Dynamical radion superfield in five-dimensional action

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1 Introduction

Brane-world

5D SUGRA on S^1/Z_2

Randall-Sundrum metric:

$$
\begin{aligned}
ds^2 &= e^{-2ky}\eta_{mn}dx^m dx^n - dy^2 \\
&= e^{-2kR\vartheta}\eta_{mn}dx^m dx^n - R^2 d\vartheta^2,
\end{aligned}
$$

where $\vartheta = y/R$. $(0 \leq \vartheta \leq \pi)$

The radius R is a dynamical d.o.f. $\Rightarrow$ **Radion**

In SUSY models, the radion forms a chiral supermultiplet, which can play an important role in the mediation of SUSY breaking.

Our purpose

To obtain the superspace action including the radion superfield

Naive Ansatz: $R \to r(x)$

$$ds^2 = e^{-2kr(x)\vartheta} g^{(4)}_{mn}(x) dx^m dx^n - r^2(x) d\vartheta^2. \qquad (1)$$

This is not a solution of E.O.M.,

i.e., $r(x)$ is not a mass-eigenstate.

In general, a 5D field B is mode-expanded as

$$B(x,y) = \sum_n f_{(n)}(y) b_{(n)}(x).$$

If $f_{(n)}(y)$ are not solutions of the mode-equation, $b_{(n)}(x)$ are not mass-eigenstates.

$\Rightarrow$ Higher modes do not decouple in low-energies.

Thus, the ansatz (1) should be corrected.

(non SUSY: Charmousis, Gregory, Rubakov (2000); Csáki, Graesser, Kribs (2001)
pure SUGRA: Bagger, Nemeschansky, Zhang (2001))

Strategy

1. Clarify how the radion mode appears in the superspace action.

2. Promote the radion to a superfield.

2 Radion field in superspace action

We will start from the 5D superconformal (SC) gravity action (Kugo & Ohashi).

Field content:

Gravitational multiplet: $\mathcal{G} = (e_\mu{}^\nu, \psi^i_\mu, \cdots)$

Vector multiplets:

$$\text{Gauge multiplet:} \quad \mathcal{V} = (V, \Phi_S)$$
$$\text{Graviphoton:} \quad (W^0_m, W^0_y)$$

Hypermultiplets: $\mathcal{H}$

$$\text{Matter fields:} \quad (H, H^C)$$
$$\text{Compensator:} \quad \Sigma = 1 - \theta^2 \mathcal{F}_\Sigma, \quad \Sigma^C = 0$$

Boundary fields:

Chiral superfields: S, Vector superfields: U

In our previous work (JHEP0410 (2004) 013),

SC action with fixed $\mathcal{G}$ $\Rightarrow$ Superspace action

$$(\langle e_m{}^n \rangle = e^\sigma \eta^{mn}, \quad \langle e_y{}^4 \rangle = 1, \quad \langle \psi_\mu \rangle = 0, \cdots)$$

Introduction of radion:

Now we will introduce the radion mode $b(x)$ in the action. To do this,

$$\sigma(y) \;\to\; F(b(x), y), \qquad (F(\langle b \rangle, y) = \sigma(y))$$
$$\langle e_y{}^4 \rangle \;\to\; G(b(x), y). \qquad (G(\langle b \rangle, y) = 1)$$

Conditions on F and G:

Condition (I)

The superspace action must reproduce SC action written by the component fields. For example,

$$
\begin{aligned}
& e^{2\sigma}\langle {e_y}^4\rangle \int d^4\theta\ \bar{\Phi}\Phi \\
= \ & e^{2\sigma}\langle {e_y}^4\rangle \eta^{mn}\left\{-\frac{\bar{\varphi}\partial_m\partial_n\varphi}{4} - \frac{\partial_m\partial_n\bar{\varphi}\varphi}{4} + \frac{\partial_m\bar{\varphi}\partial_n\varphi}{2} + \cdots\right\} \\
\Rightarrow \ & e^{2\sigma}\langle {e_y}^4\rangle \eta^{mn}\partial_m\bar{\varphi}\partial_n\varphi + \cdots \\
= \ & \left(e^{4\sigma}\langle {e_y}^4\rangle\right)\left(e^{-2\sigma}\eta^{mn}\right)\partial_m\bar{\varphi}\partial_n\varphi + \cdots \\
= \ & \det\left({e_\mu}^\nu\right) g^{mn}\partial_m\bar{\varphi}\partial_n\varphi + \cdots .
\end{aligned}
$$

After the introduction of the radion,

$$e^{2\sigma}\langle {e_y}^4\rangle \to e^{2F}G.$$

Thus, $e^{2F}G$ must be x^m-independnet. Namely,

$$e^{2F}G = e^{2\sigma}.$$

In fact, this is the necessary and sufficient contidion for superspace action $\Leftrightarrow$ component action.

