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Stability of Extra Dimension in Warped Mirabelli-Peskin Model

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cf. S.Ichinose and A.M.,Physics Letters B593 (2004) 242
S.Ichinose and A.M.,hep-th/0409193

Contents

§1 Introduction

- Problems in extra dimension model:

  (i) Radius stabilization.

  (ii) Power law running of loop corrections.

- We present a toy model

{ based on 5D super Yang-Mills (Mirabelli-Peskin) model
with U(1) gauge group and Fayet-Iliopoulos D-term
in Warped extra dimension.

$\Downarrow$ 1-loop correction to $V_{eff} \Rightarrow$ (ii)

New mechanism for (i) Radius stabilization.

§2 A toy model based on Mirabelli-Peskin Model in AdS$_5$

Consider the 5D space-time with the signature $(+----)$.
The extra dimension x^5 is compactified on S^1/Z_2 of size l.

- Anti-deSitter space metric:

$$ds^2 = \frac{1}{k^2 z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2) \quad (\mu, \nu = 0, 1, 2, 3)$$

$$z \equiv \exp(k|x^5|)/k = \exp(kr|y|)/k$$

$$y \equiv (\pi/l)x^5, -\pi \le y \le \pi$$

- Radion chiral multiplet T:

$$T = r + ib + \theta\psi_R + \theta^2 F_T$$

r: the radius of S^1, $\quad l \equiv \pi\langle r\rangle$

b: the fifth component of graviphoton

ψ_R: the fifth component of right-handed gravitino

F_T: a complex auxiliary field

We do not incorporate other 5D SUGRA multiplets.

$\Rightarrow$ Rigid SUSY on nontrivial gravitational background.

- Two branes: $\begin{cases} \text{"Non-TeV" brane at } z = 1/k \ (y = 0) \\ k \text{ : AdS curvature radius} \\ \text{TeV brane at } z = 1/q \ (y = \pi) \\ q = k \exp(-kl) \end{cases}$

$$M_{Pl}^2 = \frac{3M_5^3}{k}\left(1 - \frac{q^2}{k^2}\right) = \frac{3M_5^3}{k}\left(1 - e^{-2k\langle r\rangle}\right) \equiv \langle f(r)\rangle$$

- Assumption: $q = \mathrm{O(TeV)} \ll k < M_{Pl}$

- Radion kinetic term: $\dfrac{\partial^2 f}{\partial T^\dagger \partial T}\partial^\mu T^\dagger \partial_\mu T$

- Bulk fields (5D vector supermultiplet)

$$\left.\begin{array}{ll} A^M(M=0,1,2,3,5) & \text{vector} \\ \Phi & \text{scalar} \\ \lambda^i(i=1,2) & \text{symplectic Majorana} \\ X^a(a=1\sim 3) & \text{auxiliary scalar} \end{array}\right\} \overset{Z_2}{\Rightarrow} \left\{\begin{array}{l} V(Z_2 \text{ even}) \\ \\ \Sigma(Z_2 \text{ odd}) \end{array}\right.$$

$$\mathcal{N}=1$$

$V \equiv (A_\mu, \lambda_L, D)$ 4D vector supermultiplet

$\Sigma \equiv (\Phi + iA_5, -i\sqrt{2}\lambda_R, \mathcal{G})$ 4D chiral scalar supermultiplet

$$D \equiv X^3 - \mathcal{D}_5\Phi,\ \mathcal{D}_M \equiv \partial_M - igA_M,\ \mathcal{G} \equiv X^1 + iX^2$$

- No hypermultiplet!

- Matter fields confined on the "Non-TeV" brane:

$$\boxed{S_i \equiv (\phi_i, \psi_i, F_i) \quad (i = 1 \sim 3)}$$ chiral scalar multiplets

- Superpotential:

$$W(S) = \frac{1}{3}\left(\frac{\lambda}{3!}S_1 S_2 S_3 + \frac{\lambda'}{3}S_1^2 S_3 + \frac{\lambda''}{3}S_2^2 S_3\right)$$

$$W(S)|_{\theta^2} = \frac{1}{2}[\lambda(\phi_1\phi_2F_3 + \phi_2\phi_3F_1 + \phi_3\phi_1F_2)$$

$$+(\lambda'\phi_1^2 + \lambda''\phi_2^2)F_3 + \text{terms containing fermions}]$$

- Assumptions:

$$\boxed{\begin{array}{l}\langle\phi_1\rangle = \langle\phi_2\rangle = \eta, \langle\phi_3\rangle = 0, \langle F_1\rangle = \langle F_2\rangle = 0, \langle F_3\rangle = f \\ \lambda = \lambda' = \lambda''\end{array}}$$

