One-loop Renormalisation of $\mathcal{N} = \frac{1}{2}$ Supersymmetry

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Introduction

- Introduction to $\mathcal{N} = \frac{1}{2}$.
- $U(N)$ version of theory - and why we can't use it.
- $U(1) \times SU(N)$ theory.
- Calculation and removal of divergences.
- Summary.

$\mathcal{N} = \frac{1}{2}$ Supersymmetry

- $\mathcal{N} = \frac{1}{2}$ SUSY arises when superspace is deformed, i.e. the superspace coordinates no longer anticommute, but instead satisfy the condition

$$\{\theta^\alpha, \theta^\beta\} = C^{\alpha\beta}, \qquad \alpha = 1, 2.$$

- This condition means that products of functions of θ should be reordered according to the star product

$$f(\theta) * g(\theta) = f(\theta) exp\left(- \frac{C^{\alpha\beta}}{2} \frac{\overleftarrow{\partial}}{\partial\theta^\alpha} \frac{\overrightarrow{\partial}}{\partial\theta^\beta} \right) g(\theta).$$

- The star product leads to new terms in the action involving the non-anti-commutativity parameter C. These terms sometimes arise in the form $C^{\mu\nu}$, where

$$C^{\mu\nu} = C^{\alpha\beta}\epsilon_{\beta\gamma}(\sigma^{\mu\nu})_{\alpha}{}^{\gamma},$$
$$\sigma^{\mu\nu} = \tfrac{1}{4}(\sigma^{\mu}\bar{\sigma}^{\nu} - \sigma^{\nu}\bar{\sigma}^{\mu}),$$
$$\sigma^{\mu} \equiv (1, \sigma^{i}), \qquad \bar{\sigma}^{\mu} \equiv (1, -\sigma^{i}).$$

- The supersymmetry transformations are modified from the $\mathcal{N} = 1$ case.

U(N) and SU(N) Theories

- Original lagrangian for pure $\mathcal{N}=\frac{1}{2}$ supersymmetric gauge theory is

$$\begin{aligned} S = & \int \quad d^4x \Big[tr\{-\tfrac{1}{2}F^{\mu\nu}F_{\mu\nu} - 2i\bar{\lambda}\bar{\sigma}^{\mu}(D_{\mu}\lambda) + D^2\} \\ & - \quad 2igC^{\mu\nu}tr\{F_{\mu\nu}\bar{\lambda}\bar{\lambda}\} + g^2|C|^2 tr\{(\bar{\lambda}\bar{\lambda})^2\}\Big]. \end{aligned}$$

- This action is invariant under the transformations

$$\begin{aligned} \delta A_{\mu}^{A} &= -i\bar{\lambda}^{A}\bar{\sigma}_{\mu}\epsilon, \\ \delta\lambda_{\alpha}^{A} &= i\epsilon_{\alpha}D^{A} + (\sigma^{\mu\nu}\epsilon)_{\alpha}\left[F_{\mu\nu}^{A} + \tfrac{1}{2}iC_{\mu\nu}d^{ABC}\bar{\lambda}^{B}\bar{\lambda}^{C}\right], \\ \delta\bar{\lambda}_{\dot{\alpha}}^{A} &= 0, \\ \delta D^{A} &= -\epsilon\sigma^{\mu}D_{\mu}\bar{\lambda}^{A}. \end{aligned}$$

- Important relations for the groups $U(N)$ and $SU(N)$

$$U(N): \qquad [R^A, R^B] = if^{ABC}R^C, \qquad \{R^A, R^B\} = d^{ABC}R^C.$$
$$SU(N): \qquad [R^a, R^b] = if^{abc}R^c, \qquad \{R^a, R^b\} = d^{abc}R^c + \frac{1}{N}\delta^{ab}.$$

- $U(N)$ cannot be renormalised.
- Can $SU(N)$ be renormalised?

  -Yes, in the pure gauge situation
  (I. Jack, D.R.T. Jones and LW, Phys. Lett. **B**611 (2005) 199).

  -No, not in the $SU(N)$ gauge theory coupled to chiral matter.

