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The Electroweak Phase Transition on Orbifolds with Gauge-Higgs Unification

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Introduction

Theories in extra dimensions are promising alternatives to more standard four-dimensional extensions of the Standard Model (SM).

An interesting scenario is the possibility of identifying the Higgs field with the internal component of a gauge field (gauge-Higgs unification (GHU)) [Manton (1979); Fairlie (1979); Forgacs and Manton (1980)]:

$$A_M \longrightarrow \begin{cases} A_\mu \Rightarrow \text{4D EW bosons + K.K. towers} \\ A_i \Rightarrow \text{4D scalars (the Higgs) + K.K. towers} \end{cases}$$

In 5D GHU models the electroweak symmetry breaking occurs radiatively and is equivalent to a Wilson line symmetry breaking

[Scherk and Shwarz (1979); Hosotani (1983, 1989); Witten (1985); Ibáñez, Nilles and Quevedo (1987)].

As a consequence of the gauge inveriance the Higgs potential is entirely radiatively generated and non-local in the extra dimensions.

$$\Downarrow$$

GHU theories are a possible framework to solve the little hierarchy problem.

Wilson line dynamics at T=0

We consider a 5D gauge theory compactified on $S^1/\mathbb{Z}_2$

The one-loop contribution of a massless bosonic (fermionic) degree of freedom with charge q to the Higgs effective potential is

$$V(\alpha) = (-)^{2\eta-1} \frac{3}{4\pi^2 L^4} \sum_{\tilde{k}=1}^{\infty} \frac{1}{\tilde{k}^5} \cos(2q\tilde{k}\pi\alpha)$$

where the Higgs VEV is $\langle H \rangle = \frac{2i\alpha}{g_4 R}$ and $L = 2\pi R$, and $\eta = 0\ (1/2)$.

- Bosonic contributions tend to restore the symmetry
- Fermionic contributions tend to break the symmetry

$\Downarrow$

for a suitable field content a **spontaneous symmetry breaking** occurs.

Finite temperature effective potential

The finite temperature effective potential is obtained by compactifying the Euclidean time direction on a circle of radius $1/(2\pi T)$.

Bosons	$\longrightarrow$	periodic boundary conditions
Fermions	$\longrightarrow$	anti-periodic boundary conditions

The one-loop contribution to the effective potential becomes

$$V(T,\alpha) = (-)^{2\eta-1}\frac{3LT^5}{4\pi^2}\sum_{\tilde{k}=1}^{\infty}\sum_{\tilde{m}=-\infty}^{\infty}\frac{(-)^{2\eta\tilde{m}}}{[\tilde{m}^2+(LT\tilde{k})^2]^{\frac{5}{2}}}\cos(2q\tilde{k}\pi\alpha)$$

- Using a class of background field gauge-fixing Lagrangians it can be shown that the one-loop effective potential is gauge invariant both at zero and finite temperature.

The high temperature behaviour

The bosonic contribution to the Higgs thermal mass at high temperature is

$$M_H^2(T,0) = \frac{g_4^2}{16\pi^2}\left(\frac{2}{3}\pi^2 q^2 n_B\right)\frac{T}{L} + \cdots$$

the fermionic one is

$$M_H^2(T,0) = -\frac{g_4^2}{16\pi^2}\left(2\sqrt{2}\pi^2 q^2 n_F\right)\frac{T}{L}(LT)^{3/2} e^{-\pi LT} + \cdots$$

At high temperature the bosonic contribution dominates

$$\Downarrow$$

Symmetry restoration (EWPT)

- Difference with the 4D SM in which $M_H^2 \sim T^2$ for both fermions and bosons.

Properties of the phase transition

The contribution to the eff. potential of a charge q_B massless boson for $LT > 1$ is

$$V_B(T,\alpha) \simeq -\frac{T}{\pi^2 L^3} \sum_{\tilde{k}=1}^{\infty} \frac{\cos(2\pi \tilde{k} q_B \alpha)}{\tilde{k}^4}$$