Condition (II)

The bulk will remain AdS_5 with k when $\langle b\rangle \to \langle b\rangle +$ constant. This condition can be written as

$$G(b(x), y) = -\frac{1}{k}\partial_y F(b(x), y).$$

Solving the above conditions,

$$F = \frac{1}{2}\ln\left(e^{2\sigma} + \tilde{b}\right),$$
$$G = \frac{1}{1 + e^{-2\sigma}\tilde{b}},$$

where $\tilde{b} \equiv b - \langle b \rangle$.

Proper length:

$$r(x) = \frac{1}{\pi}\int_0^{\pi R} dy\ G(b(x), y)$$
$$= R - \frac{1}{2k\pi}\ln\left(\frac{1 + e^{2k\pi R}\tilde{b}(x)}{1 + \tilde{b}(x)}\right),$$

or equivalently,

$$\tilde{b}(x) = e^{-k\pi R}\frac{\sinh k\pi\left(R - r(x)\right)}{\sinh k\pi r(x)}.$$

Namely,

$$G = \left\{1 + e^{-2\sigma}e^{-k\pi R}\frac{\sinh k\pi\left(R - r(x)\right)}{\sinh k\pi r(x)}\right\}^{-1}.$$

Bulk action:

$$\mathcal{L} = \mathcal{L}_{\rm kin}^{\rm rad} + \left\{ \int d^2\theta \, \frac{1}{4} G_{\rm c} \mathcal{W}\mathcal{W} + \text{h.c.} \right\}$$
$$+ e^{2\sigma} \int d^4\theta \; G^{-2} \left(\partial_y V + i\Phi_S - i\bar{\Phi}_S \right)^2$$
$$- e^{2\sigma} \int d^4\theta \; \Big\{ 2M_5^3 \left(\bar{\Sigma}\Sigma \right)^{\frac{3}{2}}$$
$$- G^{\frac{3}{2}} \left(\bar{H} e^{2gV} H + \bar{H}^C e^{-2gV} H^C \right) \Big\}$$
$$+ e^{3\sigma} \left\{ \int d^2\theta \; H^C \left(\frac{\overleftrightarrow{\partial}_y}{2} + m G_{\rm c} - 2ig\Phi_S \right) H + \text{h.c.} \right\},$$

where

$$G_{\rm c} \equiv G - i \frac{W_y^0}{M_5},$$
$$\mathcal{L}_{\rm kin}^{\rm rad} \equiv \frac{3M_5^3 (k\pi)^2}{16} \left(1 - e^{-2k\pi R} \right)^2 \frac{e^{-2\sigma} G^2(r)}{\sinh^4 k\pi r} \eta^{mn} \partial_m r \partial_n r.$$

Origin of the radion:

Warp factor $\quad e^{2F} = e^{2\sigma} G^{-1}$

Fünfbein $\quad \langle e_y{}^4 \rangle \to G$

Graviphoton multiplet $\quad G_{\rm c}$

3 Promotion to the radion superfield

If we define a complex scalar τ as

$$\tau \equiv r + i\frac{w}{M_5}$$

where r is a proper length and

$$w \equiv \frac{1}{\pi}\int_0^{\pi R} dy\, W_y^0, \quad (\langle W_y^0 \rangle = 0 \text{ is assumed.})$$

the kinetic term for τ becomes the Kähler form.

This suggests that

$r(x)$ should be associated with $w(x)$ in the form of $\tau(x)$.