- Gauge group: U(1)
- 5D actions:

$$S_{gauge} = \int d^4x dz \sqrt{G} \left[-\tfrac{1}{2}(F_{MN})^2 + (\mathcal{D}_M \Phi)^2 + i\bar{\lambda}^i \gamma^M \nabla_M \lambda^i \right.$$

$$\left. +(X^a)^2 + g\bar{\lambda}^i \Phi \lambda^i - 2m_\Phi^2 \Phi^2 - 2im_\lambda \bar{\lambda}^i (\sigma_3)^{ij} \lambda^j \right]$$

$$G = \det(G_{MN}),\ \nabla_M \equiv \mathcal{D}_M + \Gamma_M$$

Γ_M: spin connection

$$m_\Phi^2 = -4k^2 + 4k\{\delta(z - 1/k) - \delta(z - 1/q)\}$$

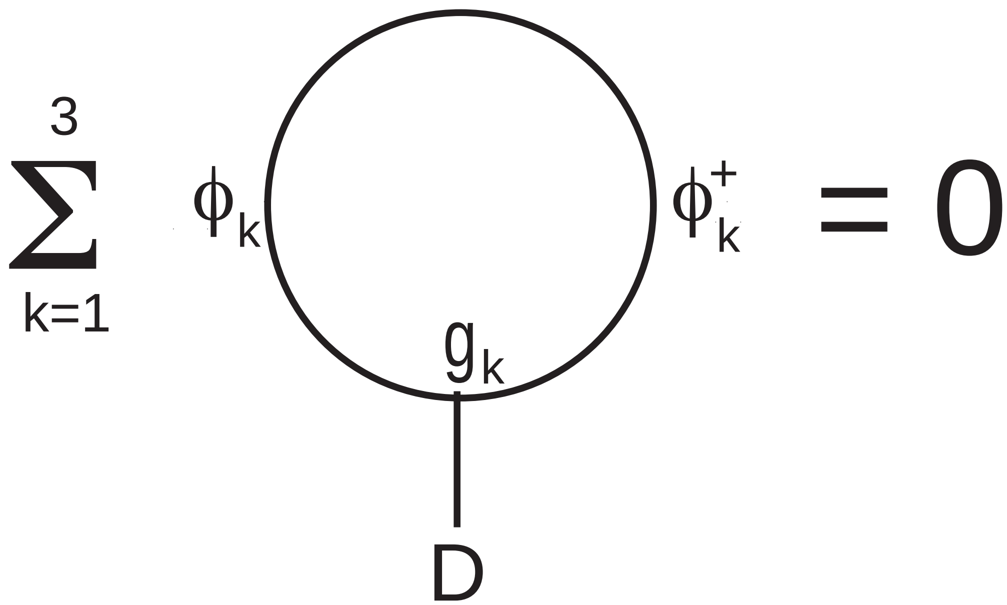
$$m_\lambda = (1/2)k\epsilon(z - 1/k)$$

(Field redefinition has been done.)

$$
\begin{aligned}
S_{matter} \;=\; \int d^4x dz \Bigg[\sum_{i=1}^{3} [\mathcal{D}_\mu \phi_i^\dagger \mathcal{D}^\mu \phi_i + \psi_i^\dagger i\bar{\sigma}^\mu \mathcal{D}_\mu \psi_i + F_i^\dagger F_i \\
-\sqrt{2} g_i (\phi_i^\dagger \lambda_L^T \sigma^2 \psi_i + \psi_i^\dagger \sigma^2 \lambda_L^* \phi_i) + g_i \phi_i^\dagger D \phi_i \Bigg] + \xi D \\
+ \Bigg[\frac{1}{2} \{ \lambda(\phi_1\phi_2 F_3 + \phi_2\phi_3 F_1 + \phi_3\phi_1 F_2) + \lambda(\phi_1^2 + \phi_2^2) F_3 \\
+ \text{terms containing fermions} \} + \text{h.c.}] \Bigg] \delta(z - 1/k)
\end{aligned}
$$

ξD: Fayet-Illiopoulos (FI) D-term

(If the matter fields S_i's are on the TeV brane,
the 1-loop correction $V_{1-loop} = 0$.)



- Assumption: $\boxed{2g_1 = 2g_2 = -g_3 = g}$

$$\Downarrow$$

No quadratic divergence!

- Propagator of Φ $\quad G_p(z,z')$:

$$\left[\partial_z^2 - \frac{3}{z}\partial_z + p^2 - \frac{m_\Phi^2}{(kz)^2}\right] G_p(z,z') = (kz)^3\delta(z-z')$$

- Dirichlet boundary conditions: $G_p(\frac{1}{k},v) = G_p(u,\frac{1}{q}) = 0$
- Solution:

$$\begin{aligned} G_p(u,v) \;=\; & \frac{k^3u^2v^2}{K_0(p/k))I_0(p/T) - I_0(p/k)K_0(p/T)} \\ & \times\{I_0(p/k)K_0(pu) - K_0(p/k)I_0(pu)\} \\ & \times\{I_0(p/T)K_0(pv) - K_0(p/T)I_0(pv)\} \end{aligned}$$