$U(1) \times SU(N)$

- We propose that the $U(N)$ theory is replaced by a $U(1) \times SU(N)$ theory.
- The action we suggest is

$$
\begin{aligned}
S \;=\; & \int d^4x \Big\{ -\tfrac{1}{4} F^{\mu\nu A} F^A_{\mu\nu} - i\bar{\lambda}^A \bar{\sigma}^\mu (D_\mu \lambda)^A + \tfrac{1}{2} D^A D^A \\
& - \; \tfrac{1}{2} i C^{\mu\nu} \Big[g d^{abc} F^a_{\mu\nu} \bar{\lambda}^b \bar{\lambda}^c + \frac{g^2}{g_0} d^{0bc} F^0_{\mu\nu} \bar{\lambda}^b \bar{\lambda}^c \\
& + \; 2 d^{ab0} g_0 F^a_{\mu\nu} \bar{\lambda}^b \bar{\lambda}^0 + d^{000} g_0 F^0_{\mu\nu} \bar{\lambda}^0 \bar{\lambda}^0 \Big] \\
& + \; \tfrac{1}{8} g^2 |C|^2 d^{abe} d^{cde} (\bar{\lambda}^a \bar{\lambda}^b)(\bar{\lambda}^c \bar{\lambda}^d) + \tfrac{1}{4N} \frac{g^4}{g_0^2} |C|^2 (\bar{\lambda}^a \bar{\lambda}^a)(\bar{\lambda}^b \bar{\lambda}^b) \\
& - \; \tfrac{1}{2N} g_0^2 |C|^2 (\bar{\lambda}^a \bar{\lambda}^a)(\bar{\lambda}^0 \bar{\lambda}^0). \Big\}
\end{aligned}
$$

- The action is invariant under the transformations

$$
\begin{aligned}
\delta A_\mu^A &= -i\bar\lambda^A\bar\sigma_\mu\epsilon, \\
\delta\lambda_\alpha^a &= i\epsilon_\alpha D^a + (\sigma^{\mu\nu}\epsilon)_\alpha\left[F_{\mu\nu}^a + \tfrac{1}{2}iC_{\mu\nu}(gd^{abc}\bar\lambda^b\bar\lambda^c + 2g_0 d^{ab0}\bar\lambda^b\bar\lambda^0)\right], \\
\delta\lambda_\alpha^0 &= i\epsilon_\alpha D^0 + (\sigma^{\mu\nu}\epsilon)_\alpha\left[F_{\mu\nu}^0 + \tfrac{1}{2}iC_{\mu\nu}(\frac{g^2}{g_0}d^{0bc}\bar\lambda^b\bar\lambda^c + g_0 d^{000}\bar\lambda^0\bar\lambda^0)\right], \\
\delta\bar\lambda_{\dot\alpha}^A &= 0, \\
\delta D^A &= -\epsilon\sigma^\mu D_\mu\bar\lambda^A.
\end{aligned}
$$

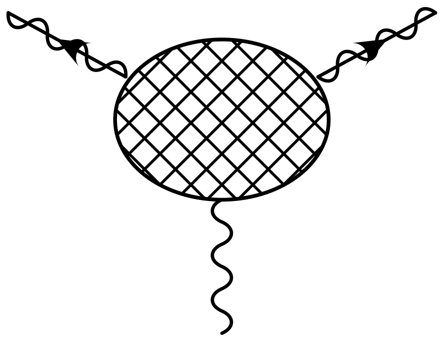
Renormalisation

- Our aim is to determine whether:

  the divergences that appear in 1-loop diagrams can simply be removed by replacing fields and couplings with bare fields and bare couplings, without the need to add further terms to make the result finite.

