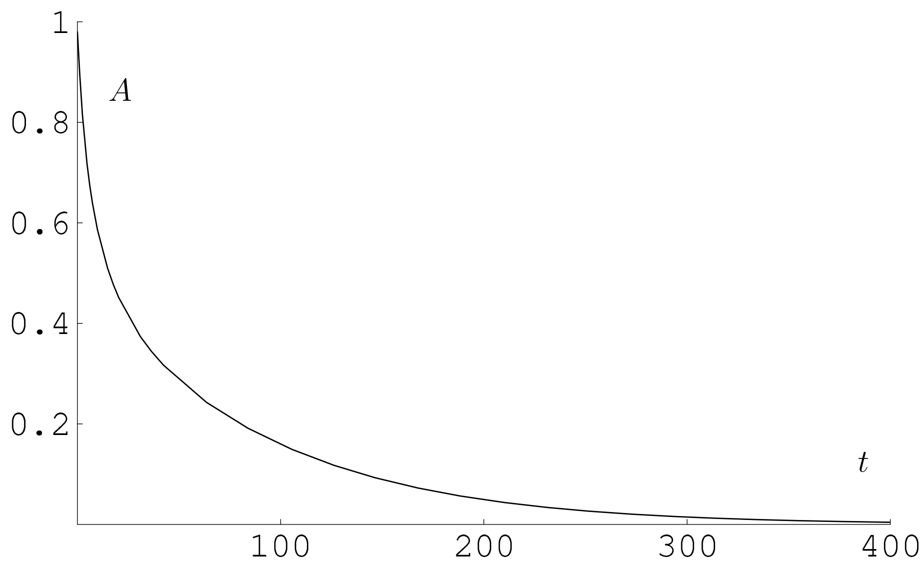
- **Example for $\Lambda_B = 0$**



- **However, since the lines of isotropic tuning are not attractors, most of the evolution is significantly anisotropic, unless the initial conditions are fine tuned**

- **Example for** $0 < \Lambda_B/M_6^4 \ll 1$





$$\textbf{with} \qquad A = \sqrt{\sum_{i=1}^{3} \frac{(\langle H \rangle - H_i)^2}{3\langle H \rangle^2}} = \sqrt{\frac{2}{3}} \left| \frac{a\dot{b}}{\dot{a}b} \right|$$

Conclusions

- **6D models with codimension-2 branes interesting because of their potential self tuning property**
- **General problem with cosmology**
  - **- either singularities**
  - **- or only tension on the brane**
- **Including an induced gravity and/or a Gauss-Bonnet term we can get a 4D Einstein equation on the brane**
- **Extra dimensional components of the equation of motion impose a bulk-brane matter tuning**
  - **- in the induced gravity case**
  - **- in the Gauss-Bonnet case with isotropic evolution**
- **There exist fixed points (attractors) for these evolutions**
  - **-** $w = 1/3$ **if** $\Lambda_B = 0$
  - **-** $w = -1$ **if** $\Lambda_B > 0$
- **In an anisotropic evolution (in the Gauss-Bonnet case) the extra dimensional constraint provides an evolution equation for the anisotropy**
- **Vast regions of parameter space available for cosmology**
- **We have the same fixed points (attractors) as before, but the lines of isotropic tuning are not attractors**