$u \equiv \mathrm{min}(z,z')$, $v \equiv \mathrm{max}(z,z')$

§3 Estimation of 1-loop diagrams

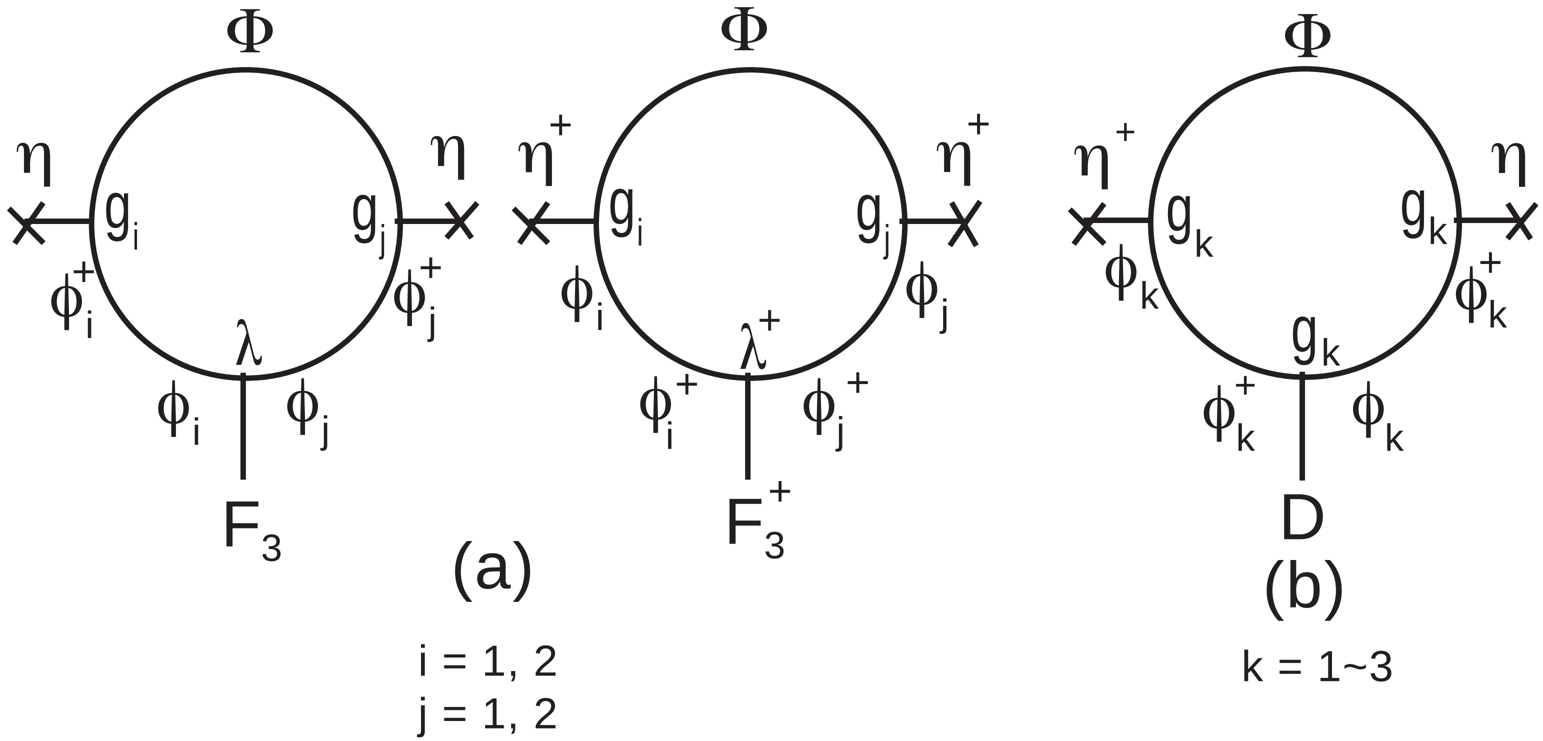


Figure 1: 1-loop diagrams dominantly contributing to V_{eff}

$$V_{eff}(\mathsf{Fig.1}) = \mathcal{A}\frac{l}{2}\int_{\mu}^{\Lambda}\frac{d^4p}{(2\pi)^4}\frac{1}{(p^2+|\lambda f|^2)^2}I(p,1/k,\Lambda/(kp),w)$$

$$\mathcal{A} \equiv \frac{1}{2}g^2(\lambda f\eta^2 + \lambda^\dagger f^\dagger \eta^{\dagger 2}) - g^3\eta^\dagger d\eta$$

$$I(p,a,b,w) \equiv \int_a^b \frac{du}{(ku)^6}\int_u^b \frac{dv}{(kv)^6}\delta(u-w)\delta(v-w)k^2$$

$$\times uv\frac{\partial^2 G_p(u,v)}{\partial u\partial v}$$

$$d \equiv \langle D\rangle, \quad \mu = \Lambda q/k, \quad \Lambda \gtrsim k, \quad w = 1/k$$

- Result:

  $w = 1/k$: Radion does not directly couple with S_i.

$$\Downarrow$$

$$V_{eff}(\text{Fig.1}) = \frac{\mathcal{A}}{128\pi^2}\left\{l\Lambda + 2lk\log\left(\frac{\Lambda}{ck}\right)\right\} + \text{finite terms}$$

$$\mathcal{A} \equiv \frac{1}{2}g^2(\lambda f\eta^2 + \lambda^\dagger f^\dagger \eta^{\dagger 2}) - g^3\eta^\dagger d\eta$$

- Linealy divergent $(\propto l\Lambda) \leftarrow \sum$ KKmode
- $l^{-1} \gg M_W \rightarrow l\Lambda \ll 10^{16}$

} Bulk effect!