this expression is equal to

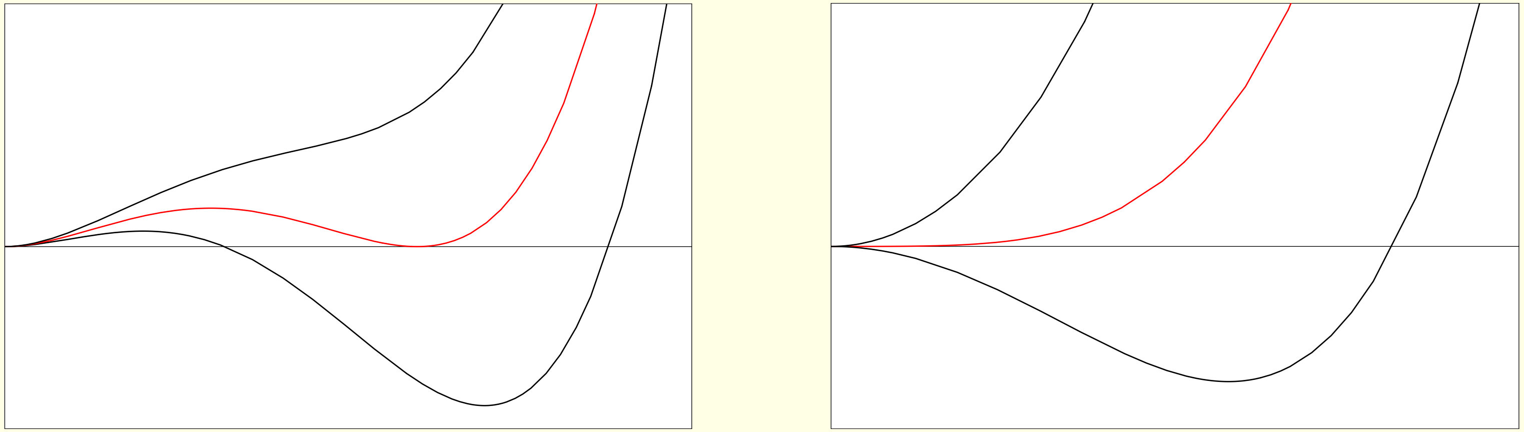
$$V_B(T,\alpha) = -\frac{\pi^2 T}{3L^3} \left[\frac{1}{30} - (q_B\alpha)^2 + 2(q_B\alpha)^3 - (q_B\alpha)^4 \right]$$

The presence of cubic terms in the expansion of the effective potential is responsible for the appearence of a **first order phase transition**

- This mechanism is similar to what happens in the 4D SM.
- This mechanism in 5D is peculiar of Wilson line phases, in the case of standard scalar fields cubic terms are not present [Dienes, Dudas, Gherghetta and Riotto (1999)]

Properties of the phase transition

First order phase transition | Second order phase transition



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The contribution of fermions and massive bosons does not give cubic terms.

The whole effective potential has the schematic form

$$\frac{L^4}{\pi^2}V(T,\alpha) = a(LT)\alpha^2 - b(LT)\alpha^3 + c(LT)\alpha^4$$

- the Higgs mass depends on the charge q_F of the fermions (for $q_F > q_B$):

$$M_H^2 \simeq \frac{g_4^2}{16\pi^2}\frac{1}{L^2}24q_F^2\zeta(3)$$

The condition for the phase transition is nearly equivalent to $a(LT_c) = 0$.

$$\Downarrow$$

- $T_C \sim 1/L$ and has a logarithmic dependence on the charges (q_B and q_F)

- the jump in the order parameter is $\alpha_{min}(LT_C) = \dfrac{b(LT_C)}{2c(LT_C)} \simeq \dfrac{6q_B}{\pi^2 q_F^2}$

A good parameter to measure the phase transition "strength" is

$$\frac{|H(T_C)|}{T_C} = \frac{2\alpha_{min}(T_C)}{g_4 R T_C}$$

To get baryogenesis at the EWPT one necessary requirement is

$$\frac{|H(T_C)|}{T_C} > 1$$

In our 5D gauge-Higgs unification models

$$\frac{|H(T_C)|}{T_C} \sim \frac{q_B}{q_F^2}$$

In this simple scenario, when high charge bulk fermions are present:

- the Higgs mass gets larger
- BUT the phase transition strength decreases

The model

We studied the EWPT in a 5D gauge theory with gauge group
$G = SU(3)_c \times SU(3)_w$ on an $S^1/\mathbb{Z}_2$ orbifold [Scrucca, Serone and Silvestrini (2003)].

The $\mathbb{Z}_2$ projection is embedded in the $SU(3)_w$ gauge group by

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \Rightarrow \quad SU(3)_w \rightarrow SU(2) \times U(1)$$

A charged 4D scalar Higgs doublet arises from A_5.