Calculation of Divergences

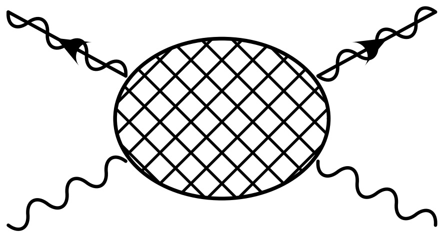
- One-loop graphs contributing to standard terms in lagrangian are as in the $\mathcal{N} = 1$ case.
- Terms containing C's are all that need to be considered.
- For example: term $C^{\mu\nu} d^{ABC} \partial_\mu A^A_\nu \bar{\lambda}^B \bar{\lambda}^C$:



This set of diagrams gives anomalous terms of the form

$$C^{\mu\nu} d^{abc} A^a_\rho \bar{\lambda}^b \bar{\sigma}^\rho_\nu \partial_\mu \bar{\lambda}^c, \qquad C^{\mu\nu} d^{a0c} A^a_\rho \bar{\lambda}^0 \bar{\sigma}^\rho_\nu \partial_\mu \bar{\lambda}^c.$$

- Similarly, the term $C^{\mu\nu} d^{ABE} f^{CDE} A^C_\mu A^D_\nu \bar{\lambda}^A \bar{\lambda}^B$



produces anomalous results of the kind

$$C^{\mu\rho} d^{cde} f^{abe} A^c_\mu A^d_\nu \bar{\lambda}^a \bar{\sigma}^\nu_\rho \bar{\lambda}^b.$$

- These terms can be removed by an unusual divergent field redefinition

$$\delta\lambda^A = -\tfrac{1}{2}NLg^2C^{\mu\nu}(gd^{abc}\sigma_\mu\bar{\lambda}^c A^b_\nu + 2g_0\sigma^\mu\bar{\lambda}^0 A^b_\nu), \quad L = \frac{1}{16\pi^2\epsilon}\ .$$

which results in a change in the action:

$$\delta S_\lambda = \int d^4x[-i\bar{\lambda}^A\bar{\sigma}^\mu\partial_\mu(\delta\lambda^A) + igf^{ABC}\bar{\lambda}^A\bar{\sigma}^\mu A^B_\mu(\delta\lambda^C)]$$

- All C-dependent terms are now in the desired form.

Bare Quantities

- The divergences are now removed by replacing fields and couplings with bare fields and bare couplings.

$$\lambda_B^a = Z_\lambda^{\frac{1}{2}} \lambda^a, \qquad \lambda_B^0 = Z_{\lambda^0}^{\frac{1}{2}} \lambda^0, \qquad A_B^{\mu a} = Z_A^{\frac{1}{2}} A^{\mu a},$$

$$A_B^{\mu 0} = Z_{A^0}^{\frac{1}{2}} A_\mu^a, \qquad g_B = Z_g g, \quad g_{0B} = Z_{g_0} g, \qquad C^{\mu\nu} = Z_c C^{\mu\nu}.$$

- The renormalisation constants for the fields are the same as in the $\mathcal{N} = 1$ supersymmetric theory and are given by

$$\begin{aligned} Z_\lambda &= 1 - g^2 L(2\alpha N + 2), \\ Z_A &= 1 + g^2 L[(3-\alpha)N - 2], \\ Z_g &= 1 + g^2 L(1 - 3N). \end{aligned}$$

- The only renormalisation constant left to determine is Z_C and we find

$$Z_C = 1$$

  That is, the non-anti-commutativity parameter C is not renormalised.

- Combining the field redefinition and the replacement of the fields and couplings with the bare quantities, we find

  $U(1) \times SU(N)$ pure gauge theory is renormalisable.

- What about gauge theory coupled to chiral matter?
- Requires usual field redefinition $\delta\lambda$.

  - Also requires redefinition of $\bar{F}$.

  - Redefinition of $\bar{F}$ also required in uneliminated component formalism of $\mathcal{N} = 1$ Susy.
- We then find that $U(1) \times SU(N)$ gauge theory coupled to chiral matter is also renormalisable.
  (For details see hep-th 0505248, I. Jack, D.R.T. Jones, LW.)

Summary

- $U(N)$ $\mathcal{N} = \frac{1}{2}$ SUSY can't be renormalised.
- $SU(N)$ $\mathcal{N} = \frac{1}{2}$ pure gauge theory can be renormalised only by implementing a divergent field redefinition.
- $U(1) \times SU(N)$ can be renormalised if a divergent field redefinition is implemented in the pure gauge theory case and in the case of the gauge theory coupled to chiral matter.