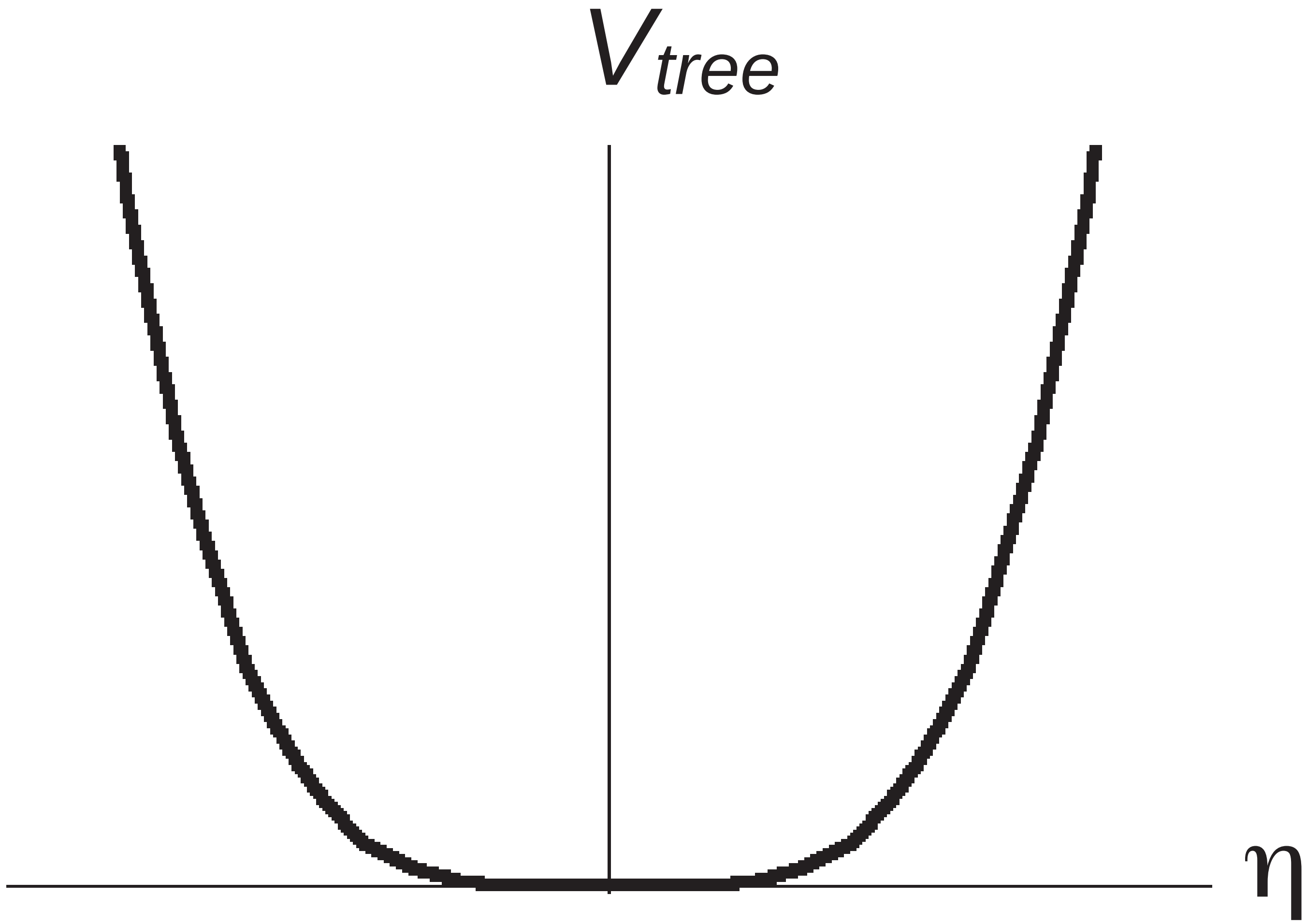
§4 Minimization of the effective potential