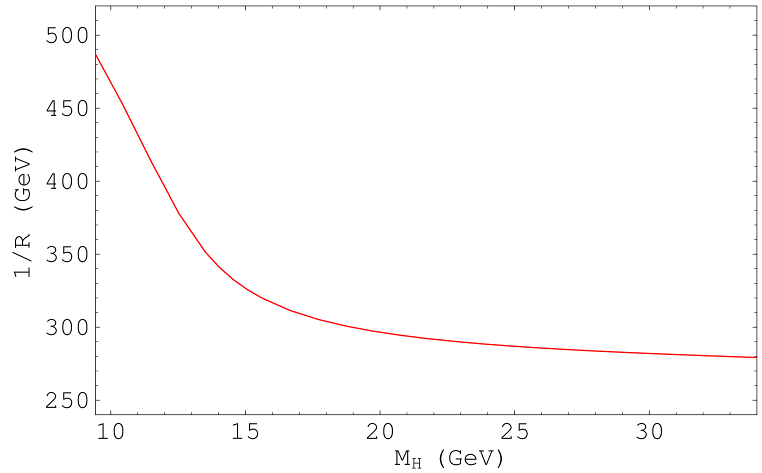
A VEV for A_5 induces a spontaneous symmetry breaking:

$$SU(2) \times U(1) \rightarrow U(1)_{EM}$$

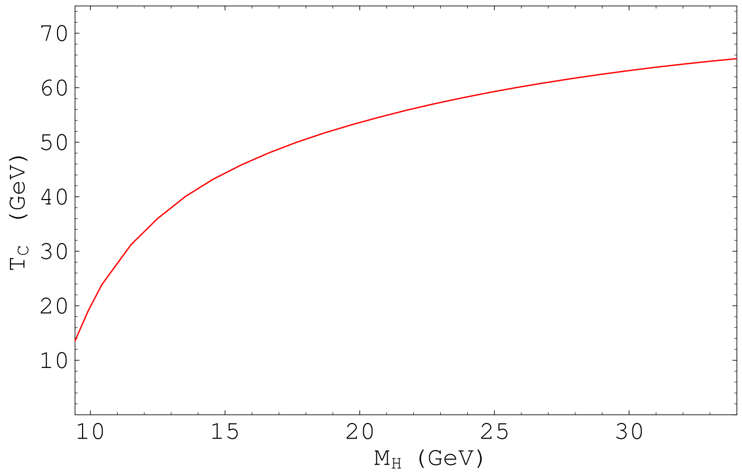
Bulk fermions are present in the representations $(\mathbf{3}, \mathbf{3})$, $(\bar{\mathbf{3}}, \mathbf{6})$, $(\mathbf{1}, \mathbf{3})$ and $(\mathbf{1}, \mathbf{6})$ of G, which couple to the 4D matter fields localized at the orbifold fixed points.