Namely,

$$G_c(r, w) = G(\tau) = \left\{ 1 + e^{-2\sigma(y)} e^{-k\pi R} \frac{\sinh k\pi(R-\tau)}{\sinh k\pi\tau} \right\}^{-1}.$$

Thus, G and W_y^0 should be understood as

$$G \equiv \mathrm{Re}\, G(\tau), \qquad W_y^0 \equiv -M_5 \mathrm{Im}\, G(\tau).$$

Promotion to superfield:

$$G_c \;\to\; G(T) = \left\{ 1 + e^{2ky} e^{-k\pi R} \frac{\sinh k\pi(R-T)}{\sinh k\pi T} \right\}^{-1},$$

$$G \;\to\; G_R \equiv \mathrm{Re}\, G(T).$$

Result:

The resulting Lagrangian is (we set $\Sigma = 1$)

$$\mathcal{L} = \left\{ \int d^2\theta \, \frac{1}{4} G(T) \mathcal{W}\mathcal{W} + \text{h.c.} \right\}$$
$$+ e^{2\sigma} \int d^4\theta \, G_{\rm R}^{-2} \left(\partial_y V + i\Phi_S - i\bar{\Phi}_S \right)^2$$
$$+ e^{2\sigma} \int d^4\theta \, G_{\rm R}^{\frac{3}{2}} \left(\bar{H} e^{2gV} H + \bar{H}^C e^{-2gV} H^C \right)$$
$$+ e^{3\sigma} \left\{ \int d^2\theta \, H^C \left(\frac{\overset{\leftrightarrow}{\partial}_y}{2} + mG(T) - 2ig\Phi_S \right) H + \text{h.c.} \right\}$$
$$- e^{2\sigma} \int d^4\theta \, 3M_5^3 \ln G_{\rm R}$$
$$+ \sum_{\vartheta^* = 0,\pi} \mathcal{L}_{\rm brane}^{(\vartheta^*)} \delta(y - R\vartheta^*),$$

where the brane Lagrangians are

$$\mathcal{L}_{\rm brane}^{(\vartheta^*)} = \left\{ \int d^2\theta \, f_{AB}^{(\vartheta^*)}(S) \mathcal{W}^A \mathcal{W}^B + \text{h.c.} \right\}$$
$$- e^{2\sigma} \int d^4\theta \, G_{\rm R}^{-1} \exp\left\{ -K^{(\vartheta^*)}(S, \bar{S}, U) \right\}$$
$$+ e^{3\sigma} \left\{ \int d^2\theta \, G^{-\frac{3}{2}}(T) P^{(\vartheta^*)}(S) + \text{h.c.} \right\}.$$

We have assumed that $\mathcal{N} = 1$ SUSY is preserved.

4 Supersymmetric radius stabilization

(Maru & Okada, 2003)

Stabilization sector:

$\mathcal{H} = (H, H^C)$ with the bulk mass m

Cosmological constant $\Lambda < 0$: $\Rightarrow$ $\sigma(y) = -ky$

Boundary terms:

$$P^{(0)} = J_0 H, \quad P^{(\pi)} = J_\pi H. \quad (J_0,\ J_\pi\text{: constants})$$

Radion potential:

$$V_{\rm rad}(\bar{\tau}, \tau) = \frac{\left|\frac{1}{2} G_0^{-\left(\frac{3}{4}+\frac{m}{2k}\right)}(\tau)\right|^2 \left|J_0 - J_\pi e^{-\left(\frac{3}{2}k+m\right)\pi\tau}\right|^2}{\int_0^{\pi R} dy\ e^{(k-2m)y} G_{\rm R}^{\frac{3}{2}} \left|G^{-\frac{m}{2k}}(\tau)\right|^2} + \mathcal{O}(\epsilon^4),$$

where $\epsilon \equiv |J_\pi| / M_5^{3/2}$.

$\Rightarrow$ The radius is stabilized with $\tau = \dfrac{\ln(J_\pi/J_0)}{\left(\frac{3}{2}k+m\right)\pi}$.

Radion mass:

$$m_{\rm rad}^2 = \left(\mathcal{K}_{T\bar{T}}^{(4)}\right)^{-1} \left.\frac{\partial^2 V_{\rm rad}}{\partial\tau\partial\bar{\tau}}\right|_{\tau=R}$$

$$= \frac{\epsilon^2 k^2}{6}\left(1 - \frac{2m}{k}\right)\left(\frac{3}{2} + \frac{m}{k}\right)^2 e^{-2k\pi R} \frac{1 - e^{-2k\pi R}}{1 - e^{-(k-2m)\pi R}} + \mathcal{O}(\epsilon^4).$$

$= \{$Result obtained by solving E.O.M.$\}$

5 Summary

- We derived 5D $\mathcal{N} = 1$ superspace action including the dynamical radion superfield.
- We can read off the couplings of the radion superfield to the matter superfields from our action.
- The correct radion mass is obtained by our action.

Future work

- Extention of our action including the compensators
- Scherk-Schwarz breaking as spontaneous ~~SUSY~~ (in the flat case)