$$V_{eff} = V_{tree} + V_{1-loop} + V_{FI}$$

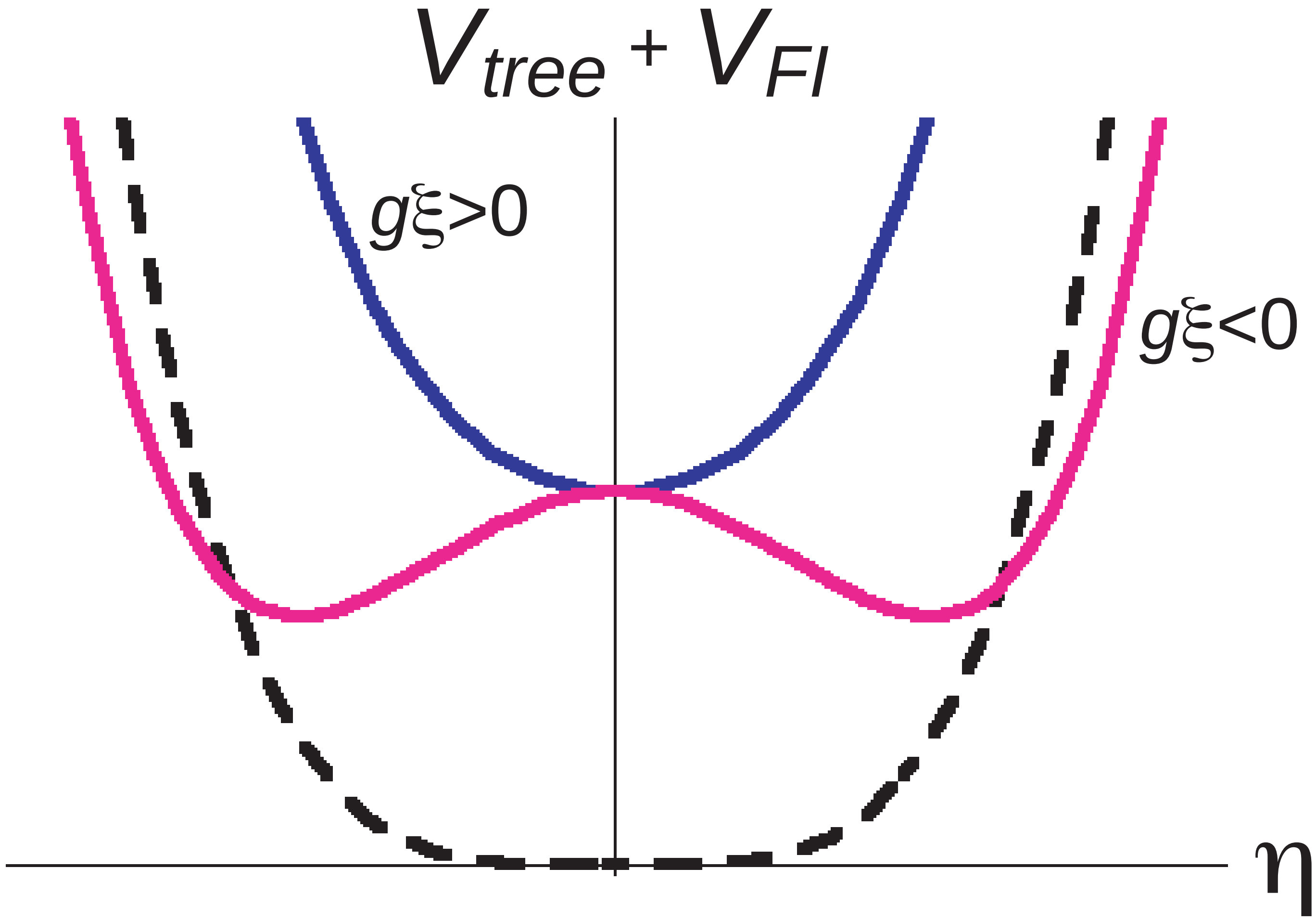
$$\begin{aligned} V_{tree} &= -f^\dagger f - \frac{1}{2}\lambda f \eta^2 - \frac{1}{2}\lambda^\dagger f^\dagger \eta^{\dagger 2} - \frac{1}{2}d^2 - g\eta^\dagger d\eta \\ V_{1-loop} &= \alpha\{(\lambda^\dagger f^\dagger \eta^{\dagger 2} + \lambda f \eta^2)/2 - g\eta^\dagger d\eta)\} \\ V_{FI} &= -\xi d \end{aligned}$$

$$\alpha \equiv \frac{l\Lambda g^2}{128\pi^2}$$

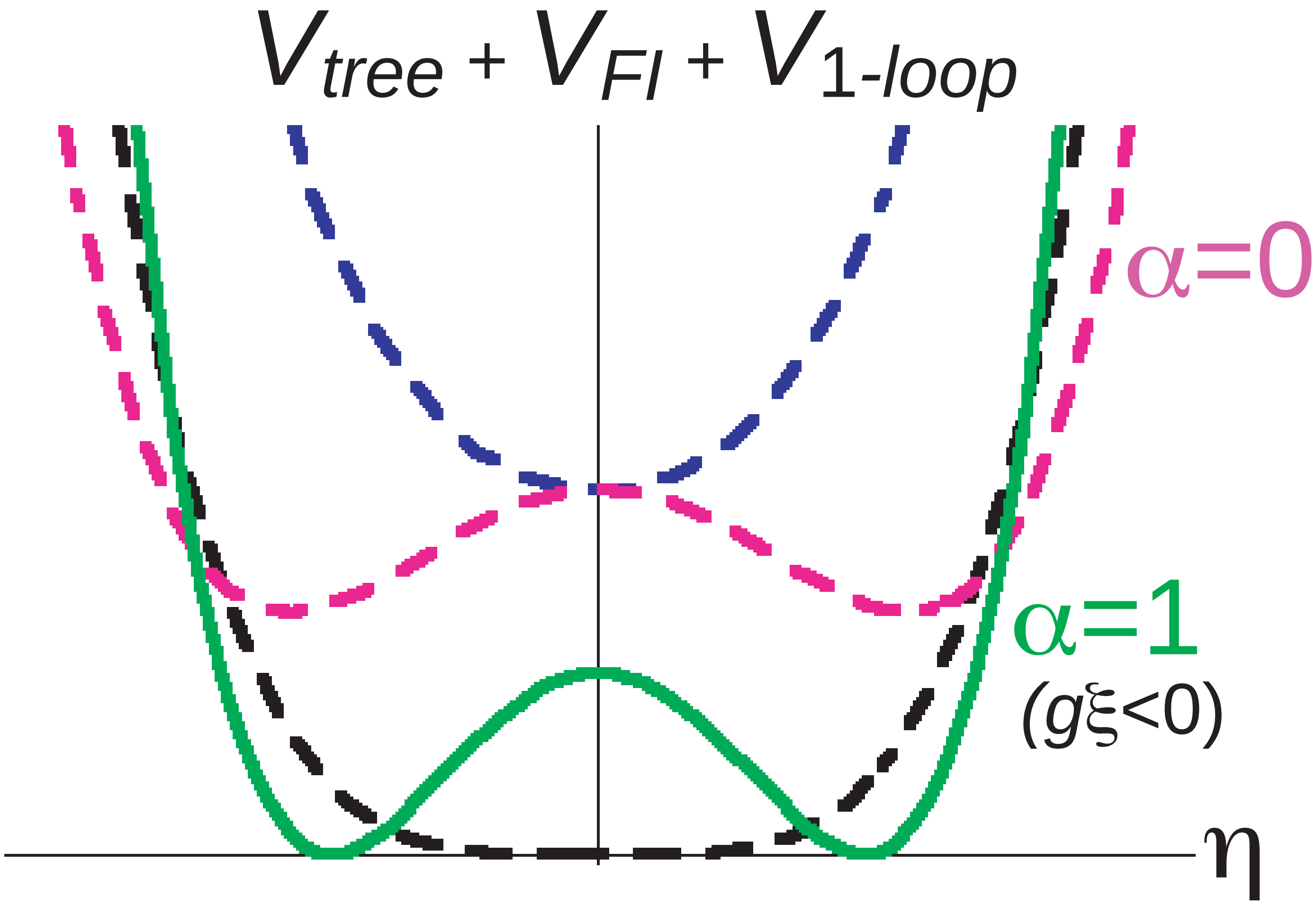
(f, d, η, λ are taken to be real for simplicity below.)