Numerical results: basic model

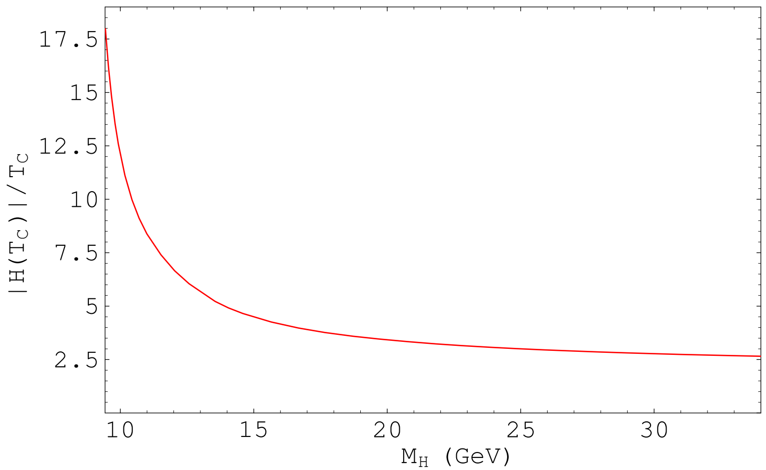
Compactification radius



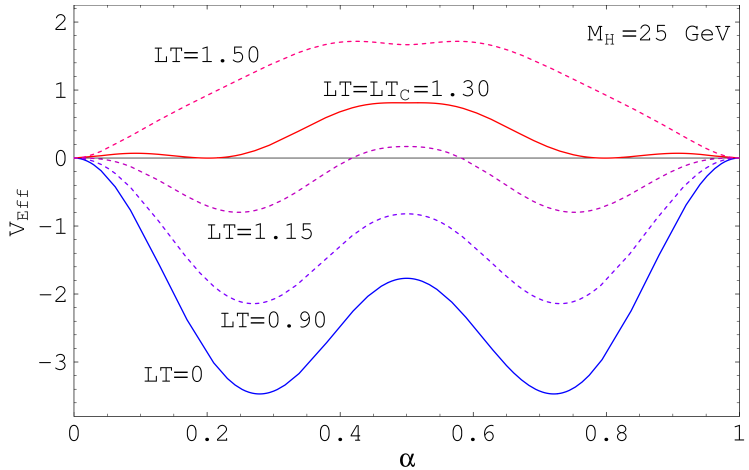
Critical temperature



Phase transition strength

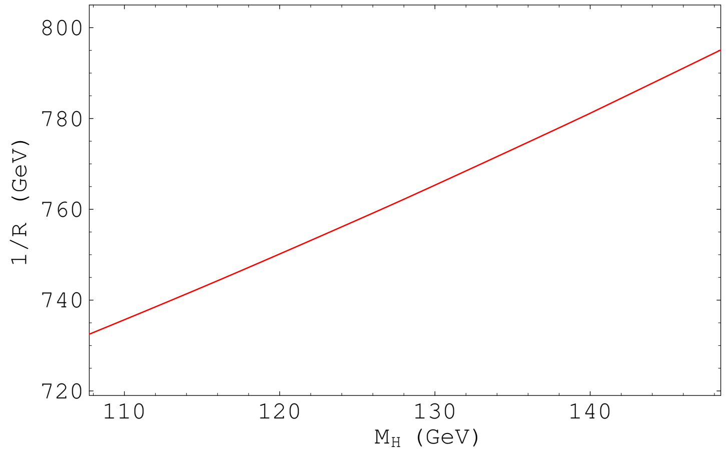


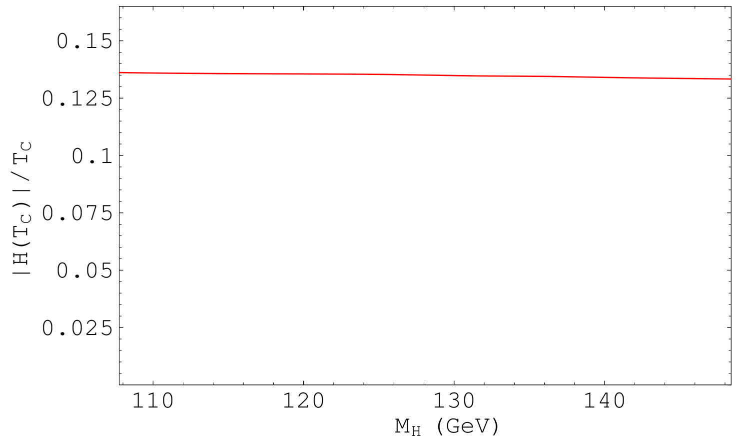
Effective potential



Numerical results: high rank fermions

Compactification radius

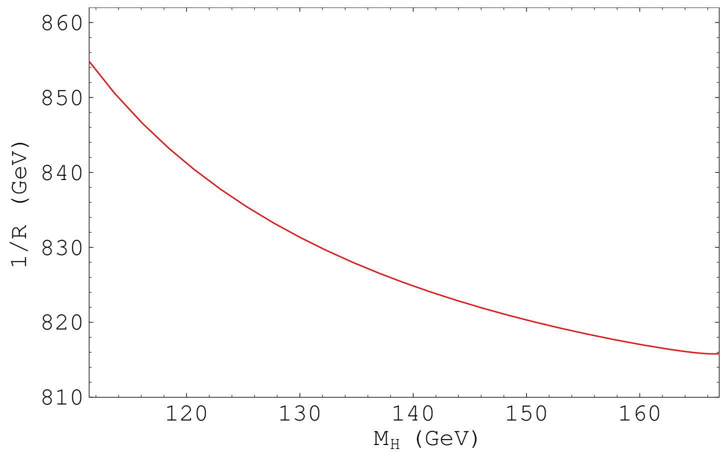


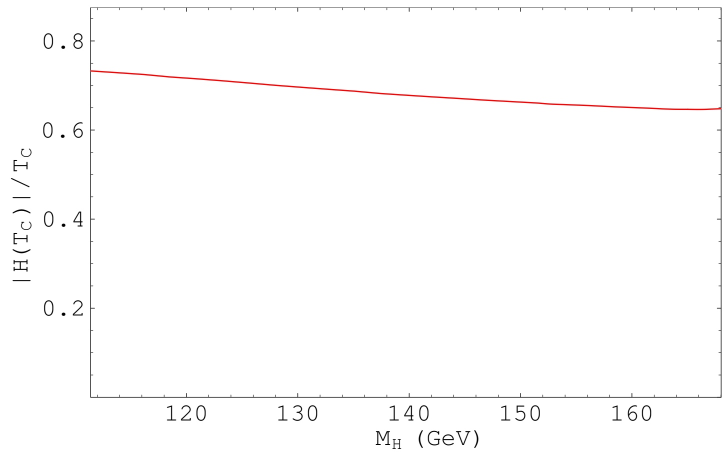


Phase transition strength

Numerical results: localized gauge kinetic terms

Compactification radius





Phase transition strength

Conclusions

We have studied the behaviour of Wilson line phases at finite temperature

Main features:

- At $T = 0$ a spontaneous symmetry breaking occurs
- An EW phase transition appears at $T \sim 1/L$ and is typically of first order

In the basic model:

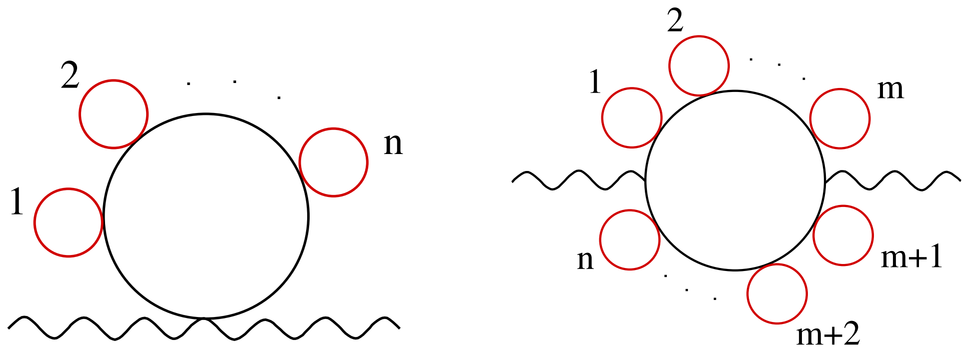
- The phase transition is very strong
- BUT the Higgs mass is usually too low ($M_H < M_W/2$)

Possible extensions:

- Bulk fermions in large rep. of $SU(3)_w$ $\Rightarrow$ { reasonable Higgs masses; BUT very weak phase transition }
- Large localized gauge kinetic terms $\Rightarrow$ { reasonable Higgs masses; moderately strong phase transition }

Higher-loop corrections

To study the stability of the one-loop potential under radiative corrections, we computed the leading IR divergent contributions (Daisy diagrams) in 5D scalar QED.



Due to their IR divergent behaviour Daisy diagrams (I_n) can affect the effective potential at small α.

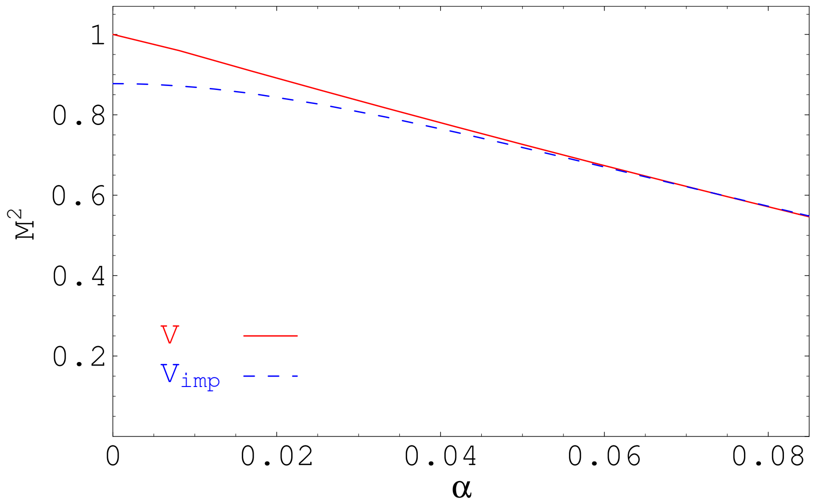
Their effective expansion parameter is $$\gamma \equiv \lim_{n\to\infty} \frac{I_n}{I_{n-1}} \simeq -\frac{\lambda L T}{48\pi^2}\frac{1}{\alpha^2}$$

- $\gamma < 0$ $\Rightarrow$ some cancellation between daisy diagrams is expected

Daisy diagrams can be resummed by adding and subtracting to the Lagrangian the one-loop (thermal) mass correction for any field [Parwani (1993)].

A good estimate of daisy-contributions is performed comparing the Wilson line thermal mass with and without resummation.

For $\lambda \sim 1$ and $LT \sim 1$:



- Deviations are small for $LT \lesssim 1$. They become relevant ($\sim 30\%$) for $LT \sim 10$, thus well above the typical phase transition temperature.