V_{tree}
η



$V_{tree} + V_{FI}$
gξ>0
gξ<0
η



$V_{tree} + V_{FI} + V_{1\text{-}loop}$
α=0
α=1
(gξ<0)
η

$g\xi > 0 \Rightarrow V^{eff}(f = 0, d = -\xi, \eta = 0, \alpha) = \xi^2/2 > 0$

[minimum for any α]

$\Downarrow$

SUSY broken, U(1) unbroken.

α: flat direction $\Rightarrow$ The radius is not stabilized.

$g\xi < 0 \Rightarrow V^{eff}(f=0, d=0, \eta=\pm\sqrt{|\xi/2g|}, \alpha=1) = 0$

[global minimum]

$\Downarrow$

SUSY restored, U(1) broken.

The radius is stabilized at $l^{-1} = \Lambda g^2/128\pi^2$.

Λ	$M_{GUT} \approx 10^{16}$ GeV	$M_{Pl} \approx 2 \times 10^{18}$ GeV
l^{-1}	$\approx 10^{11}$ GeV	$\approx 2 \times 10^{13}$ GeV
k	$\approx 2 \times 10^{12}$ GeV	$\approx 5 \times 10^{14}$ GeV
M_5	$\approx 1.6 \times 10^{16}$ GeV	$\approx 1.7 \times 10^{17}$ GeV
$\langle r \rangle$	$\approx 8 \times 10^{-13}$ fm	$\approx 3 \times 10^{-15}$ fm
m_{radion}	$\approx 0.03(\lvert\xi\rvert/(100\text{TeV})^2)$GeV	$\approx 10(\lvert\xi\rvert/(100\text{TeV})^2)$GeV

- Numerology for $\alpha = 1$, $|\lambda| = |g| = 0.1$:

$M_W \ll l^{-1}, k \ll M_{GUT} \Rightarrow$ "Non-TeV" = "Intermediate"!

- $$m^2_{radion} \sim \left|\frac{\partial^2 f(r)}{\partial r^2}\right|^{-1} \frac{\partial^2 V^{eff}}{\partial r^2}\bigg|_{\alpha=1} \sim \frac{\lambda^2 \xi^2 e^{2kl}}{3kM_5^3 l^2}$$

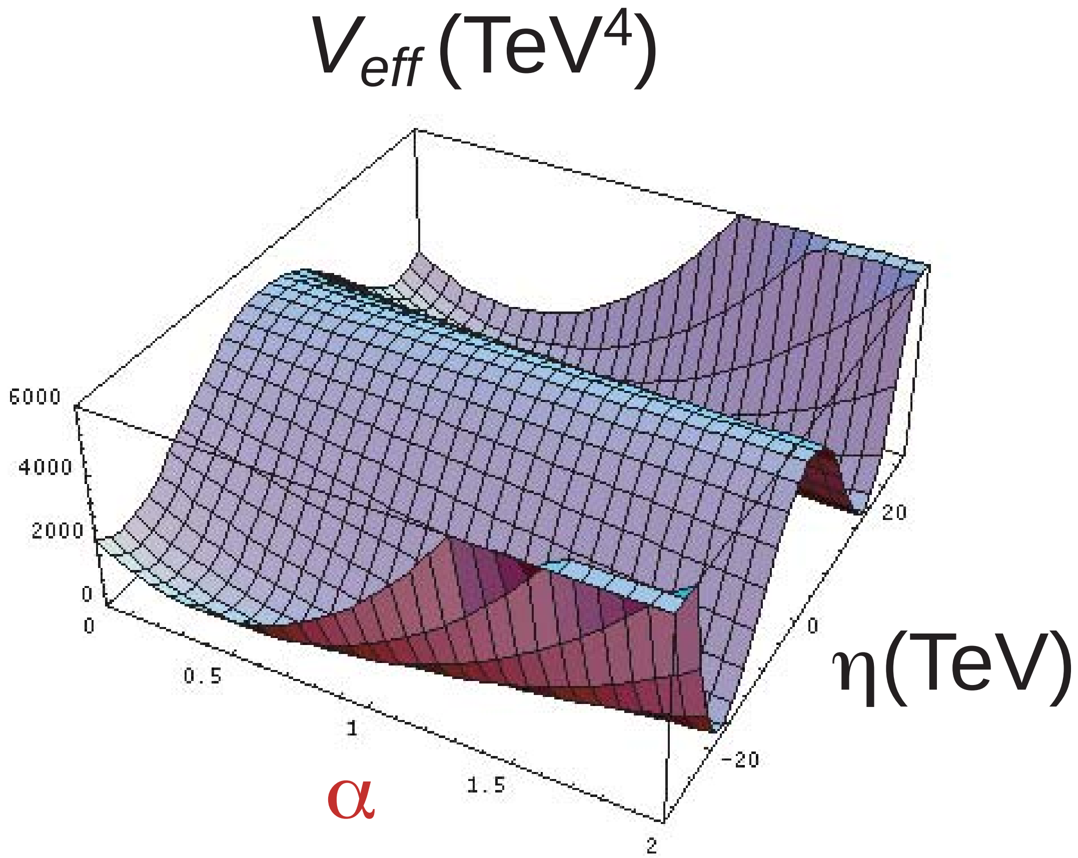
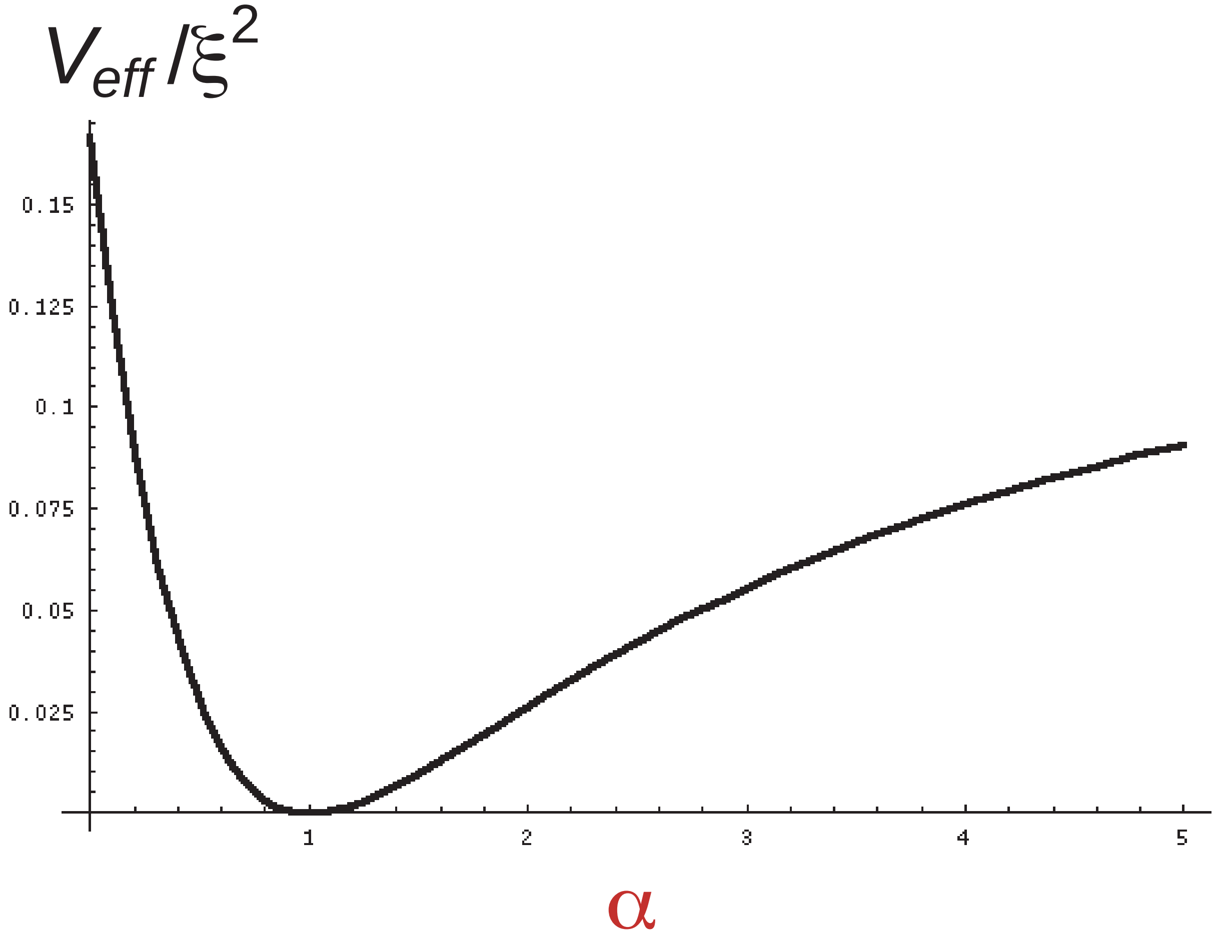


Figure 2: $V_{eff}(\eta, \alpha)$, for $f = d = 0, \lambda = g = 0.1$ and $\xi = 100$ TeV



V_{eff}/ξ^2
0.15
0.125
0.1
0.075
0.05
0.025
1
2
3
4
5
α

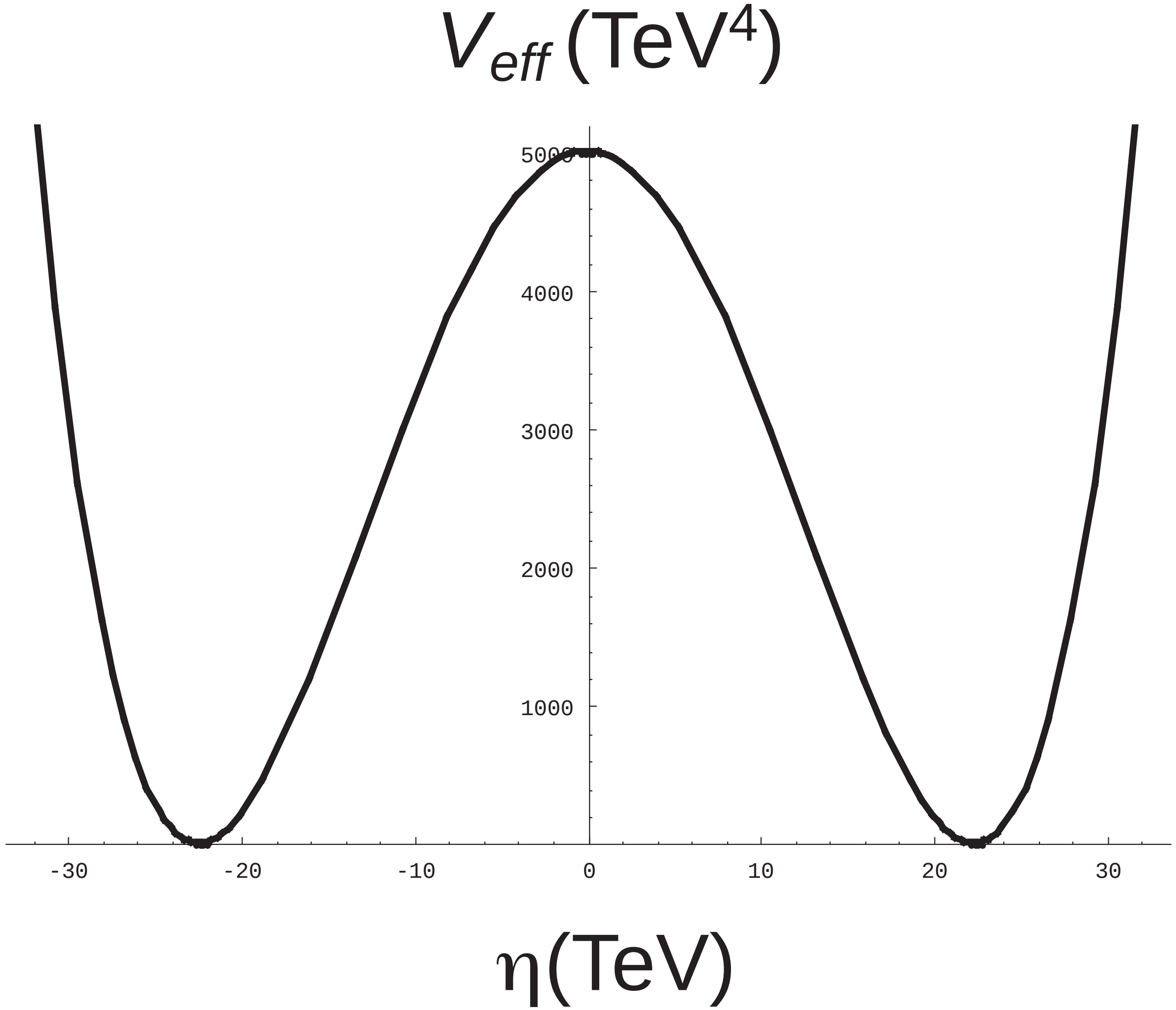


Figure 3: $g = 0.1 \quad \xi = 100\text{TeV}^2 \quad \eta = \pm\sqrt{|\xi/2g|}$

§5 Conclusion

1. We have estimated the dominant 1-loop contribution to the effective potential for a toy model based on Mirabelli-Peskin model with the gauge group U(1) in the warped background of AdS_5. If the matter supermultiplet is placed on the "Non-TeV" brane, it has been proved to be proportional to the 4D momentum cutoff times the size of extra dimension, i.e., $V_{1-loop} \propto l\Lambda$, as in the case of flat background.

2. We have incorporated the FI D-term, minimized the (tree $+$ FI $+$ 1-loop) effective potential and found the case that the radius can be stabilized at $l^{-1} \approx 10^{11-13}$ GeV with "Non-TeV" identified with "Intermediate".

3. Our model implements a new mechanism for the radius stabilization of the extra dimension in the sense that i) we have a gauge supermultiplet in the bulk and chiral super-multiplets on the brane but have no hypermultiplet in the bulk and ii) the linearly divergent quantum effect of the bulk has played an essential role.

4. It is a subject of future study to extend our model to the full 5D SUGRA.